

Problem Set 1

Due: Monday, September 11, 2023 at noon

Problem 1.1 [Hamiltonian Cycle Problem \rightarrow Traveling Salesman Problem].

Recall from lecture that a *Hamiltonian cycle* in a graph is a cycle visiting every vertex exactly once. The HAMILTONIAN CYCLE PROBLEM is the NP-complete problem of deciding whether the input graph has a Hamiltonian cycle.

The TRAVELING SALESMAN PROBLEM IN GRAPHS is to decide, given a complete graph¹ with *positive integer* edge weights and given a target integer t , does there exist a cycle² in the graph that visits every vertex at least once and has total weight $\leq t$?

Prove that the TRAVELING SALESMAN PROBLEM IN GRAPHS is NP-complete. Remember that in order to prove a problem is NP-complete, you must show that it is both NP-hard and contained in NP. To prove NP-hardness, reduce from the HAMILTONIAN CYCLE PROBLEM.

¹A *complete graph* has an edge between every pair of distinct vertices.

²Throughout this class, a *cycle* is allowed to repeat vertices and/or edges; if it doesn't repeat vertices, the cycle is called *simple*.