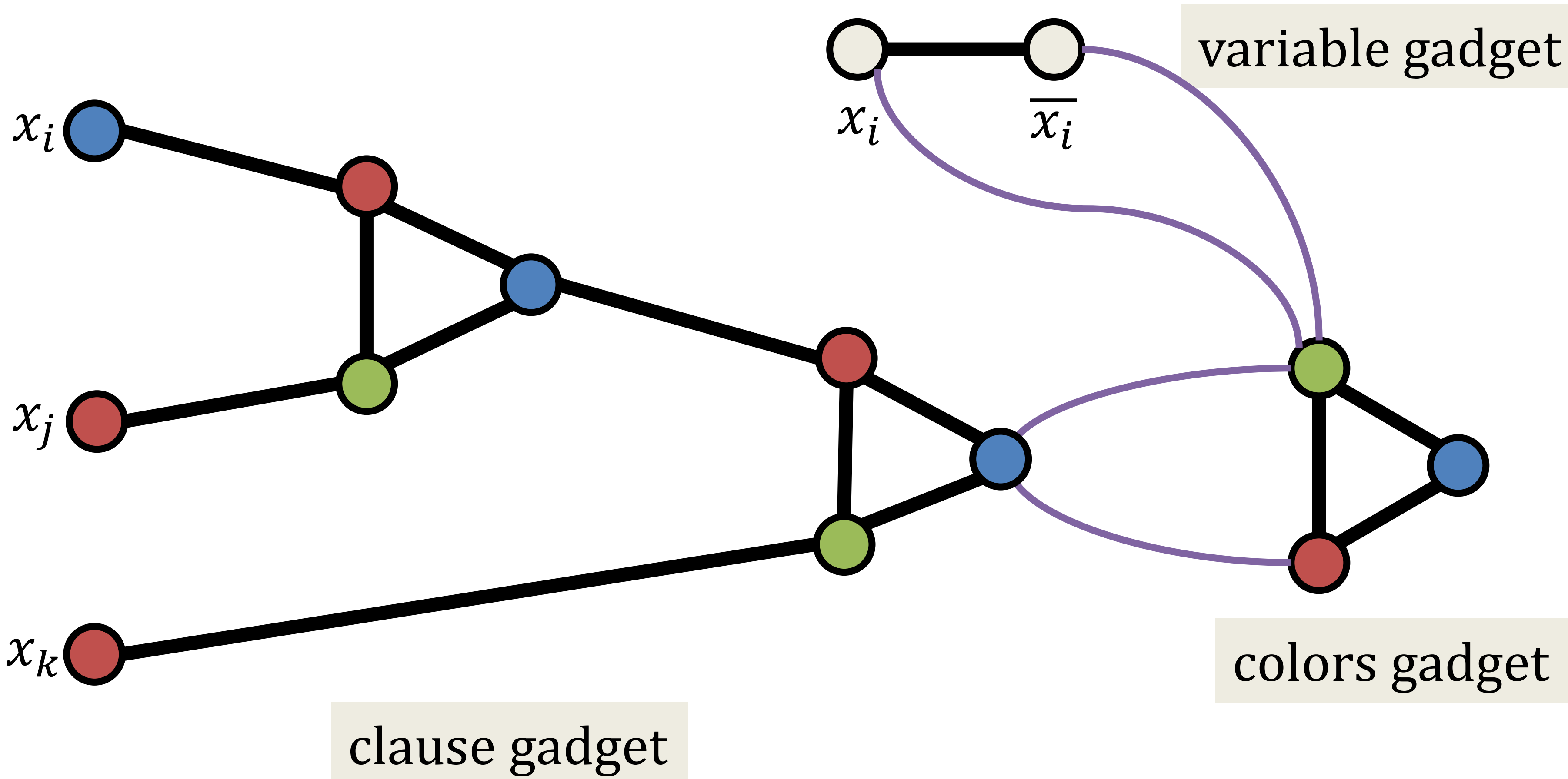
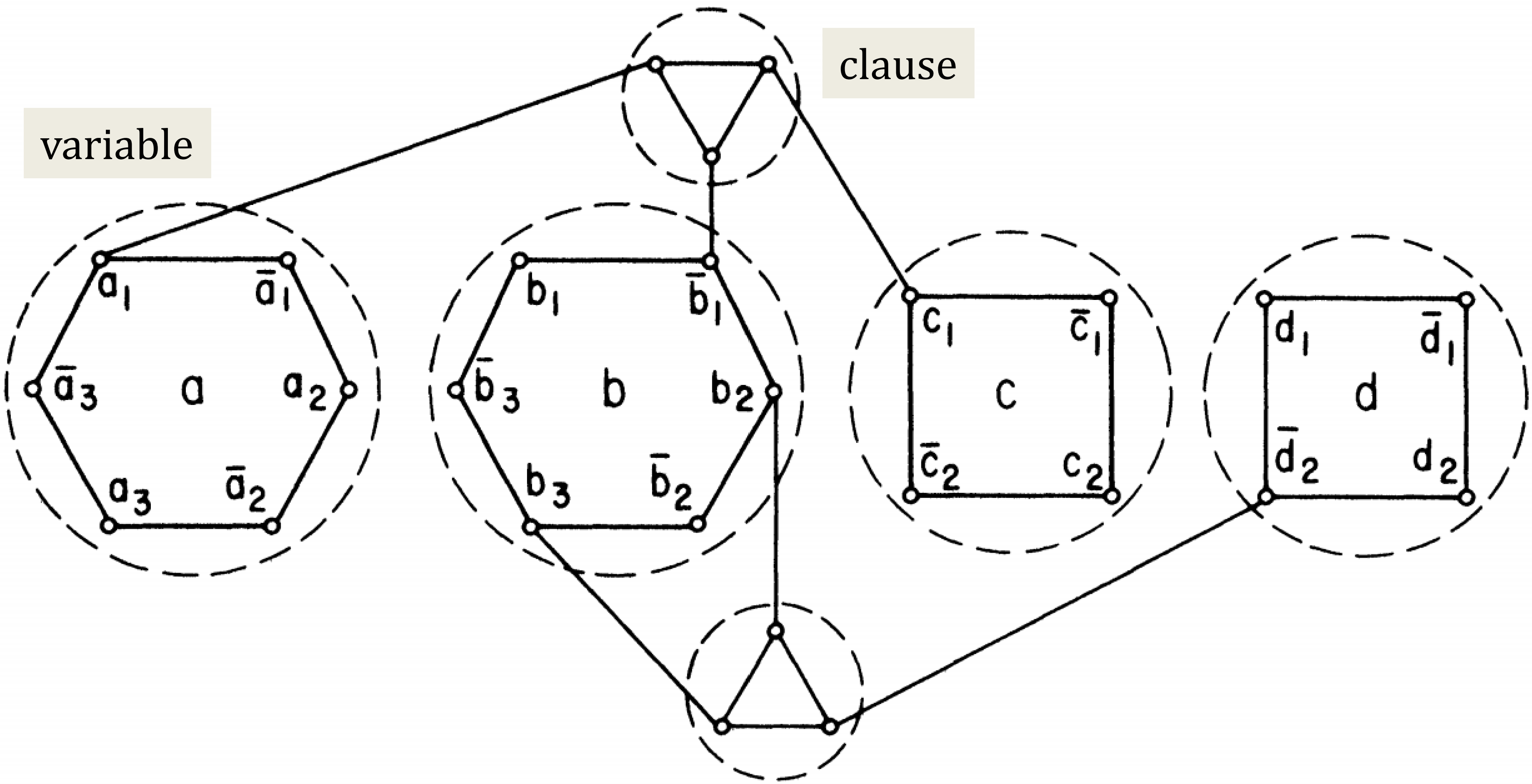
Vertex 3-Coloring

[Garey, Johnson, Stockmeyer 1976]



Planar Vertex Cover

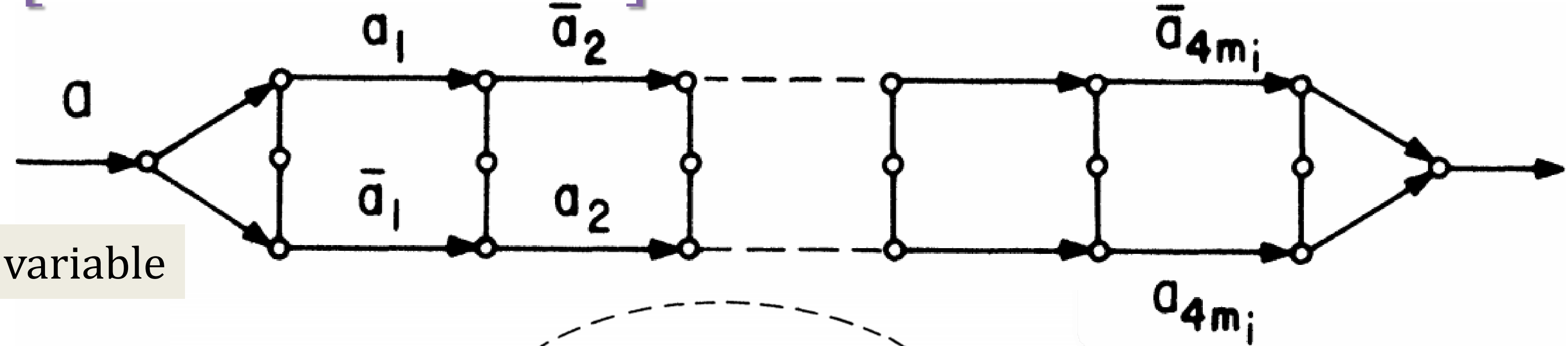
[Lichtenstein 1982]



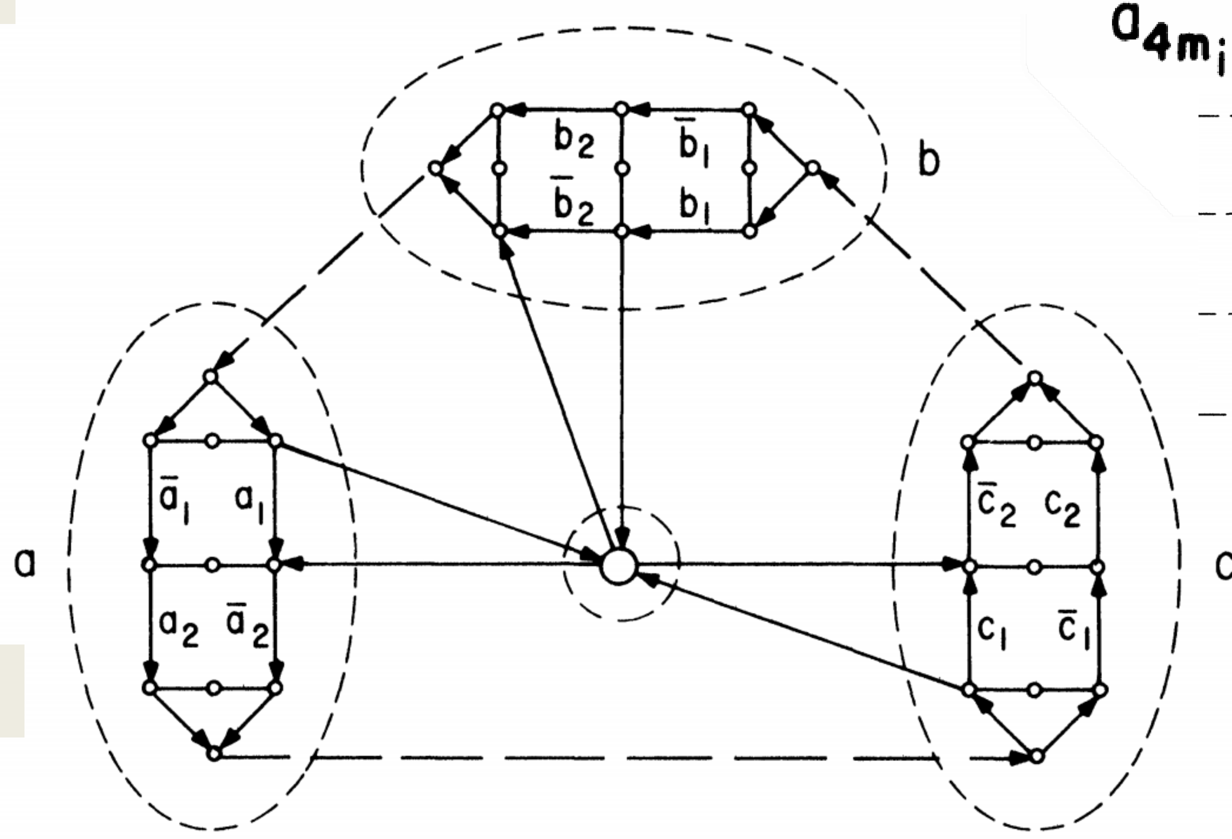
Example : $B = (a + \bar{b} + c)(b + b + \bar{d})$

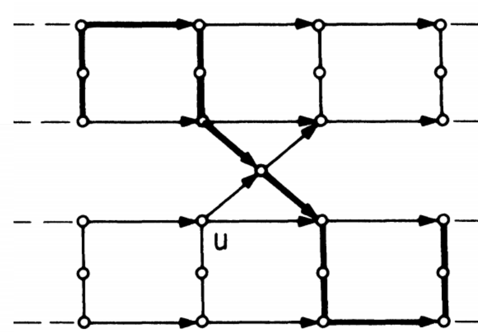
Planar (Directed) Hamiltonian Cycle

[Lichtenstein 1982]



variable



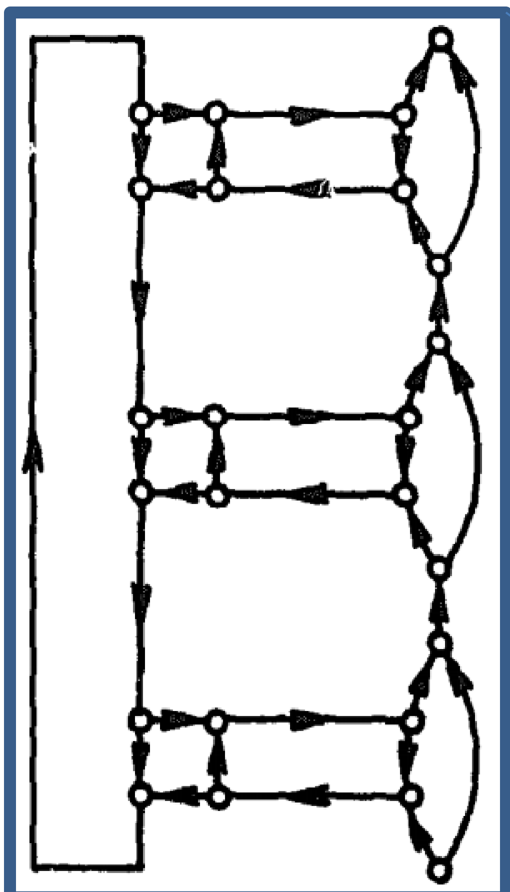


clause

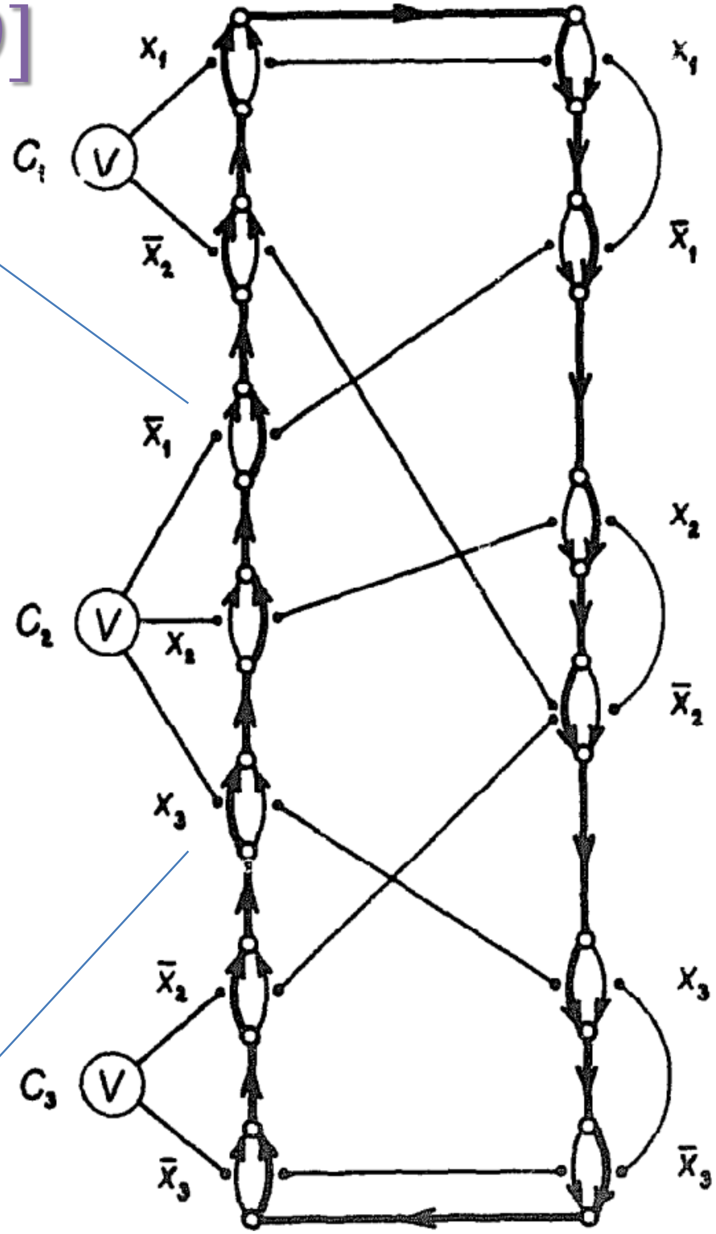
$a \vee \bar{b} \vee c$

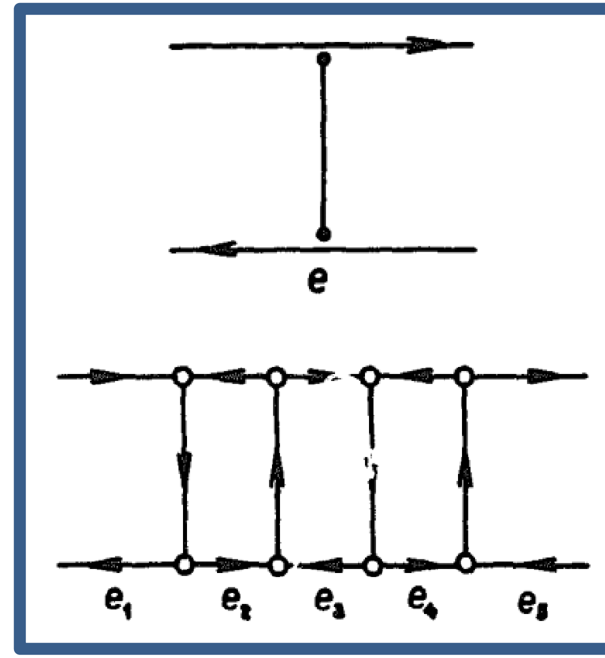
Planar Directed Max-Degree-3

[Plesńik 1979]



clause gadget



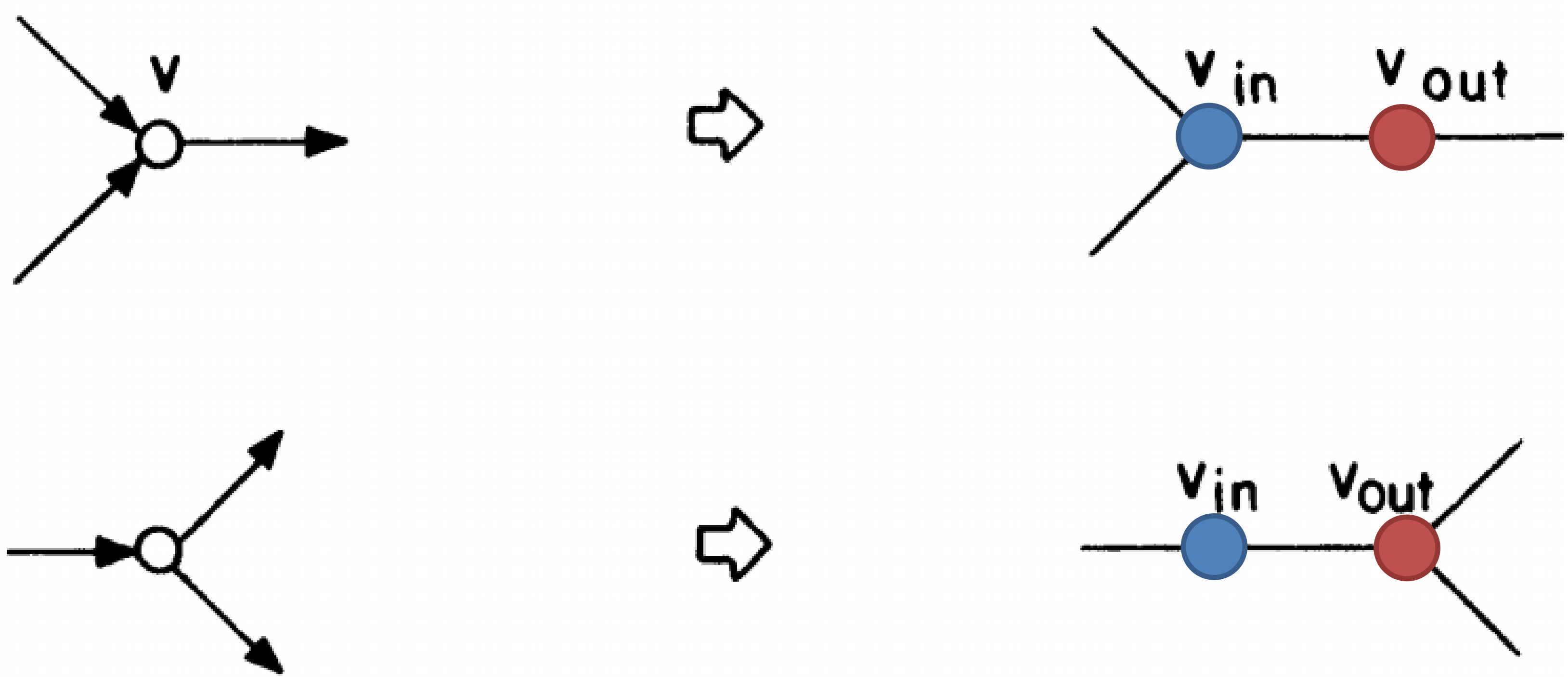


XOR gadget

$(x_1 \vee \overline{x_2})$
$\wedge (\overline{x_1} \vee x_2 \vee x_3)$
$\wedge (\overline{x_2} \vee \overline{x_3})$

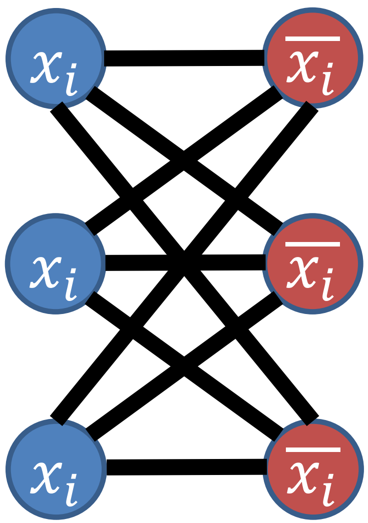
Planar Bipartite Max-Degree-3

[Itai, Papadimitriou, Szwarcfiter 1982]

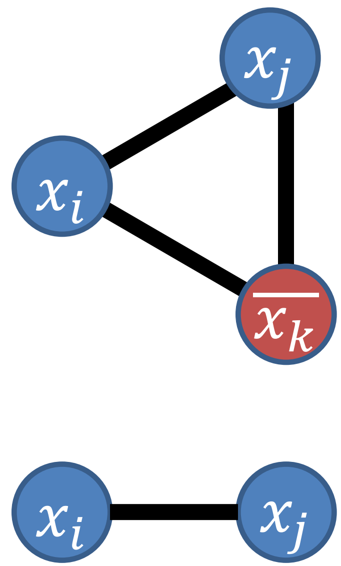


Independent Set

[Papadimitriou & Yannakakis 1991]



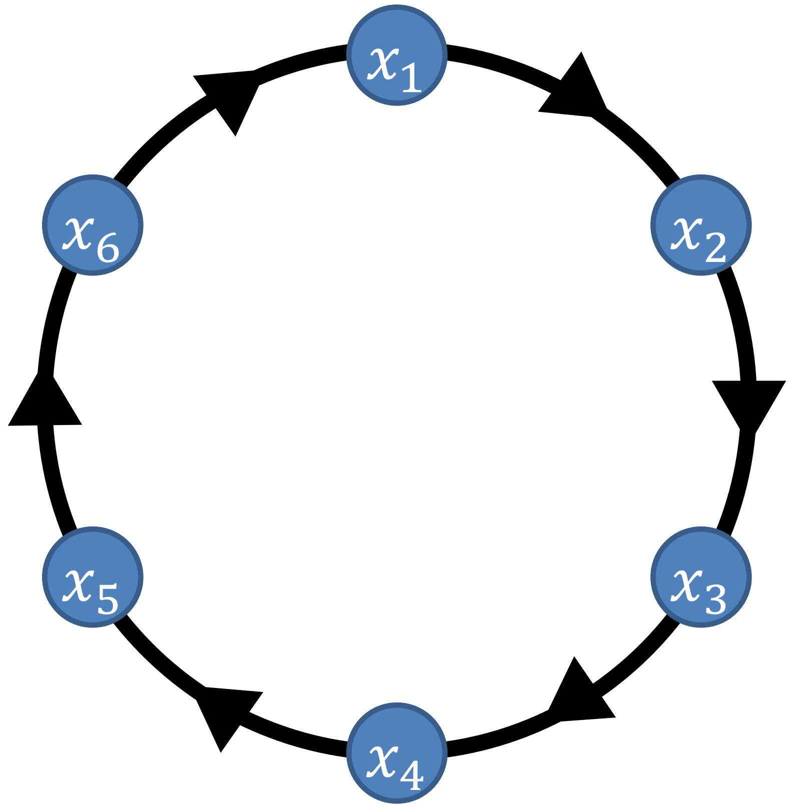
variable



clause

3SAT-3

e.g. [Papadimitriou & Yannakakis 1991]

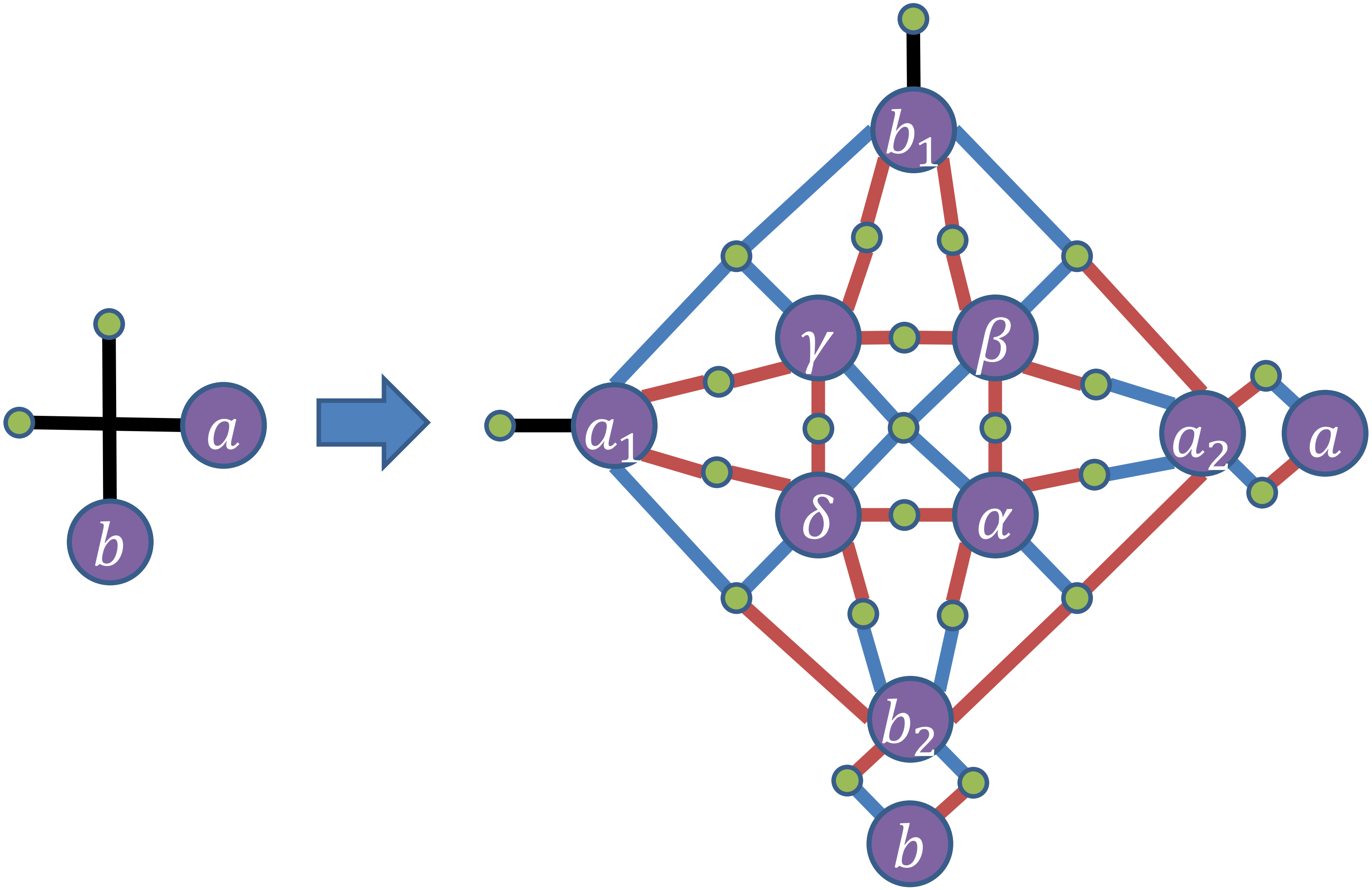


$x_i \Rightarrow x_{i+1}$

$\neg x_i \vee x_{i+1}$

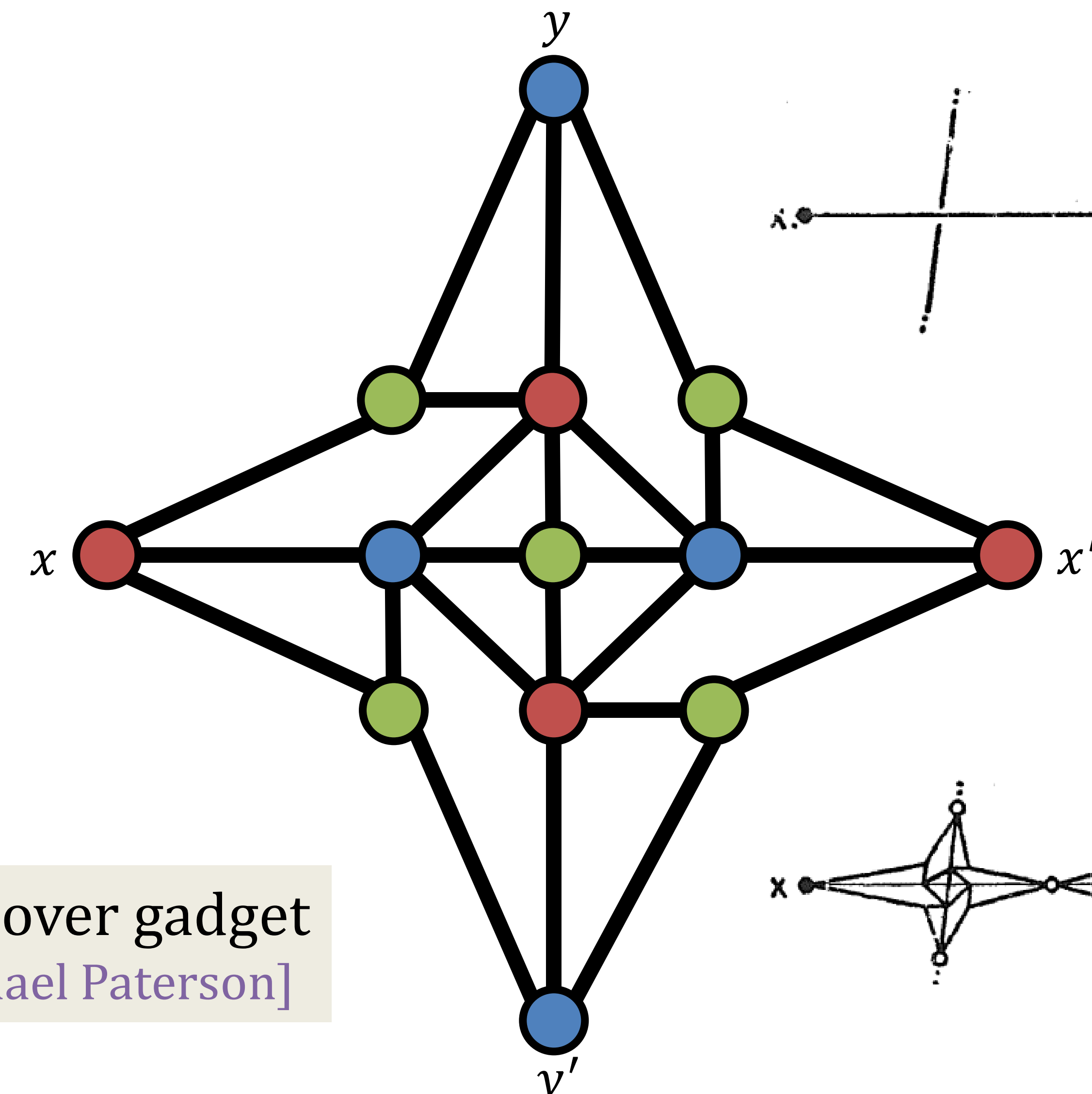
Planar 3SAT is NP-hard

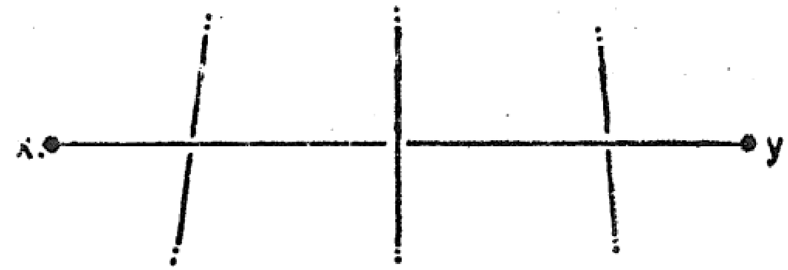
[Lichtenstein 1982]

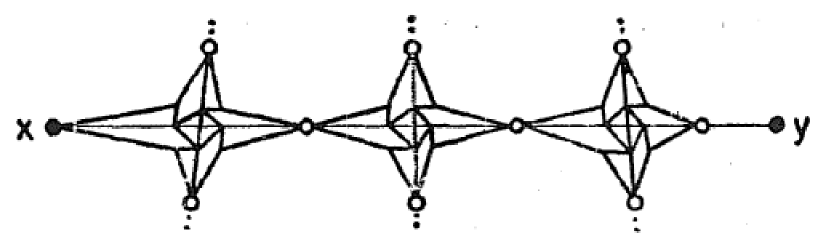


Planar 3-Coloring

[Garey, Johnson, Stockmeyer 1976]







crossover gadget

[Michael Paterson]

Grid Tiling [Marx 2007]

(1,1) (3,1) (2,4)	(5,1) (1,4) (5,3)	(1,1) (2,4) (3,3)
(2,2) (1,4)	(3,1) (1,2)	(2,2) (2,3)
(1,3) (2,3) (3,3)	(1,1) (1,3)	(2,3) (5,3)

$k = 3, n = 5$

⇨

(1,1) (3,1) **(2,4)**	(5,1) **(1,4)** (5,3)	(1,1) **(2,4)** (3,3)
(2,2) (1,4)	(3,1) **(1,2)**	**(2,2)** (2,3)
(1,3) **(2,3)** (3,3)	(1,1) **(1,3)**	**(2,3)** (5,3)

$k = 3, n = 5$

Marx 2014

Clique → Grid Tiling

	(v_i, v_i)			

Each diagonal cell defines a value v_i ...

Marx 2014

Clique → Grid Tiling

	$(v_i, .)$			
$(., v_i)$	(v_i, v_i)	$(., v_i)$	$(., v_i)$	$(., v_i)$
	$(v_i, .)$			
	$(v_i .,)$			
	$(v_i, .)$			

. . . which appears on a "cross"

Marx 2014

Clique → Grid Tiling

	$(v_i, .)$			
$(., v_i)$	(v_i, v_i)	$(., v_i)$	$(., v_i)$	$(., v_i)$
	$(v_i, .)$			
	$(v_i, .)$		(v_j, v_j)	
	$(v_i, .)$			

v_i and v_j are adjacent for every $1 \leq i < j \leq k$.

Marx 2014

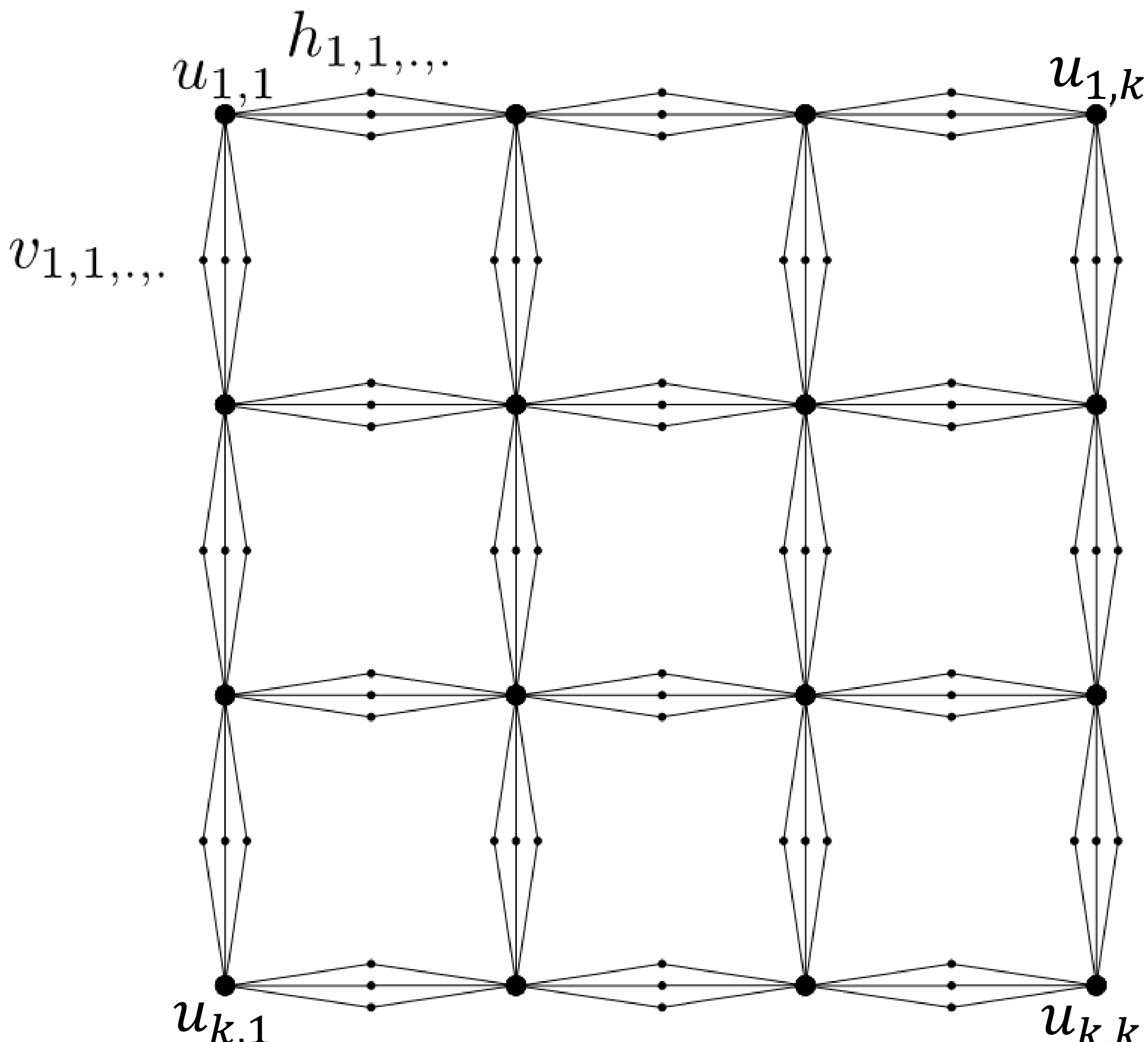
Clique → Grid Tiling

	$(v_i, .)$		$(v_j, .)$	
$(., v_i)$	(v_i, v_i)	$(., v_i)$	(v_j, v_i)	$(., v_i)$
	$(v_i, .)$		$(v_j, .)$	
$(., v_j)$	(v_i, v_j)	$(., v_j)$	(v_j, v_j)	$(., v_j)$
	$(v_i, .)$		$(v_j, .)$	

v_i and v_j are adjacent for every $1 \leq i < j \leq k$.

Marx 2014

Grid Tiling → k-Outerplanar List Coloring



Cygan, Fomin, Kowalik, Lokshtanov, Marx, Pilipczuk, Pilipczuk, Saurabh 2015

Grid Tiling with $\leq$

(5,1) (1,2) (3,3)	(4,3) (3,2)	(2,3) (2,5)
(2,1) (5,5) (3,5)	(4,2) (5,3)	(5,1) (3,2)
(5,1) (2,2) (5,3)	(2,1) (4,2)	(3,1) (3,2) (3,3)

$k = 3, n = 5$

(5,1) **(1,2)** (3,3)	**(4,3)** (3,2)	(2,3) **(2,5)**
(2,1) (5,5) (3,5)	**(4,2)** (5,3)	(5,1) **(3,2)**
(5,1) **(2,2)** (5,3)	(2,1) **(4,2)**	(3,1) (3,2) **(3,3)**

$k = 3, n = 5$

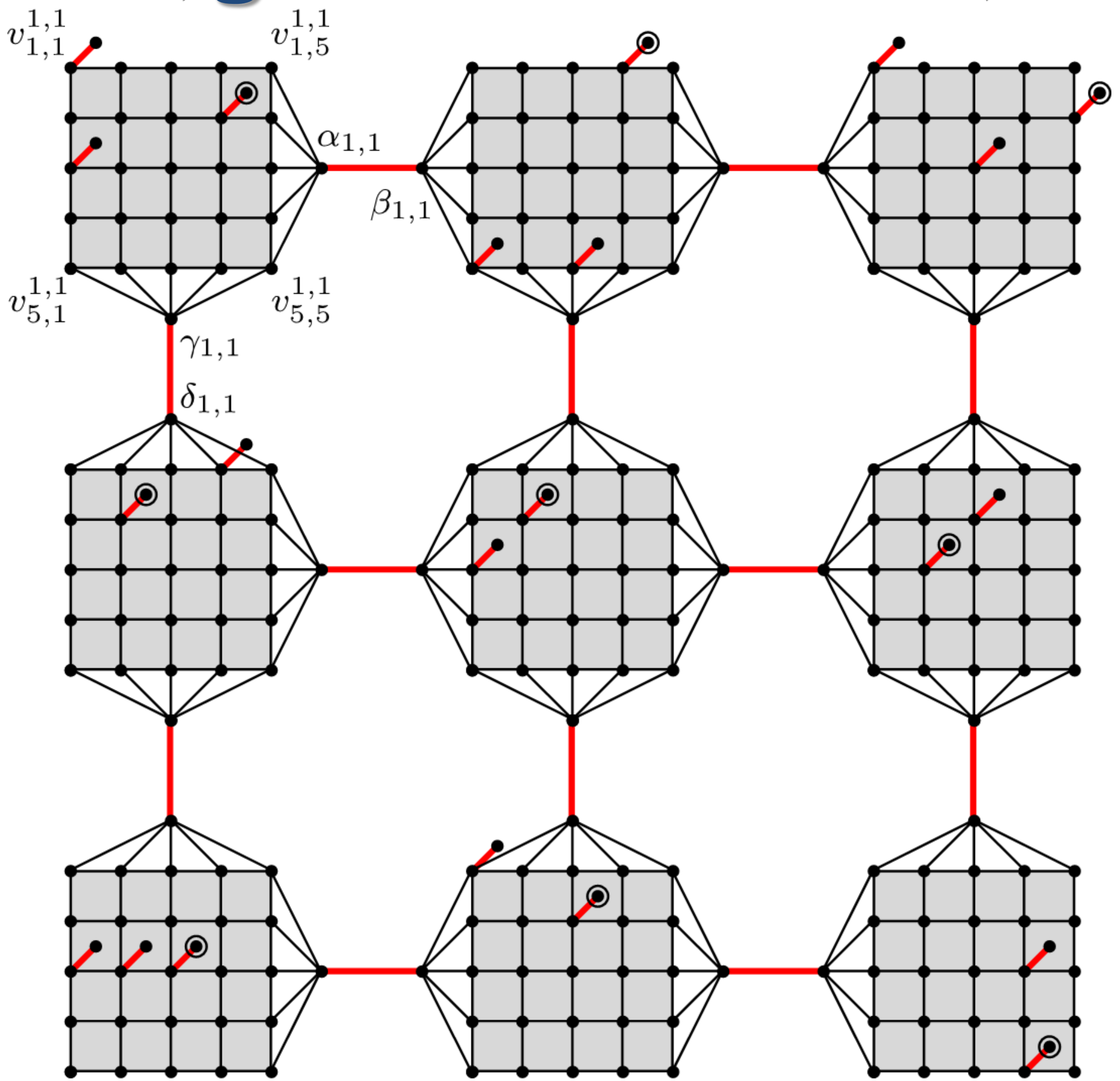
Marx 2014

Grid Tiling → Grid Tiling with ≤

$S'_{4i-3,4j-3}$: $(iN - z, jN + z)$	$S'_{4i-3,4j-2}$: $(iN + a, jN + z)$	$S'_{4i-3,4j-1}$: $(iN - a, jN + z)$	$S'_{4i-3,4j}$: $(iN + z, jN + z)$
$S'_{4i-2,4j-3}$: $(iN - z, jN + b)$	$S'_{4i-2,4j-2}$: $((i+1)N, (j+1)N)$	$S'_{4i-2,4j-1}$: $(iN, (j+1)N)$	$S'_{4i-2,4j}$: $(iN + z, (j+1)N + b)$
$S'_{4i-1,4j-3}$: $(iN - z, jN - b)$	$S'_{4i-1,4j-2}$: $((i+1)N, jN)$	$S'_{4i-1,4j-1}$: (iN, jN)	$S'_{4i-1,4j}$: $(iN + z, (j+1)N - b)$
$S'_{4i,4j-3}$: $(iN - z, jN - z)$	$S'_{4i,4j-2}$: $((i+1)N + a, jN - z)$	$S'_{4i,4j-1}$: $((i+1)N - a, jN - z)$	$S'_{4i,4j}$: $(iN + z, jN - z)$

Cygan, Fomin, Kowalik, Lokshtanov, Marx, Pilipczuk, Pilipczuk, Saurabh 2015

Grid Tiling with ≤ → Scattered Set

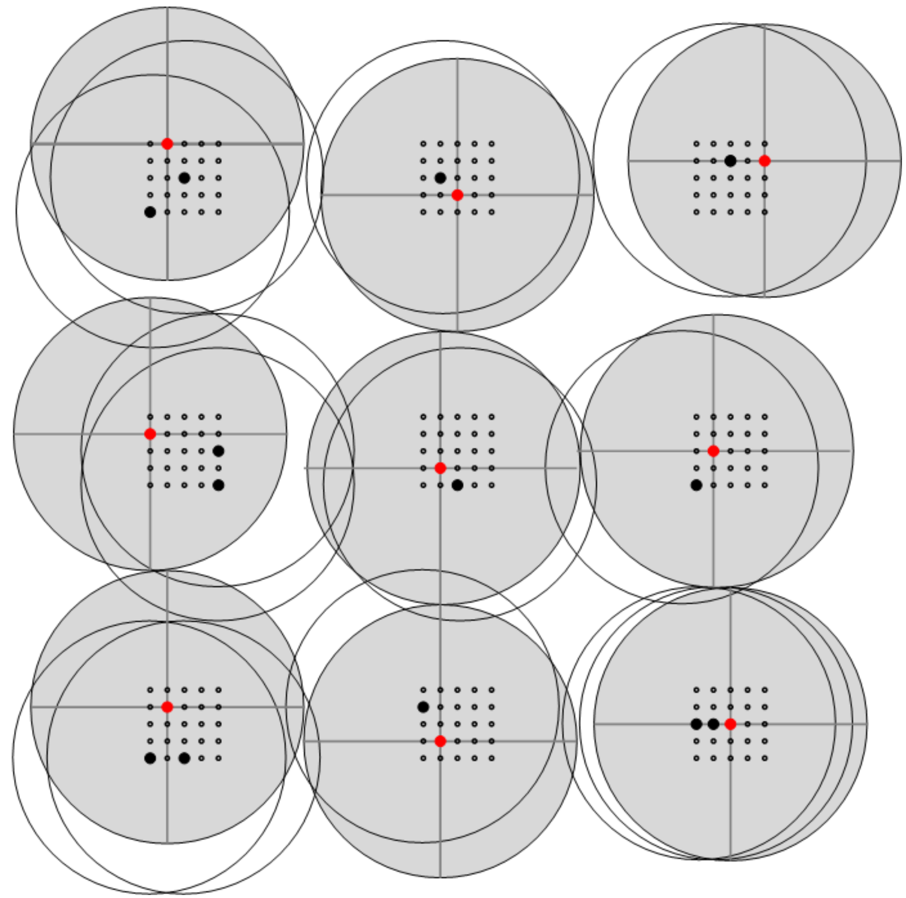


red path length $= 100 \times$ grid size

Cygan, Fomin, Kowalik, Lokshtanov, Marx, Pilipczuk, Pilipczuk, Saurabh 2015

Grid Tiling with ≤ → Unit-Disk Independent Set

$S[1,3]$: (1,1) (2,5) (3,3)	$S[2,3]$: (3,2) (2,3)	$S[3,3]$: (5,4) (3,4)
$S[1,2]$: (5,1) (1,4) (5,3)	$S[2,2]$: (3,1) (2,2)	$S[3,2]$: (1,1) (2,3)
$S[1,1]$: (1,1) (3,1) (2,4)	$S[2,1]$: (2,2) (1,4)	$S[3,1]$: (1,3) (2,3) (3,3)



Cygan, Fomin, Kowalik, Lokshtanov, Marx, Pilipczuk, Pilipczuk, Saurabh 2015