6.890	Lecture 18	Nov. 4, 2014

0-player games (simulations)

- polynomial # moves → P
- polynomial space → PSPACE
- infinite space → undecidable

Conway's Game of Life: [Conway 1970]

- cellular automata
- live cell lives ⇔ exactly 2 or 3 live neighbors
- dead cell becomes live ⇔ exactly 3 live neighbors
- PSPACE-complete in finite board
  - Turing machine simulation [Paul Rendell 2000]
    (pushdown automaton with 2 stacks)
- undecidable in infinite board (dead outside input)
  - growing Turing machine [Paul Rendell 2000]
  - 2-counter machine = Minsky machine ← exponential slowdown!
    [Conway, Berlekamp, Guy – Winning Ways 1982]
    - wire, terminator, turn, delay
    - shift: many offsets cause glider destruction
    - AND & OR gates
    - kickback → thinning → crossover
    - split & NOT (complicated)
    - counter registers, test, create, push/pull
    - precise glider positioning away from guns
    - self-destruction via boomerangs

Deterministic Constraint Logic: (DCL)

- edges can also be active or inactive
  ↳ just flipped
- vertex active if its active incoming edges' weight ≥ 2
- in each round:
  - reverse inactive edges pointing to active vertices & reverse active edges pointing to inactive vertices
  - these are the new active edges

- PSPACE-complete even for planar AND/OR graphs
  - guarantee gadget inputs reverse at $t \equiv 0 \pmod 4$
  - quantifier gadgets use new "switch" & degree-2 vertices to control timing
  - CNF formula uses AND', OR', split' gadgets which take inputs & return acknowledgments (fixes timing & "blow-back")
    - trick to guarantee first input of AND' activates before second (if they both do)
  - remove degree-2 vertices
    - edge → 4-path & remove red-red vertices
    - remove blue-blue vertices
    - remove red-blue vertices (timing is OK)
  - crossover gadget

Multiplayer games:

- typical question: given a game position, can next-player-to-move force a win?
- in worst case, other players collude against you, effectively acting as one player

2-player games:

- call players "white" & "black" (as in Chess, Go, ...)
- polynomial # moves → ∈ PSPACE:

Q3SAT { ∃ move : ∀ responses : ∃ move : ∀ responses : ⋯
(I followed the rules ∧ I won) ∨ (you broke rules)

SAT games: [Schaefer – JCSS 1978]

- QSAT is a 2-player game: G_ω(CNF)
  - player 1 chooses x_1, player 2 chooses x_2, ⋯
  - player 1 wins ⟺ formula satisfied
- impartial games: (both players have same moves)
  - on turn, player sets any unassigned variable
- partizan games: (different moves for players)
  - white variables & black variables (50/50%)
  - on turn, player sets unassigned var. of same color
- default game: player 1 wins ⟺ formula satisified at end
- seek game: win if first to satisfy formula } treat unassign as ∅
- avoid game: lose if first to satisfy formula }

- PSPACE-complete:
  - impartial game positive 11-SAT → 11-CNF
  - impartial game positive 11-DNF SAT
  - partizan game CNF SAT
  - impartial/partizan avoid positive 2-DNF SAT
  - impartial/partizan seek positive 3-DNF SAT
  - impartial/partizan avoid positive CNF SAT
  - impartial/partizan seek positive CNF SAT

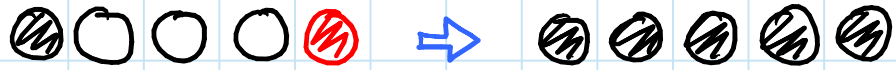
<u>Kayles</u>: (≈ indep. set) [Schaefer – JCSS 1978]

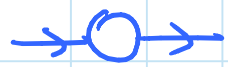
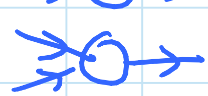
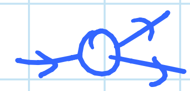
- (impartial) node Kayles:
  - on turn, player adds node to independent set
  - lose if can't move
- (partizan) bipartite node Kayles:
  - white vs. black nodes is the bipartition

<u>Geography</u>: (generalization of word game) (≈ longest path)

- given (directed) graph & start node for token
- on turn, player moves token along (directed) edge
- node geography: can't revisit nodes
  - directed PSPACE-complete [Lichtenstein & Sipser 1980]
  - undirected ∈ P [Fraenkel, Scheinerman, Ullman 1993]
- edge geography: can't revisit edges
  - directed PSPACE-complete [Schaefer – JCSS 1978]
  - undirected PSPACE-complete [Fraenkel, Scheinerman, Ullman – TCS 1993]
    - bipartite ∈ P

Reversi/Othello: [$<$ 1883]

- move = 
  - reverse in between 1 & 8 directions

- PSPACE-complete [Iwata & Kasai – TCS 1994]
- polynomial # moves: move consumes board
- reduction from directed node geography in bipartite max-degree-3 graph
  - rightward chains are threats by black: black takes α, then α', then corner, then all of bottom territory $\Rightarrow$ win
  - white wins if black can't move
  - degree-2 vertices: 
  - degree-3 vertices:  & 

if double visited then white or black wins

↳ black or white chooses

ASIDE:

Bounded NCL: NP-complete
- each edge can be reversed only once
- NP-complete for planar constraint graphs with AND, SPLIT, OR, CHOICE vertices

  differ in initial edge orientations

  can't expand without making new type of AND
- planar via crossover
- similar to proof of Constraint Graph Satisfaction

Bounded 2-player Constraint Logic (2CL)
- each edge is either white or black
- each edge can be reversed only once
- goal: each player has target edge & wins if they reverse it

- PSPACE-complete for planar constraint graphs with white AND, SPLIT, OR, CHOICE & VARIABLE vertex 
  - reduction from impartial game positive CNF SAT
  - players take turns setting variables
  - positive ⇒ white wants true, black wants false
  - black can't win (edge irreversible)
  - white wins ⇔ formula satisfied
  - crossover gadget (only use of CHOICE)
- Can make OR protected using free edge

  no constraint at degree-1 end