

Today: video games & PSPACE. First some NP:

Metatheorem 1: [Viglietta - FUN 2012 & arXiv:1201.4995]

- player traversing planar environment from start
- location traversal & single-use paths \Rightarrow NP-hard
 - \hookrightarrow player must visit some locations
 - \hookrightarrow player can traverse only once
- reduction from Planar Max-deg-3 Hamiltonicity
- vertex \rightarrow location traversal
 - \Rightarrow visit each vertex \geq once
- edge \rightarrow single-use path
 - max. degree 3 \Rightarrow never revisit vertex
- applications
 - Boulderdash
 - Lode Runner
 - Zelda II

[Aloupis, Demaine, Guo, Viglietta 2014]

Metatheorem 2: [Viglietta - FUN 2012 & arXiv:1201.4995]

- location traversal & tokens + toll roads \Rightarrow NP-hard
 - can pick (one) up \leftarrow
 - \hookrightarrow need token to traverse
- vertex \rightarrow location traversal + token
- edge \rightarrow toll road
 - traversing twice \Rightarrow stranded without token
- application: Pac-Man

Recall from L1:

- PSPACE = {problems solvable in polynomial space}
- \subseteq EXP: only exponentially many states
 - \supseteq NP: simulate all executions, take running OR
 - = NPSPACE [Savitch 1970]

Base PSPACE-complete problems:

- simulate linear-space algorithm (e.g. Turing machine)
- QSAT: (AKA QBF & TQBF)
given (fully) quantified Boolean formula, is it true?
 - e.g. $\forall x \exists y : (\bar{x} \vee y) \wedge (x \vee \bar{y})$ ($x \equiv y$)
 - can assume quantifiers in front (prenex) & alternate \forall/\exists (\exists only \Rightarrow SAT \Rightarrow NP-comp.)
- Schaefer-style dichotomy theorem:
 - $\in P \Leftrightarrow$ Horn, dual-Horn, 2-CNF, or X(N)OR (not if satisfied by all true/all false)
 - PSPACE-complete otherwise [Chen-C. Surveys 2009]
- planar QSAT [Schaefer - SICOMP 1981] [L7]
 - add \exists for new variables at end of quantifiers
- planar 1-in-3 QSAT (as in L7)

Metatheorem 3: [Viglietta - FUN 2012 & arXiv:1201.4995]

- player traversing planar environment from specified start to specified goal
- door + open pressure plate + close pressure plate
 - ↳ traversable ↳ walk on it ⇒ PSPACE-hard
 - only if open ⇒ open specific door ↳ ditto, close
- & exactly 1 open & 1 close plate per door
- reduction from Q3SAT
- clause gadget
- existential quantifier gadget
- universal quantifier gadget
- one plate of each type for each door
- applications:
 - many FPSs e.g. Doom, Quake, Heretic, Hexen, ...
 - many RPGs e.g. Eye of the Beholder
 - many adventure games e.g. SCUMM engine (Maniac Mansion, Monkey Island, Space Quest, ...)
 - Prince of Persia

Metatheorem 4: buttons instead of pressure plates

↳ optional: can press or not
↳ activates 3 doors at once

- pressure plate gadget
- in fact 2 doors per button suffice
[Bodlaender & van der Zanden - CIAC 2015]
- applications: MANY
 - ~~Sonic the Hedgehog~~ (Sega Genesis)
 - The Lost Vikings (Super NES; PC) "Erik the Swift"
 - Tomb Raider (Sega Saturn & PS1; PC)

Metatheorem 5: [Aloupis, Demaine, Guo, Viglietta 2014]

- door with traverse, open, close paths \Rightarrow PSPACE-hard
only if open \leftarrow can open \rightarrow must close

- applications:
 - Legend of Zelda: A Link to the Past
(Ocarina of Time, Majora's Mask, Oracle of Seasons, The Minish Cap, Twilight Princess \approx PushPush-1)
 - Donkey Kong Country 1, 2, 3
 - Super Mario Bros. [Demaine, Viglietta, Williams - unpublished, 2014]
 - Lemmings [Viglietta - FW 2014]
(tested in DOS version = Amiga version = unoff. editor)