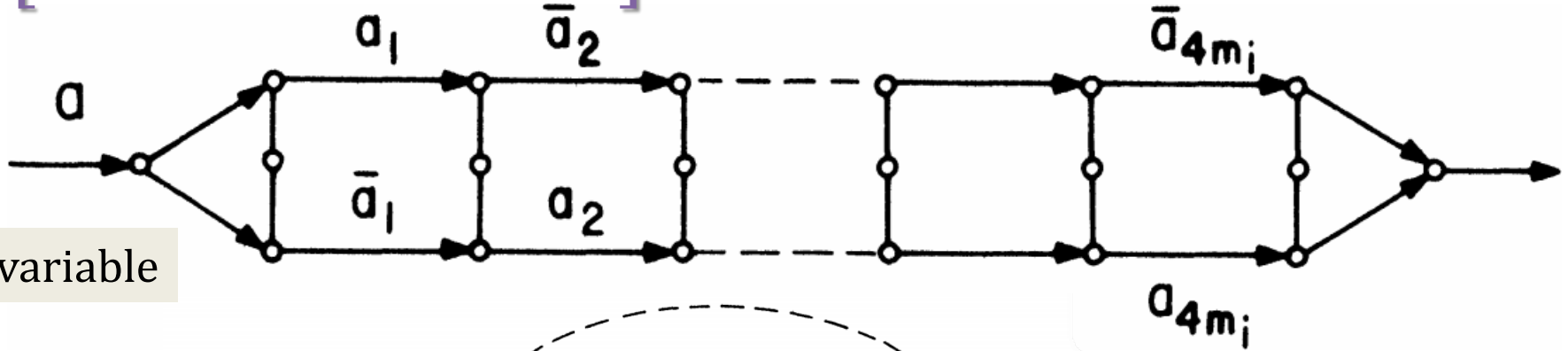


Icosian Game [Hamilton 1857]

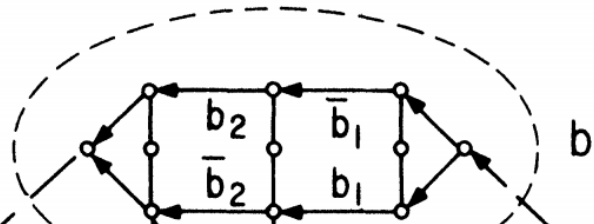


Planar (Directed) Hamiltonian Cycle

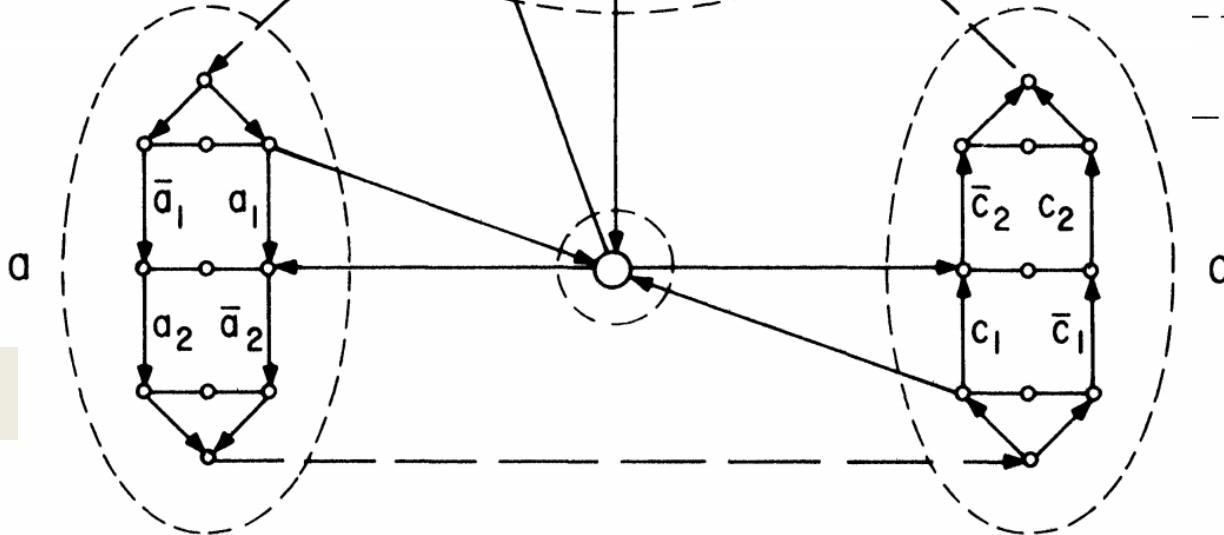
[Lichtenstein 1982]



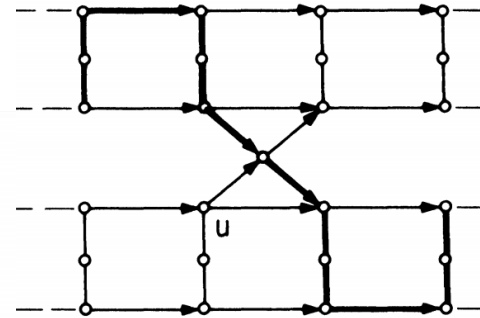
variable



clause



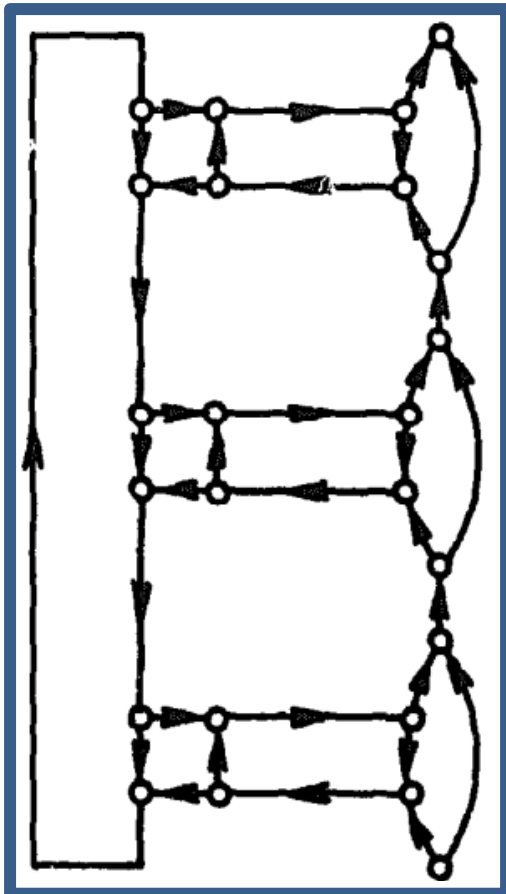
$a \vee \bar{b} \vee c$



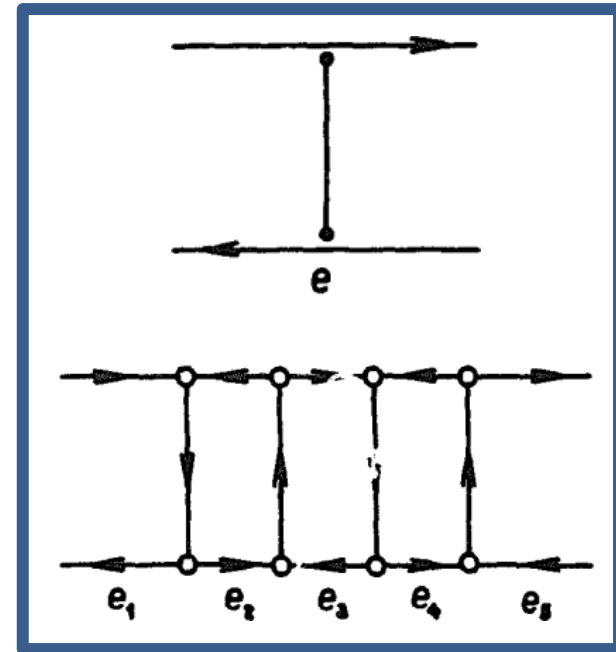
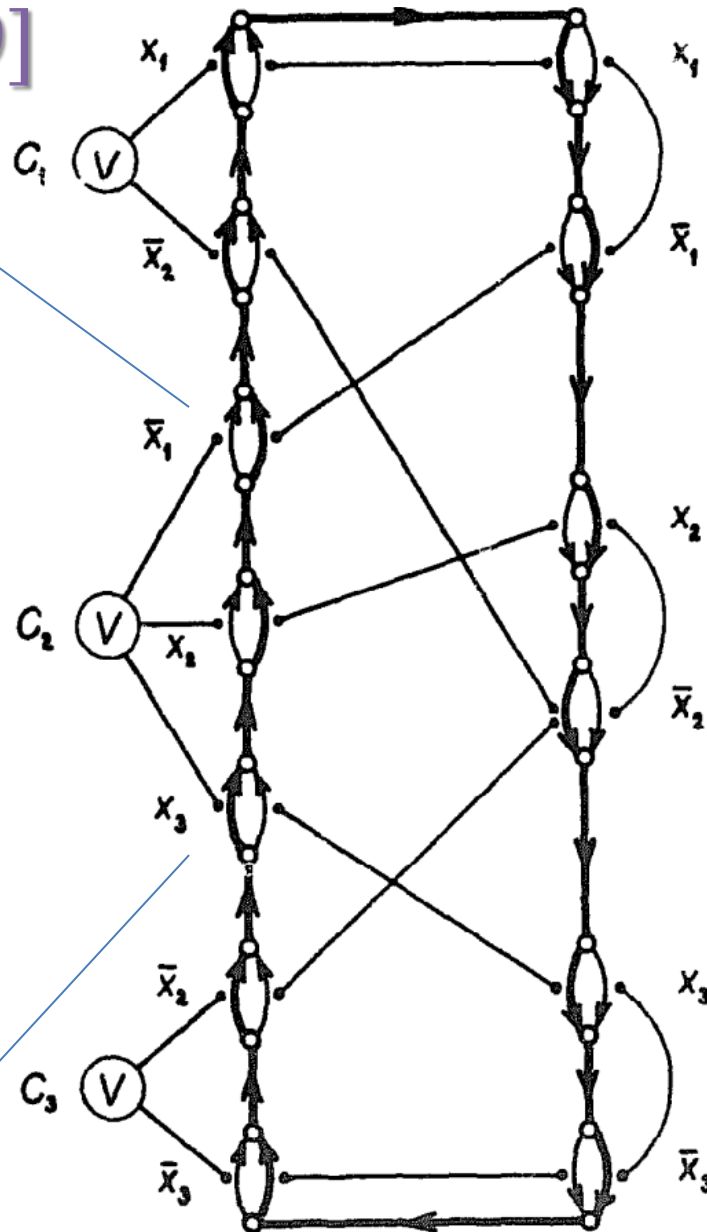
c

Planar Directed Max-Degree-3

[Plesník 1979]



clause gadget

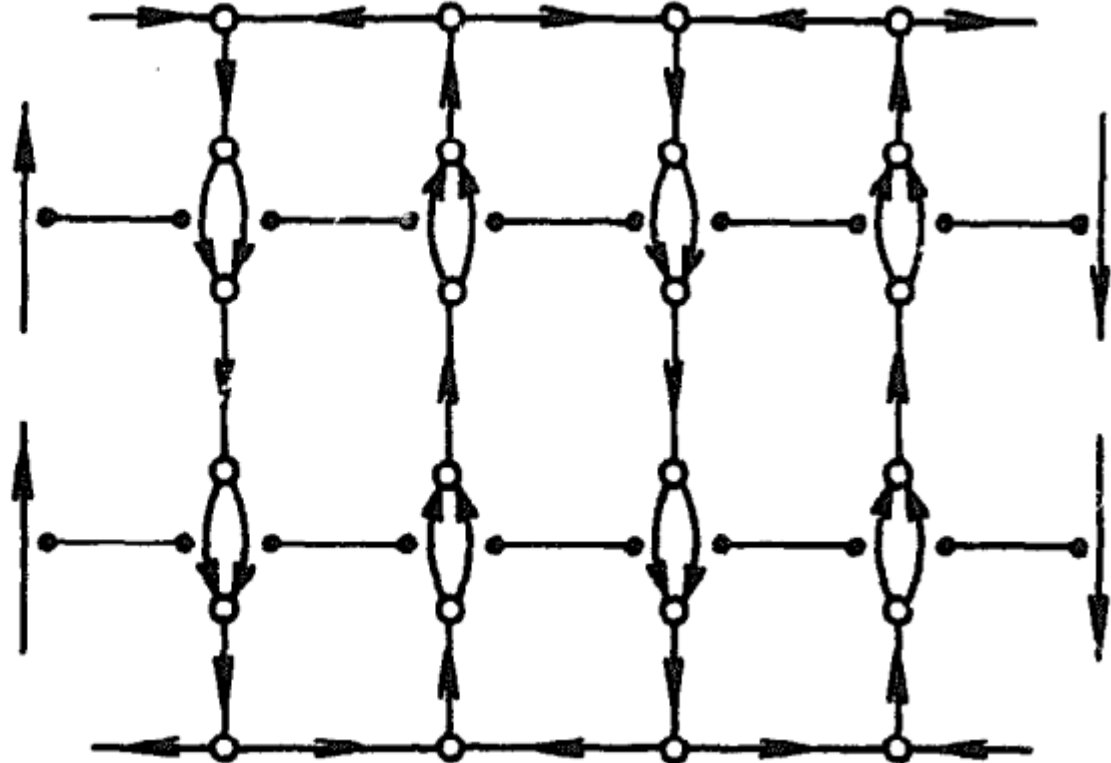
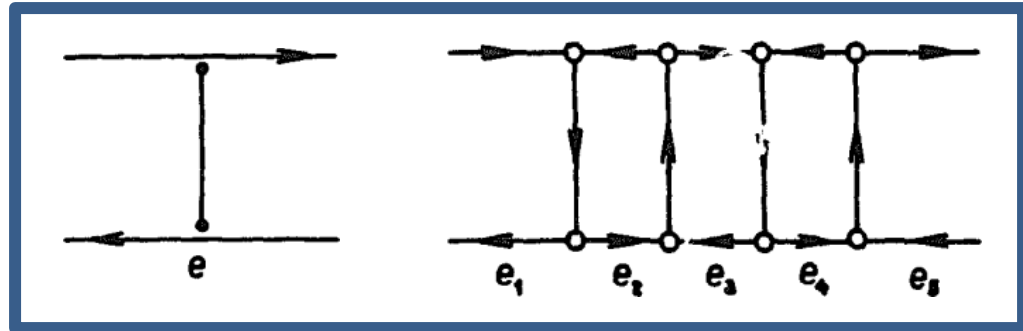


XOR gadget

$$\begin{aligned}
 &(x_1 \vee \bar{x}_2) \\
 &\wedge (\bar{x}_1 \vee x_2 \vee x_3) \\
 &\wedge (\bar{x}_2 \vee \bar{x}_3)
 \end{aligned}$$

Planar Directed Max-Degree-3

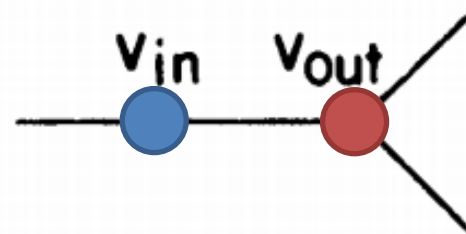
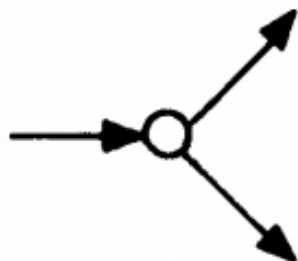
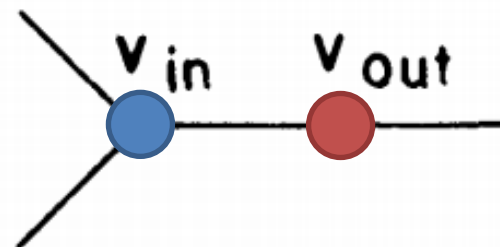
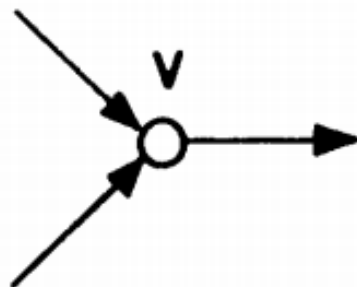
[Plesník 1979]





Planar Bipartite Max-Degree-3

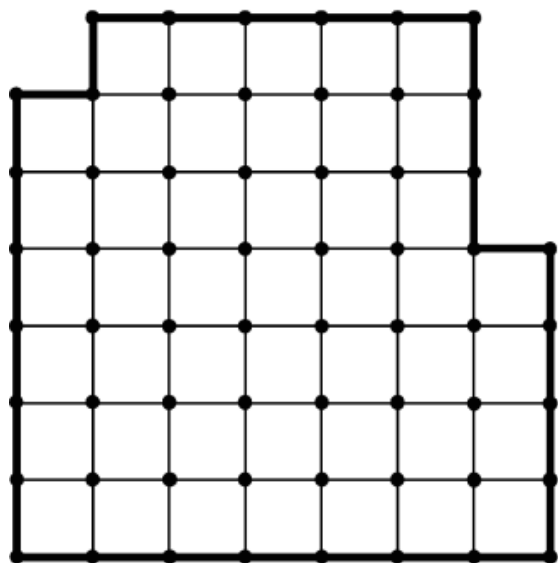
[Itai, Papadimitriou, Szwarcfiter 1982]



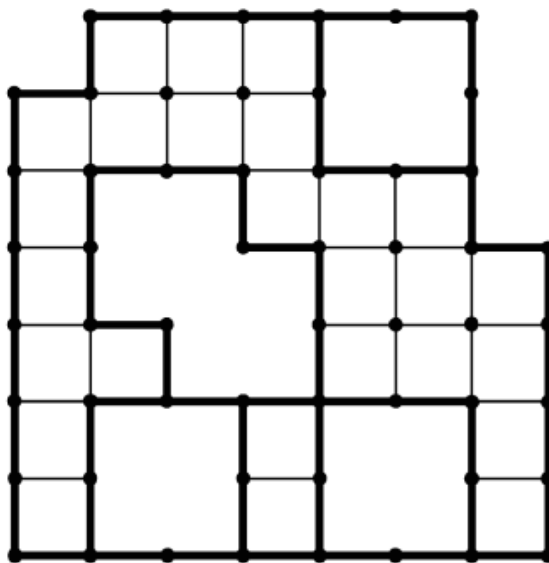


Grid Graphs

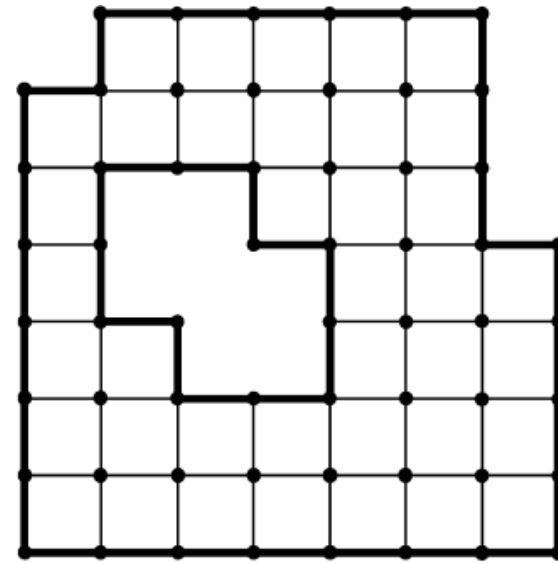
[Itai, Papadimitriou, Szwarcfiter 1982]



solid
(no holes)



holes

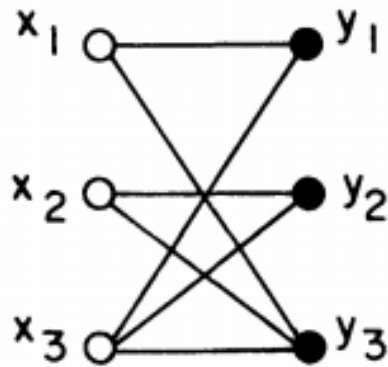


holes

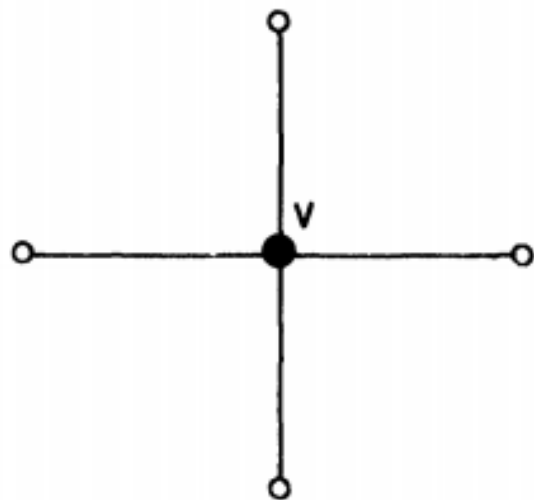
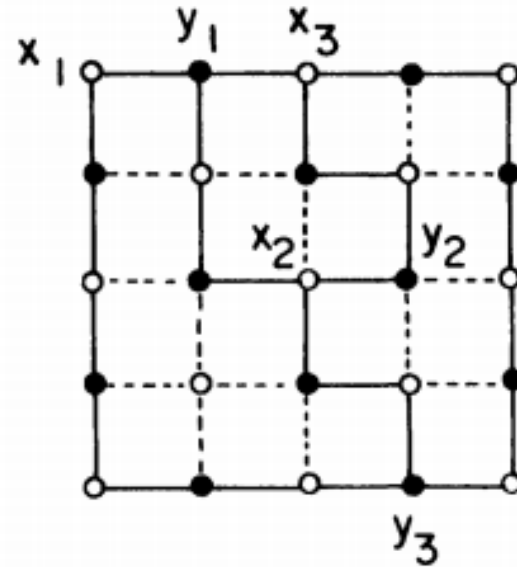
figure by Arkin, Fekete, Islam, Meijer, Mitchell,
Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009

Planar Bipartite Graph Drawing

[Itai, Papadimitriou, Szwarcfter 1982]

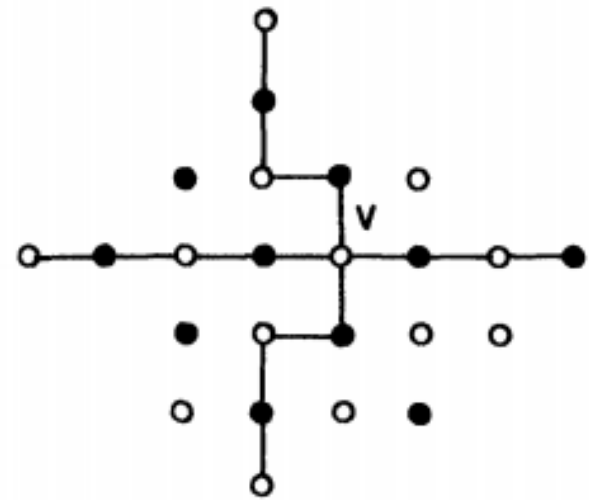


emb
⇒



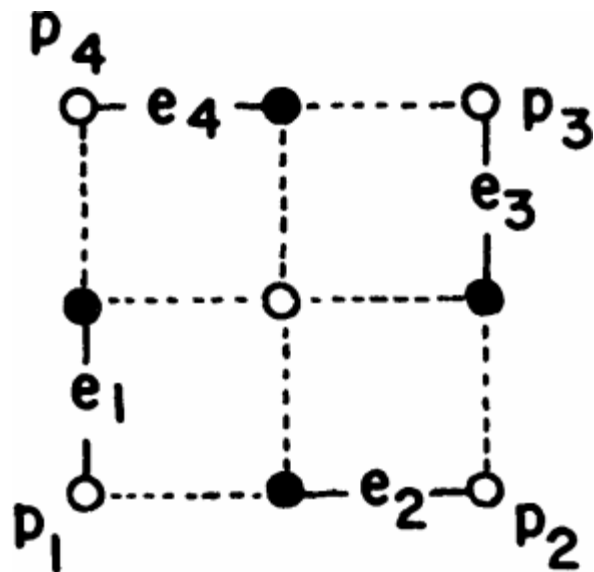
⇒

scale 3×

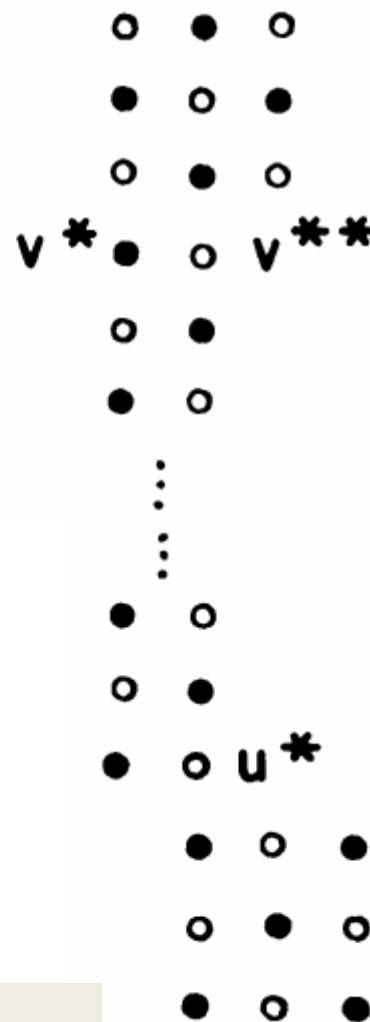


Hamiltonicity in Grid Graphs

[Itai, Papadimitriou, Szwarcfter 1982]



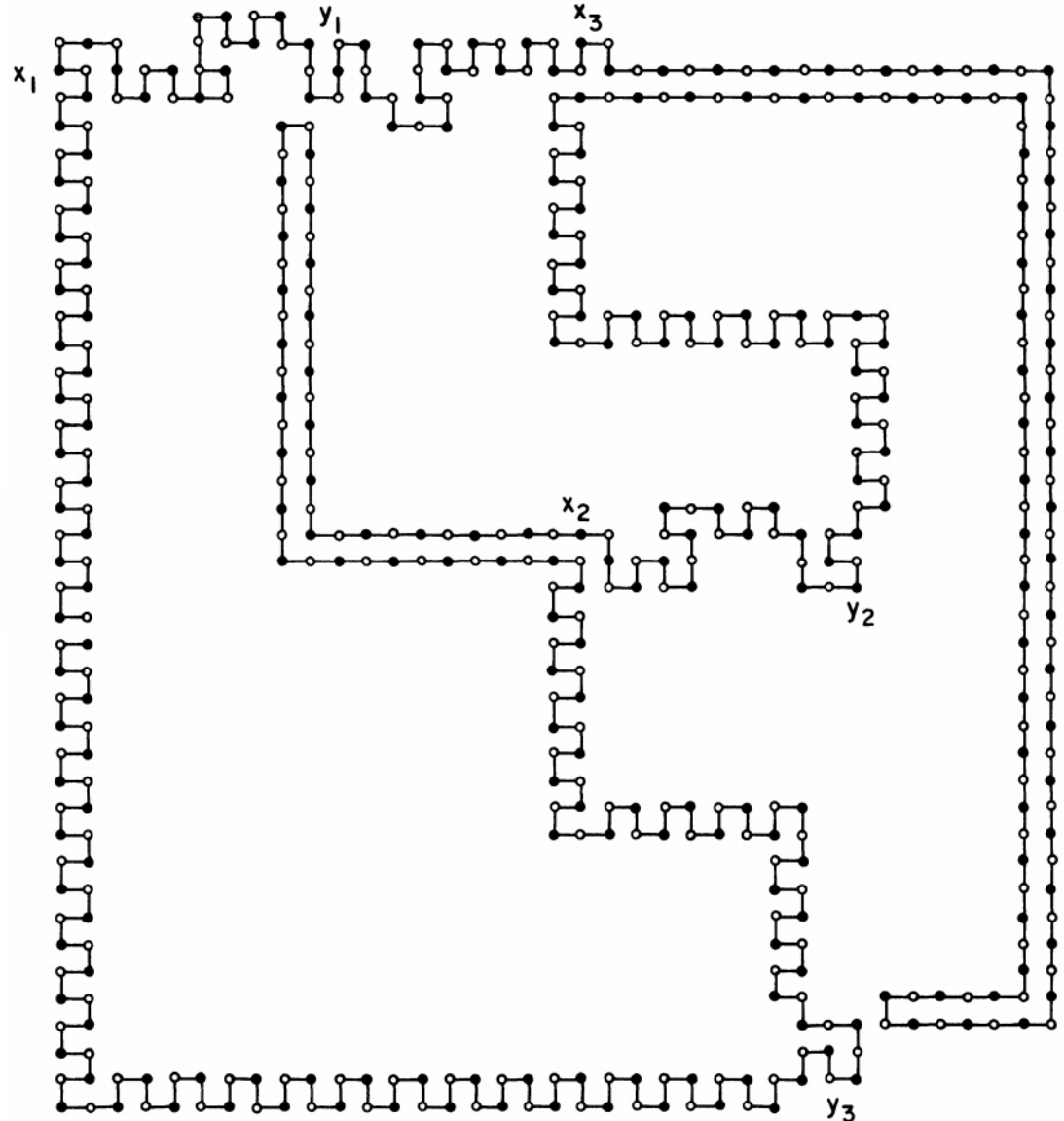
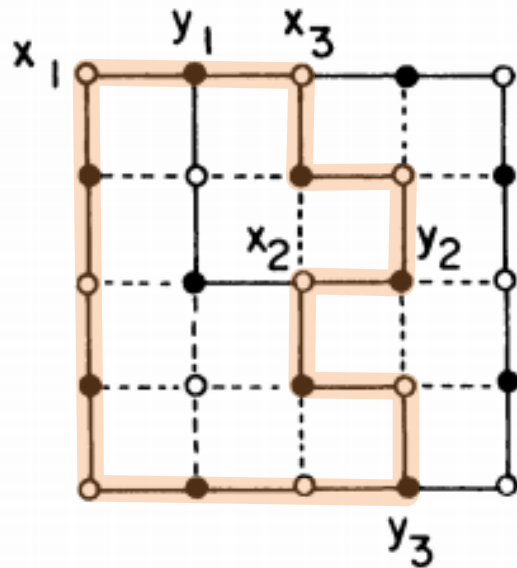
vertex gadget



vertex-edge connections

Hamiltonicity in Grid Graphs

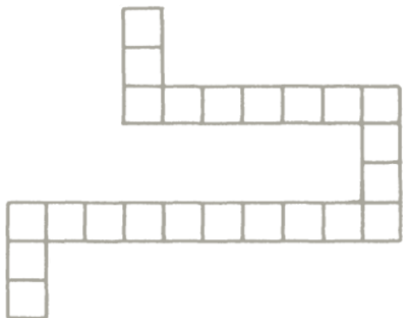
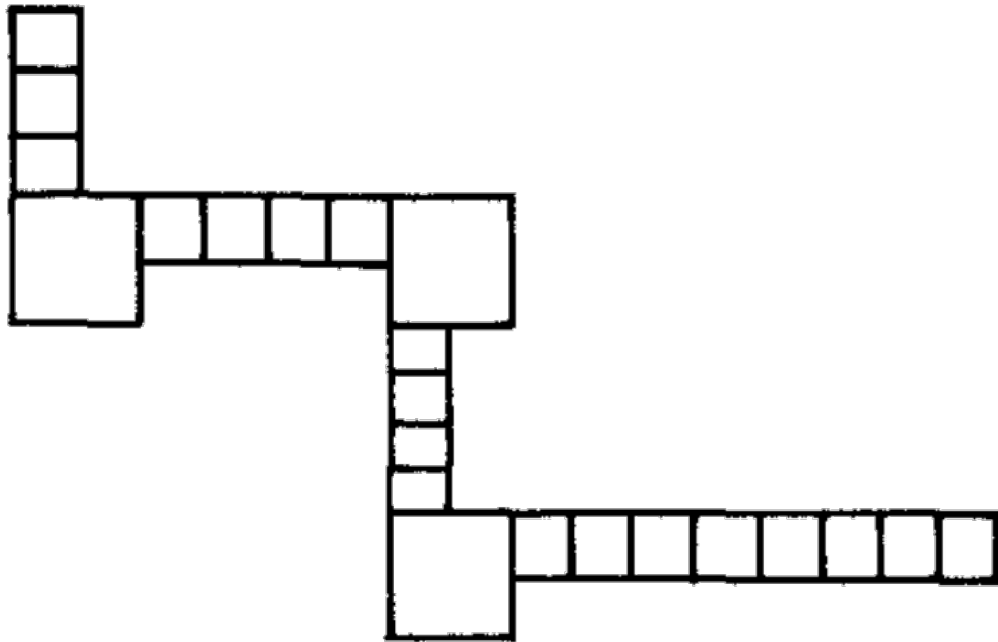
[Itai, Papadimitriou, Szwarcfter 1982]



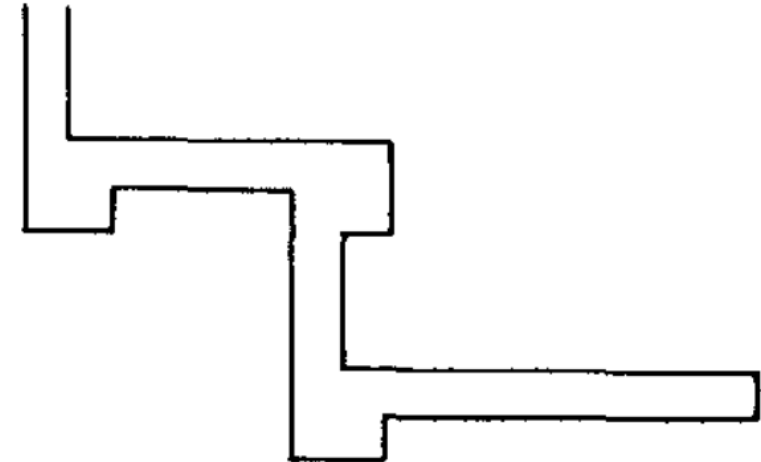


Max-Degree-3 Grid Graphs

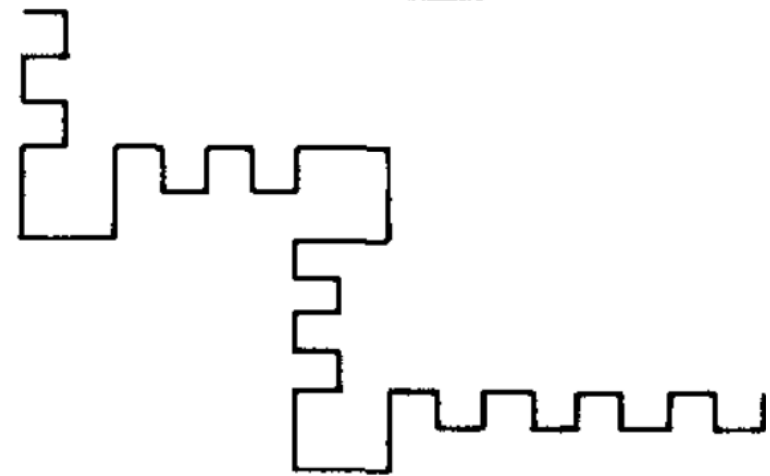
[Papadimitriou & Vazirani 1984]



edge gadget



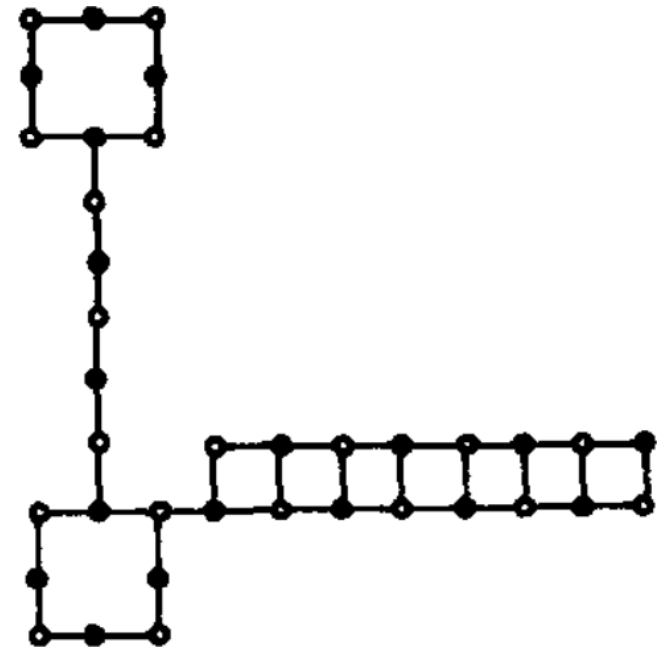
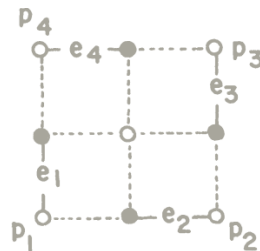
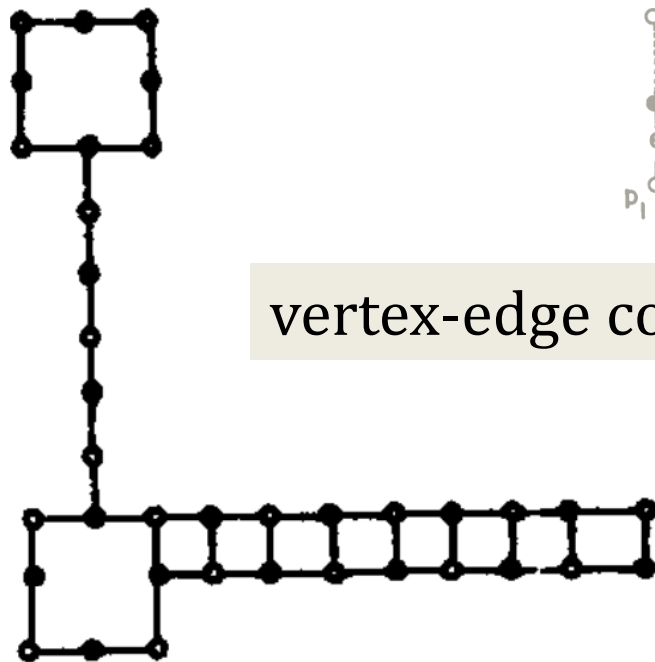
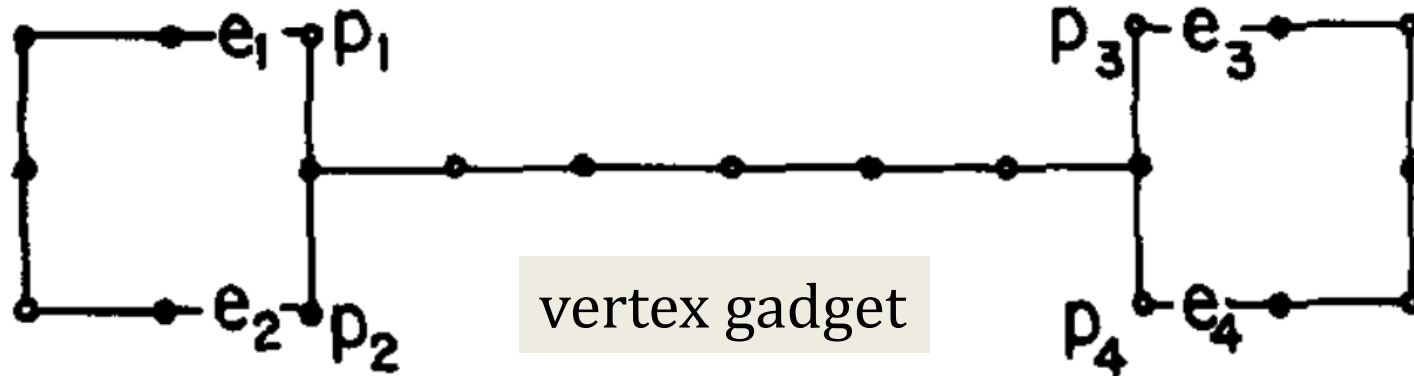
Return Path



Cross Path

Max-Degree-3 Grid Graphs

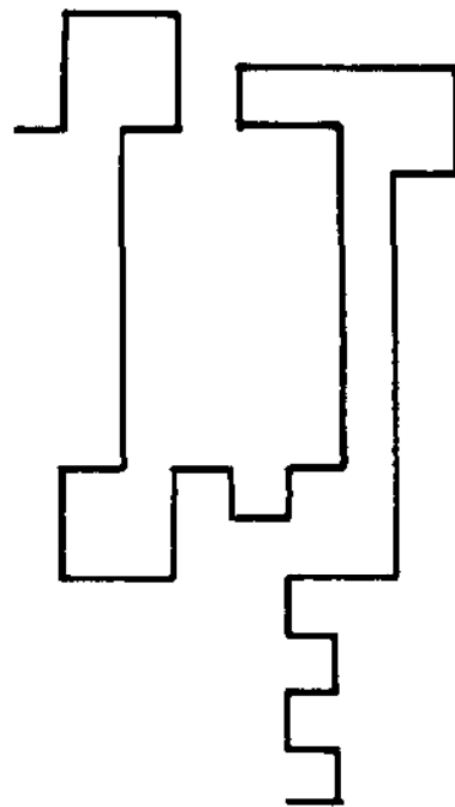
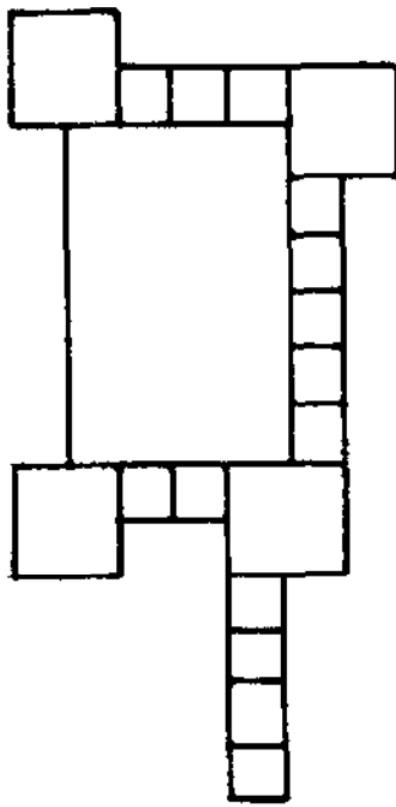
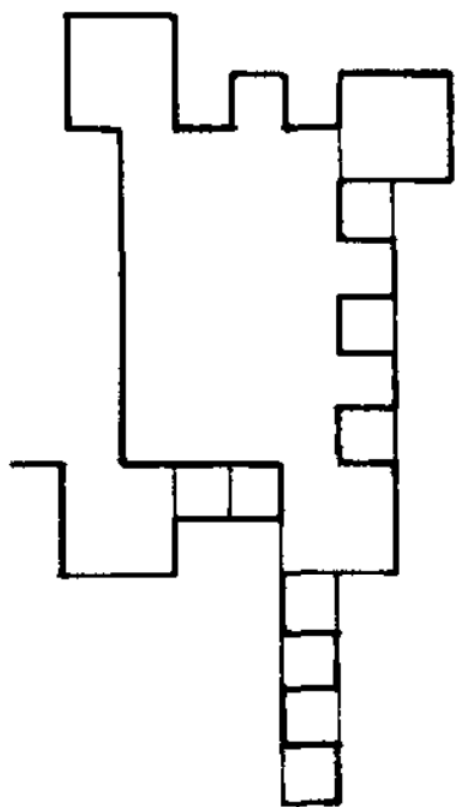
[Papadimitriou & Vazirani 1984]



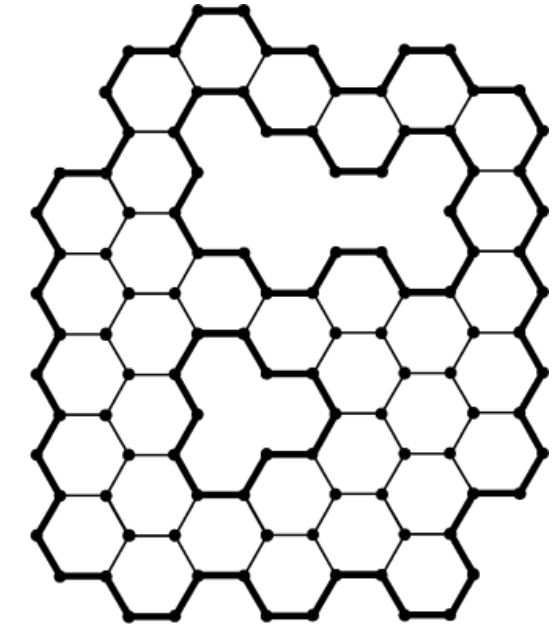
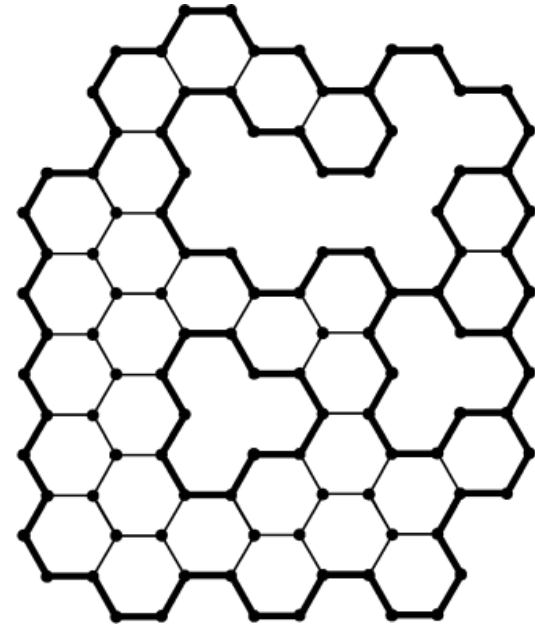
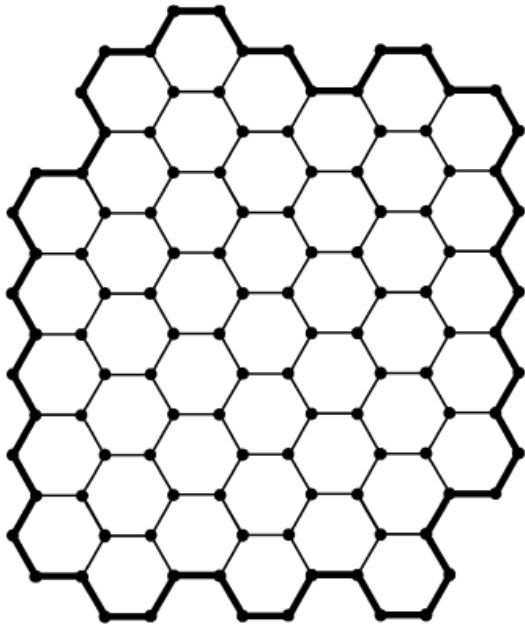
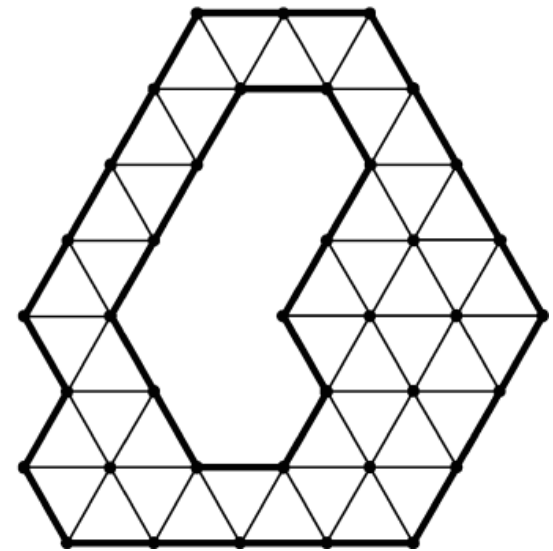
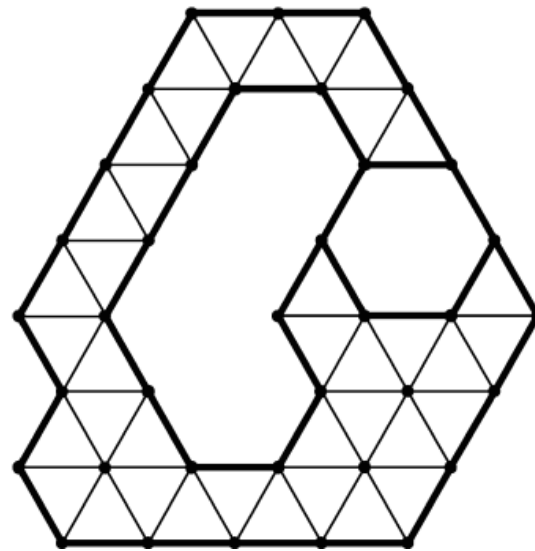
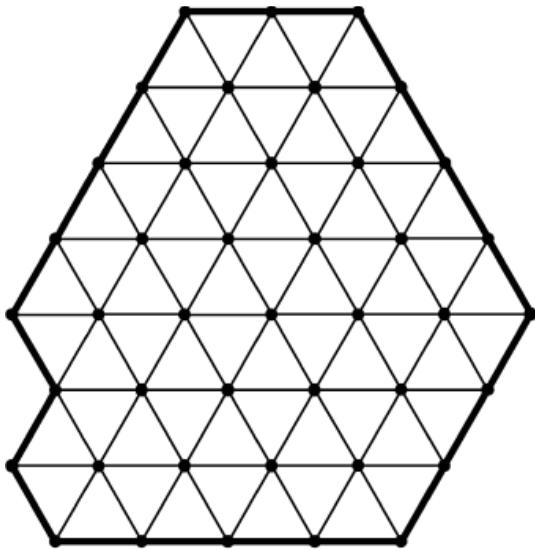


Max-Degree-3 Grid Graphs

[Papadimitriou & Vazirani 1984]



forced-edge vertex-edge connections



solid

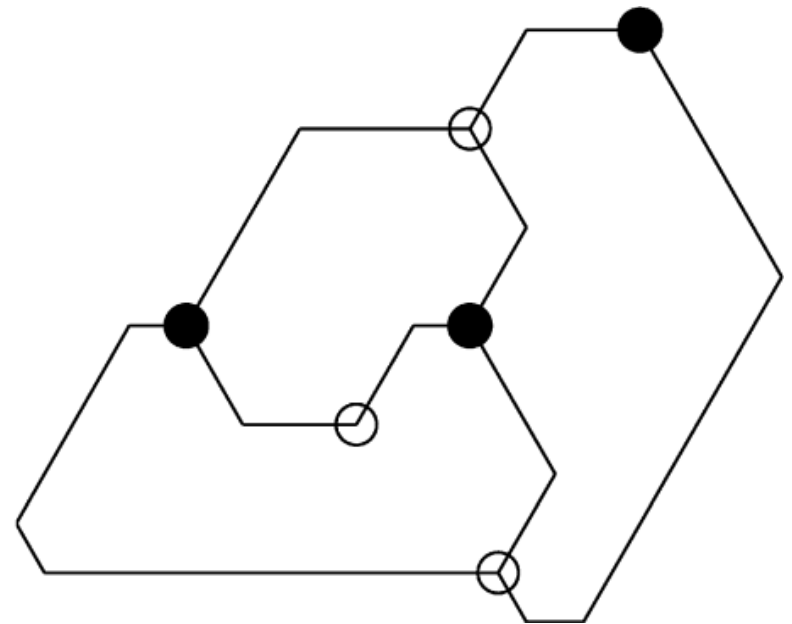
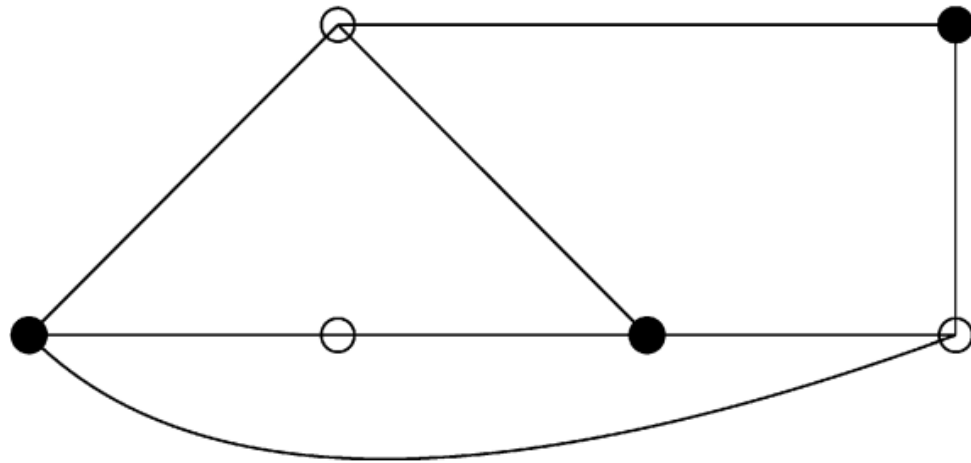
Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009

polygonal



Hamiltonicity in Triangular Grid Graphs

Arkin, Fekete, Islam,
Meijer, Mitchell, Núñez-
Rodríguez, Polishchuk,
Rappaport, Xiao 2009



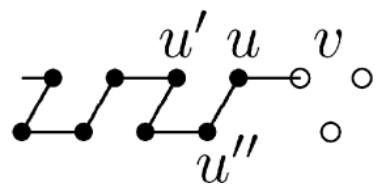
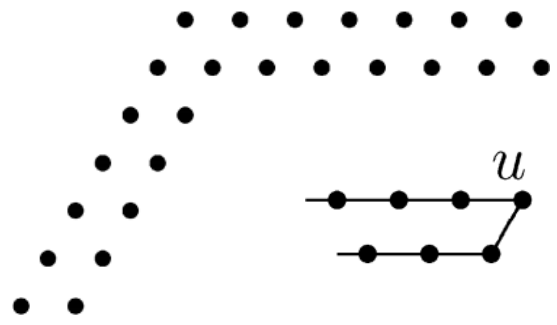
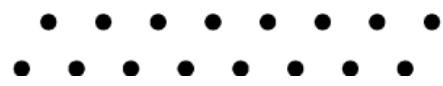
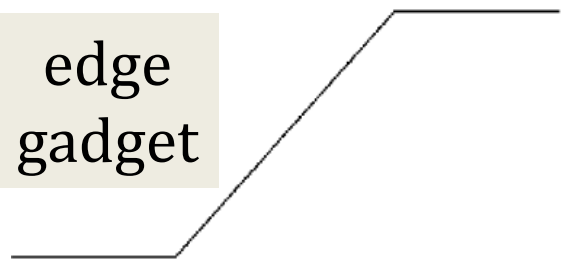


vertex gadget

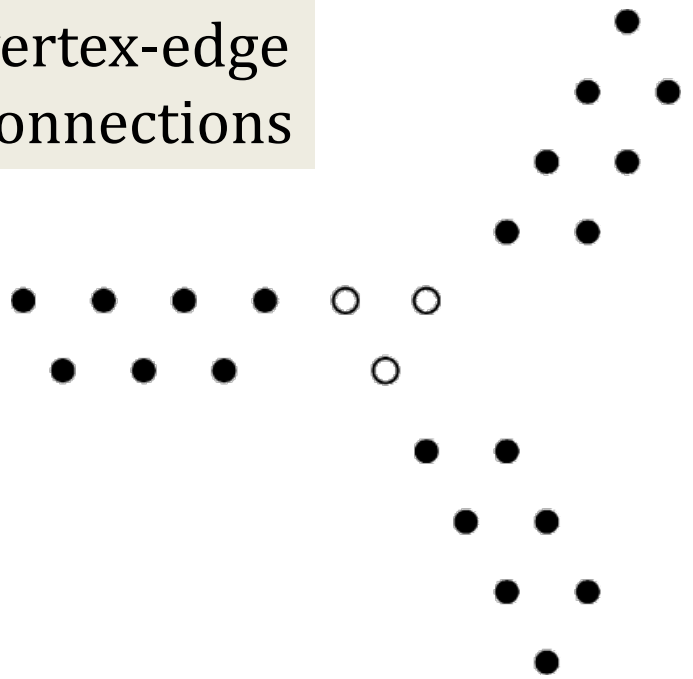
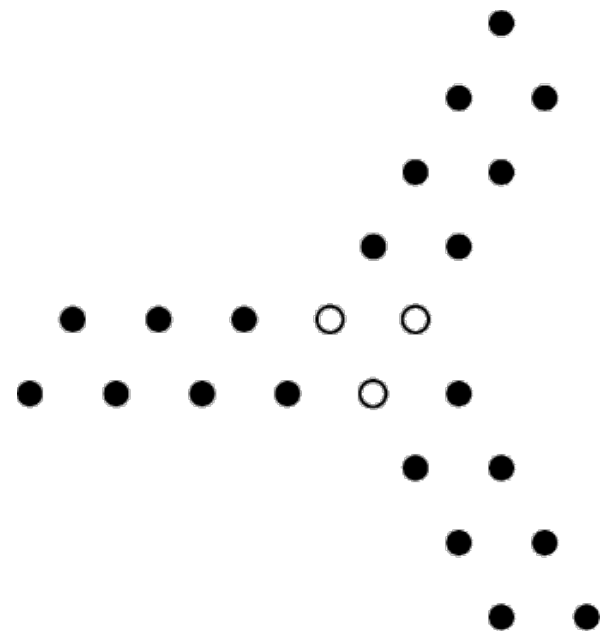


Arkin, Fekete, Islam,
Meijer, Mitchell, Núñez-
Rodríguez, Polishchuk,
Rappaport, Xiao 2009

edge
gadget

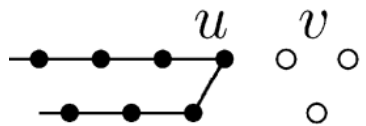
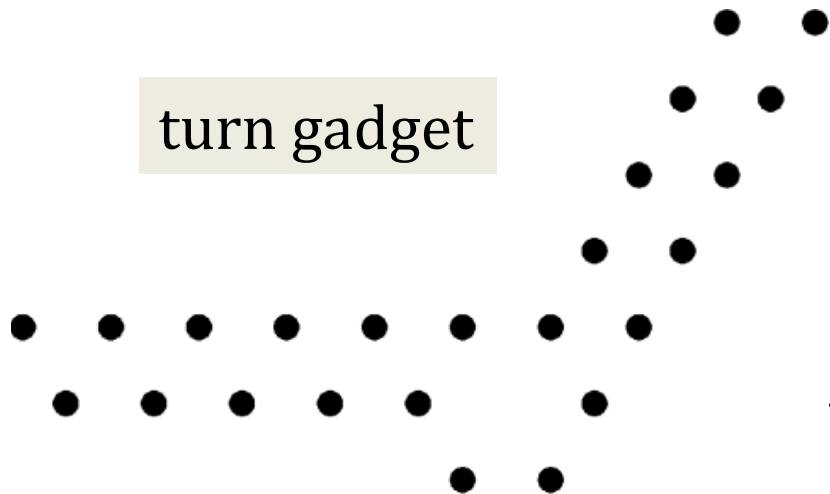


vertex-edge
connections

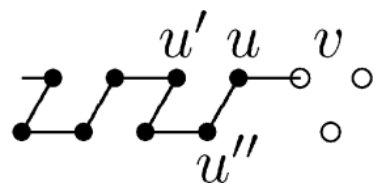




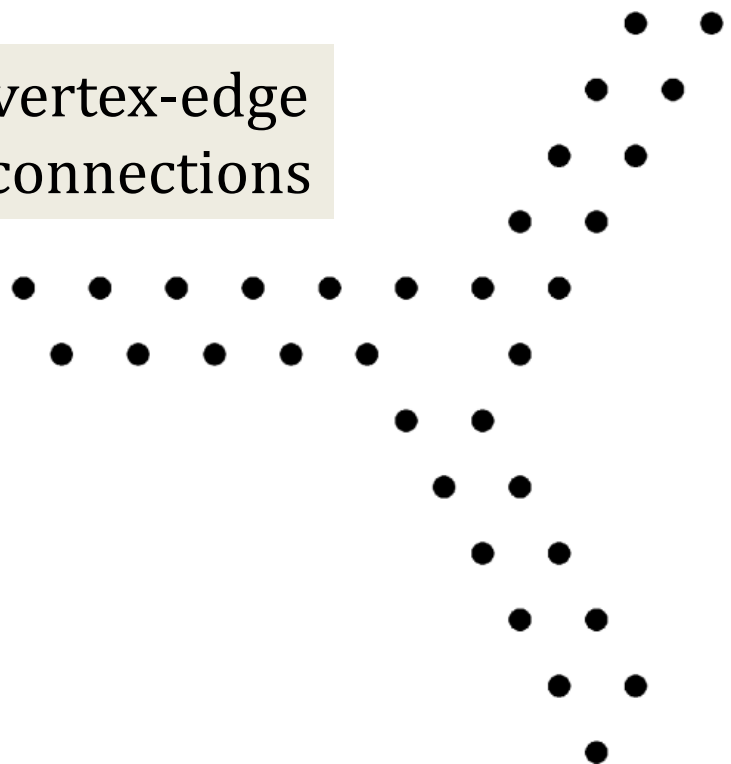
turn gadget



Arkin, Fekete, Islam,
Meijer, Mitchell, Núñez-
Rodríguez, Polishchuk,
Rappaport, Xiao 2009



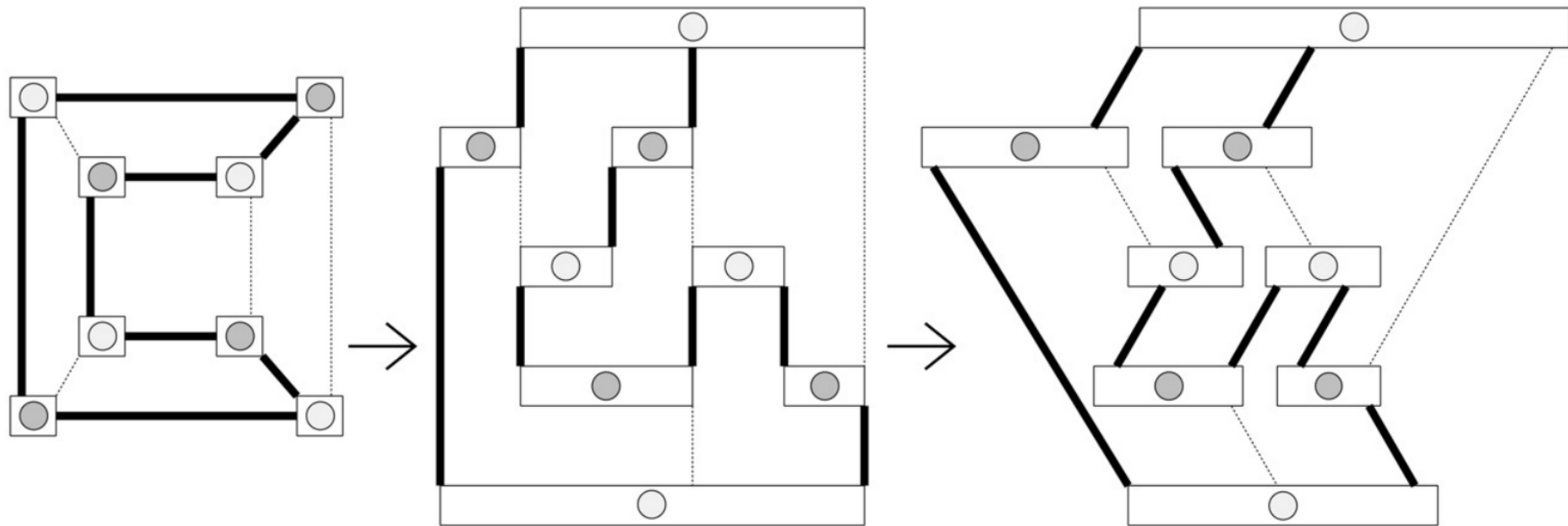
vertex-edge
connections





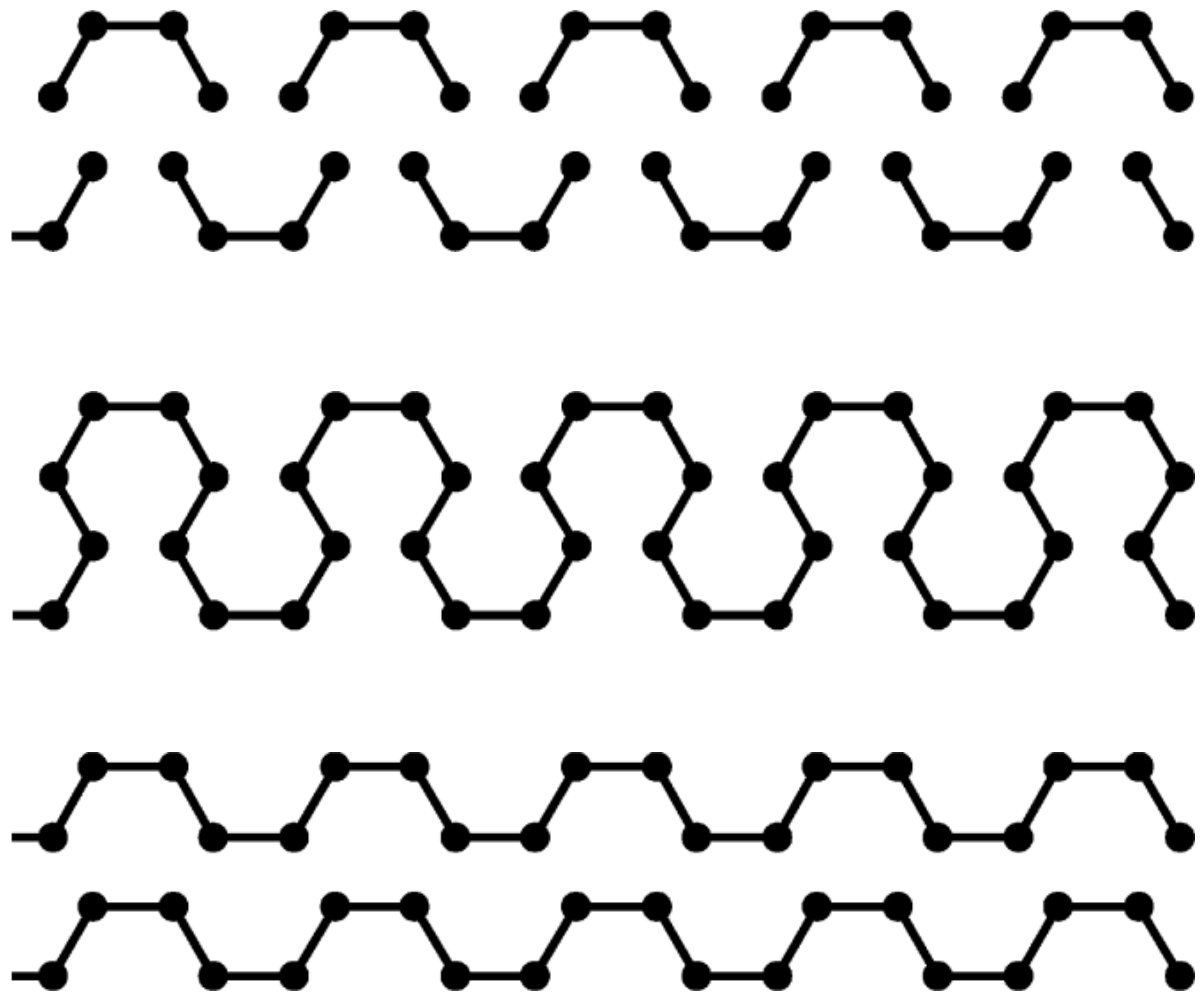
Hamiltonicity in Hexagonal Grid Graphs

Arkin, Fekete, Islam,
Meijer, Mitchell, Núñez-
Rodríguez, Polishchuk,
Rappaport, Xiao 2009





Arkin, Fekete, Islam,
Meijer, Mitchell, Núñez-
Rodríguez, Polishchuk,
Rappaport, Xiao 2009



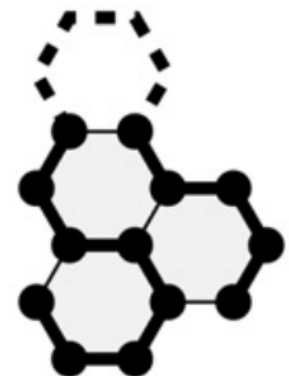
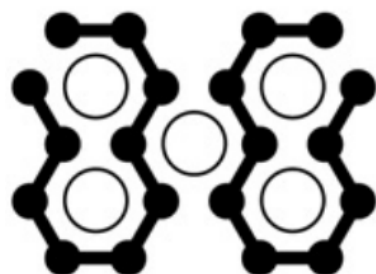
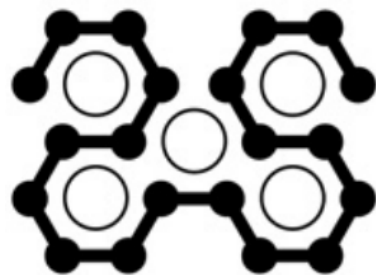
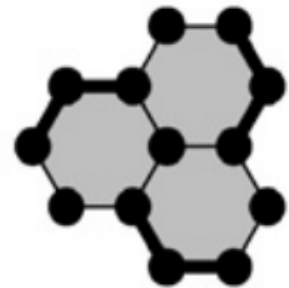
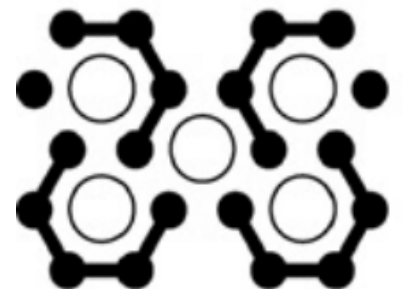
edge gadget



U-turn

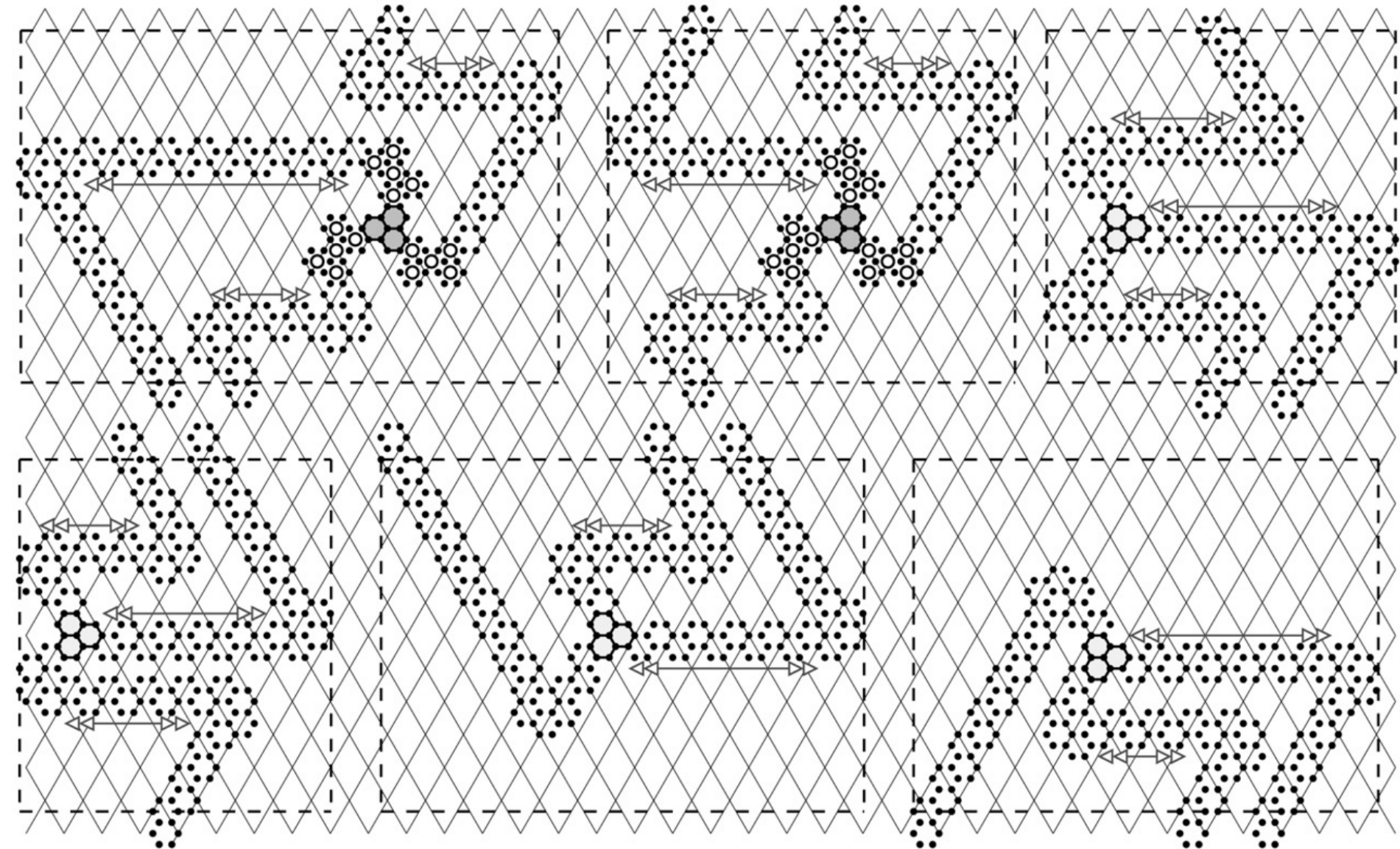
turn gadget

vertex core

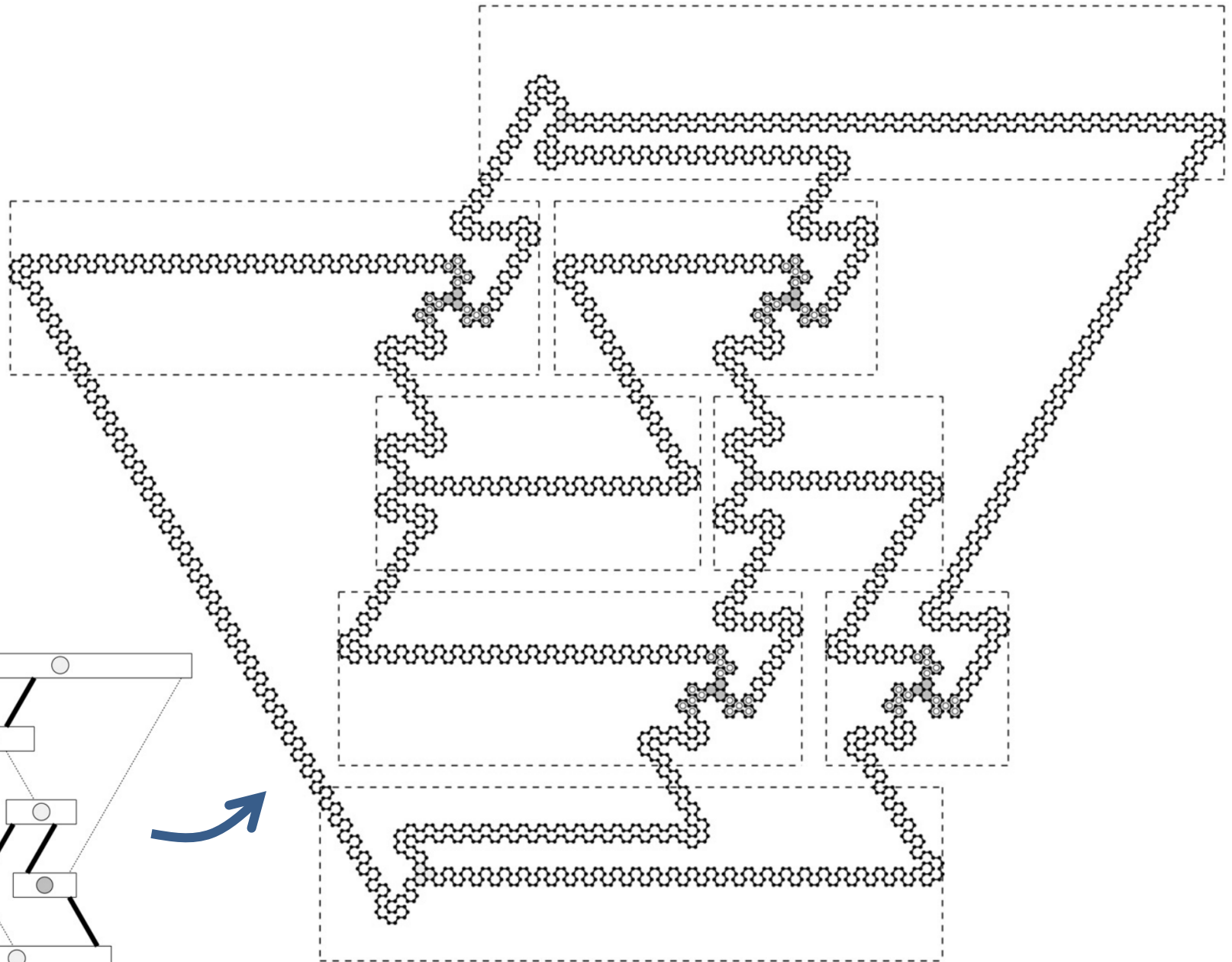
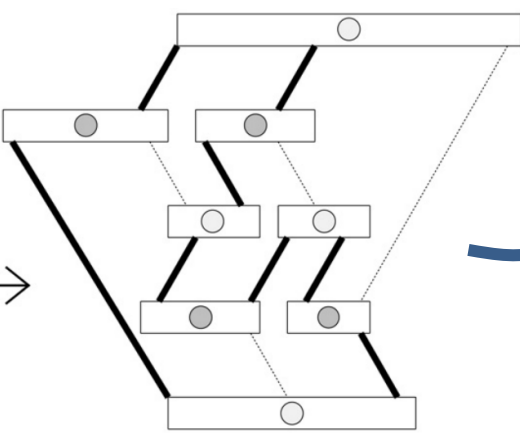


Arkin, Fekete,
Islam, Meijer,
Mitchell, Núñez-
Rodríguez,
Polishchuk,
Rappaport, Xiao
2009

vertex gadgets

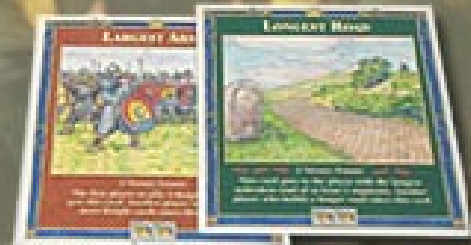


Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009



Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009

THE SETTLERS OF CATAN



> 15 million sold

[Klaus Teuber 1995]

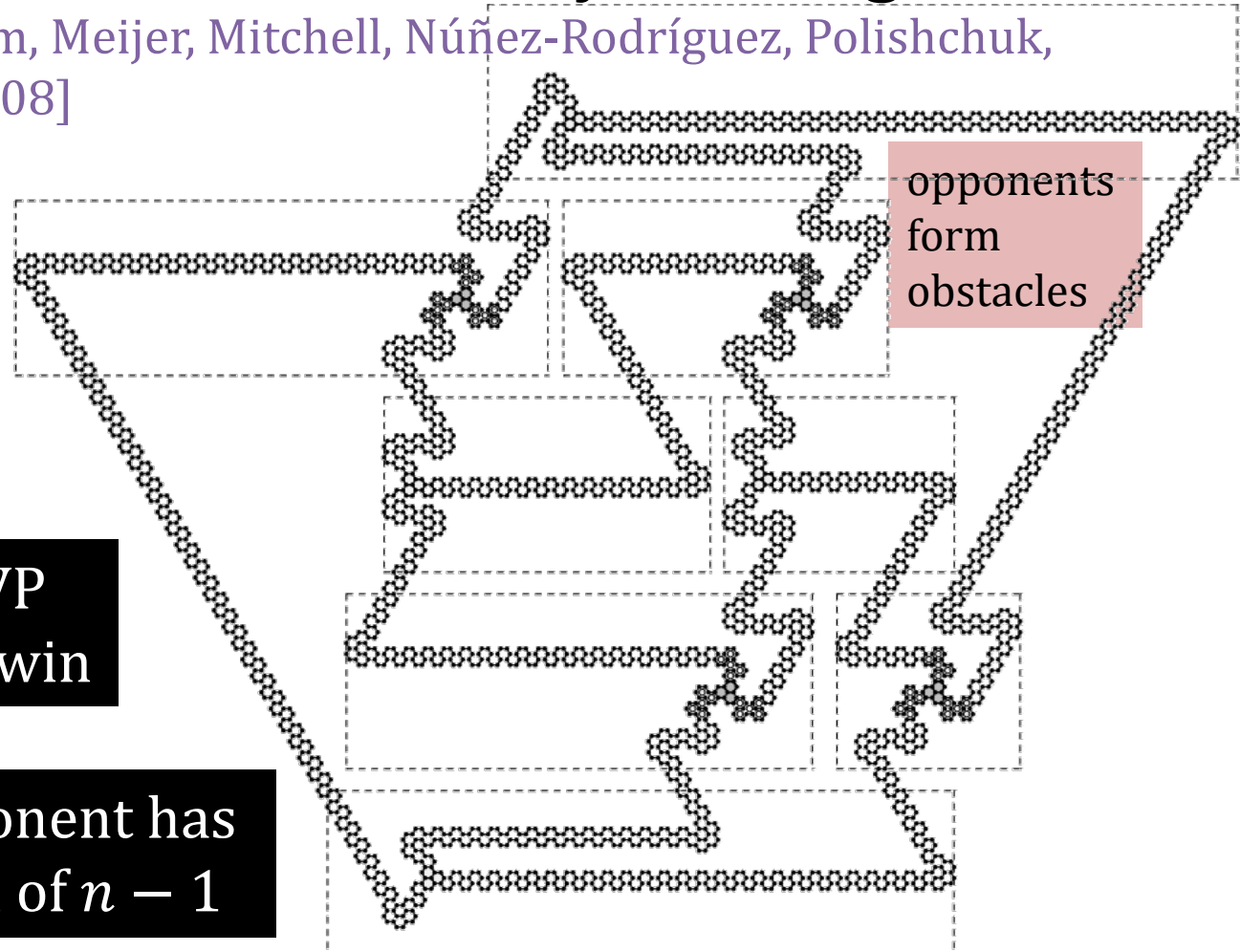
Settlers of Catan Mate-in-1 is NP-Complete [Demaine, van Eycke, McKay 2011]

- Reduction from Hamiltonicity in hex grids

[Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2008]



$\times \infty$
(buys roads)



opponents form obstacles

2 VP to win

opponent has road of $n - 1$



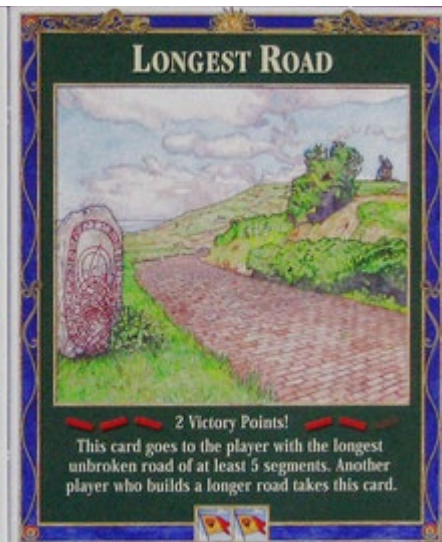
Settlers of Catan Mate-in-0 is NP-Complete

[Demaine, van Eyck, McKay 2011]

- Reduction from Hamiltonicity in hex grids

[Arkin, Fekete, Islam, Meijer, Mitchell, Núñez-Rodríguez, Polishchuk, Rappaport, Xiao 2009]

In fact, deciding whether you've already won is NP-complete!



2 VP
to win

opponent has
road of $n - 1$



Slitherlink [Nikoli 1989]

Slitherlink Easy Author: Casty

Progress: ██████████ x1

nikoli's solving history 40min-7min-1min 02:06

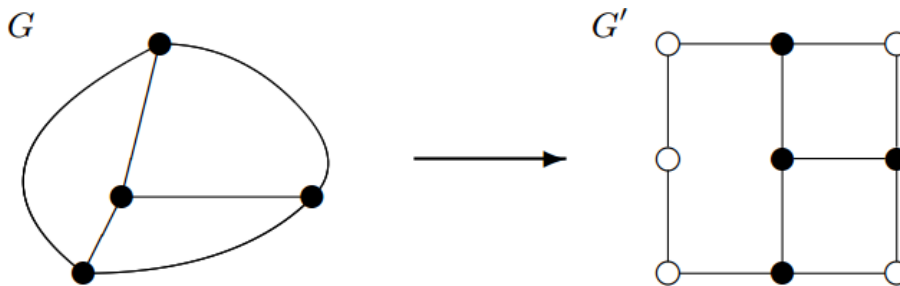


Slitherlink Easy Author: Casty

Progress: ██████████ x1

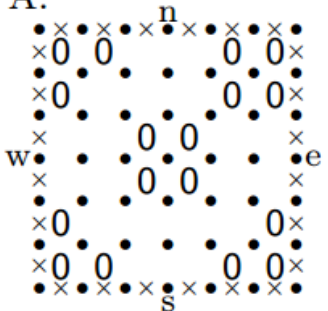
nikoli's solving history 40min-7min-1min 02:06

Slitherlink is NP-complete

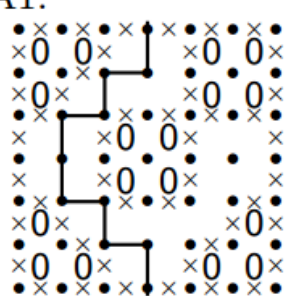


optional vertex

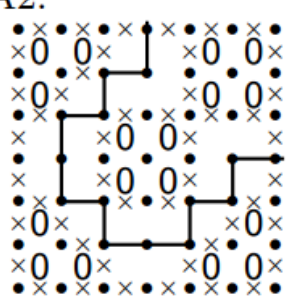
A.



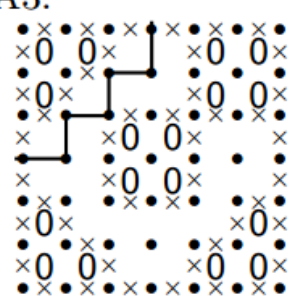
A1.



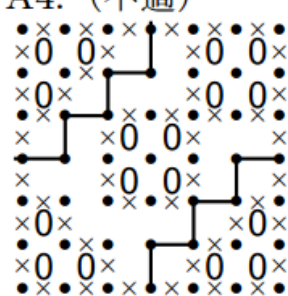
A2.



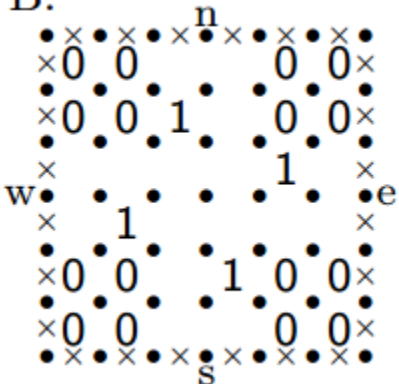
A3.



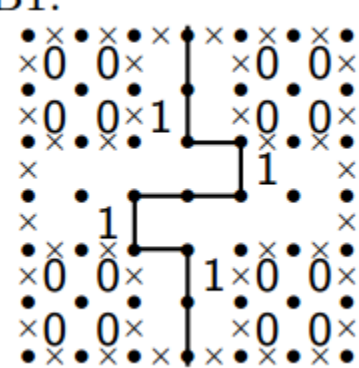
A4. (不適)



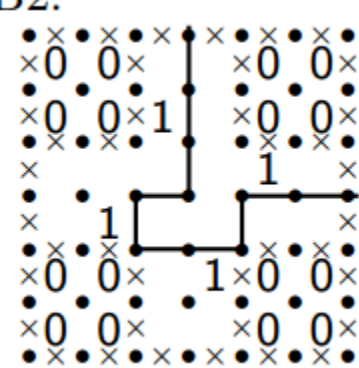
B.



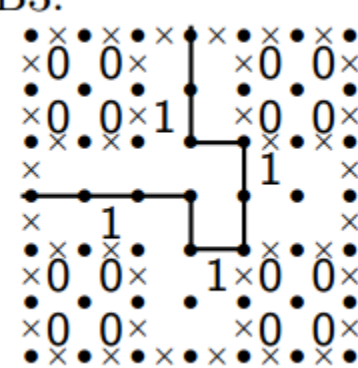
B1.



B2.



B3.



required vertex

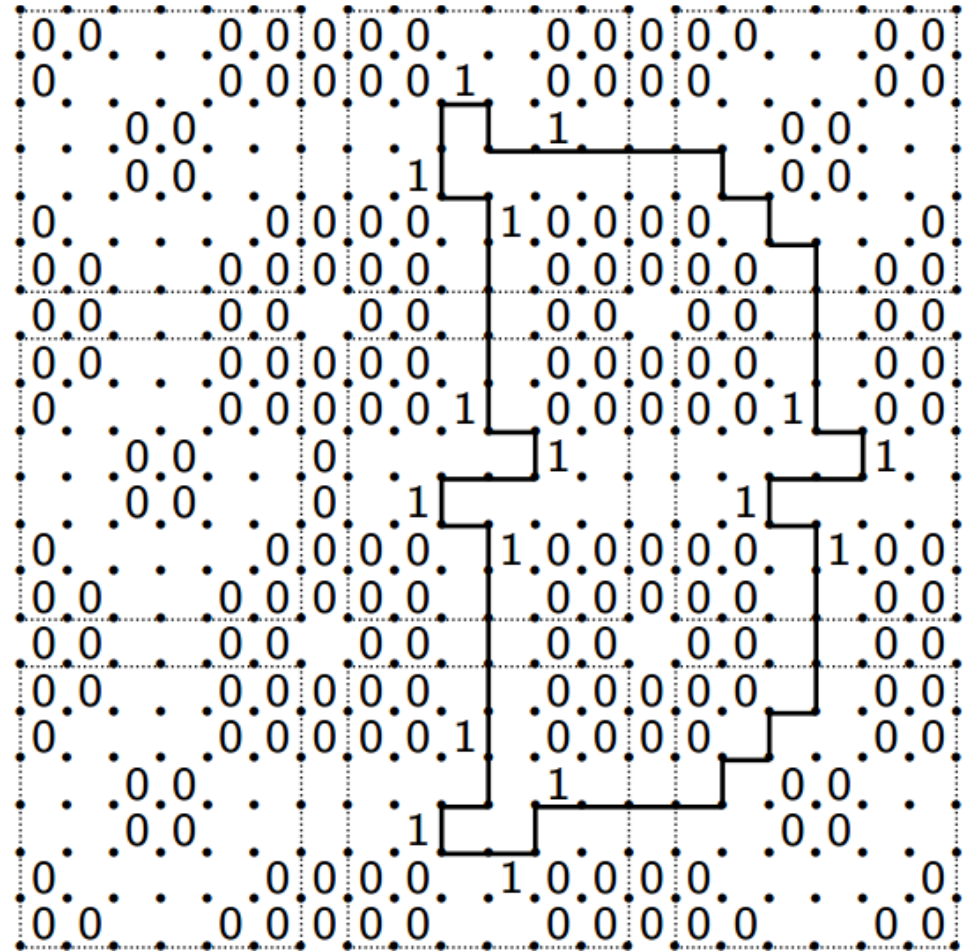
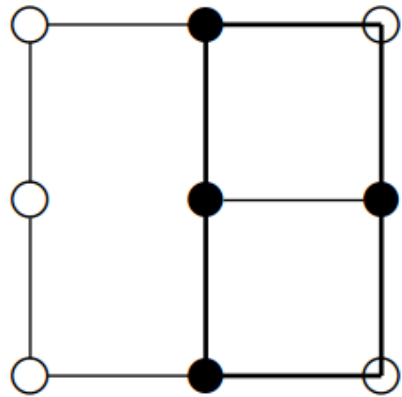
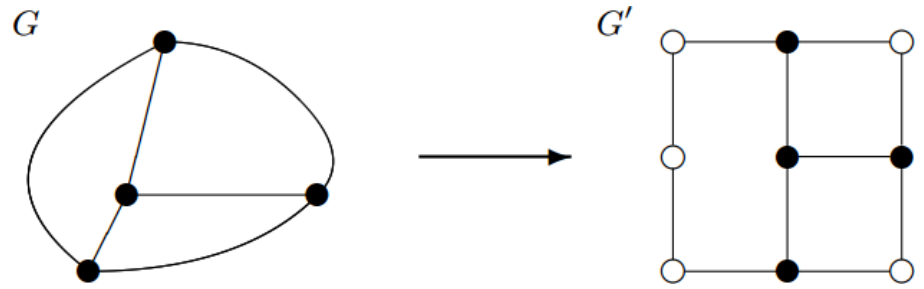
non-edge

edge

[Yato 2000]



Slitherlink is NP-complete



[Yato 2000]

Hashiwokakero [Nikoli 1990]

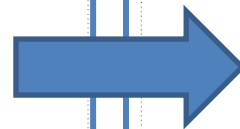
Hashiwokakero Easy Author : SAKAMOTO, Nobuyuki

Progress: ██████████ x1

A 10x10 grid of numbers for a Hashiwokakero puzzle. The numbers are arranged as follows:

1	5	3	3						
		1		2					
4	8	2	3						
	2	2		2					
3	2	1	3						
	6	5	4						
3	1	3	1						
	3	1							
3	3	6	4						

nikoli's solving history 00:31



Hashiwokakero Easy Author : SAKAMOTO, Nobuyuki

Progress: ██████████ x1

The same 10x10 grid as in the previous image, but with bridges (lines) connecting the numbers. The solution is as follows:

1	5	3	3						
		1		2					
4	8	2	3						
	2	2		2					
3	2	1	3						
	6	5	4						
3	1	3	1						
	3	1							
3	3	6	4						

nikoli's solving history 00:31

Lawn Mowing



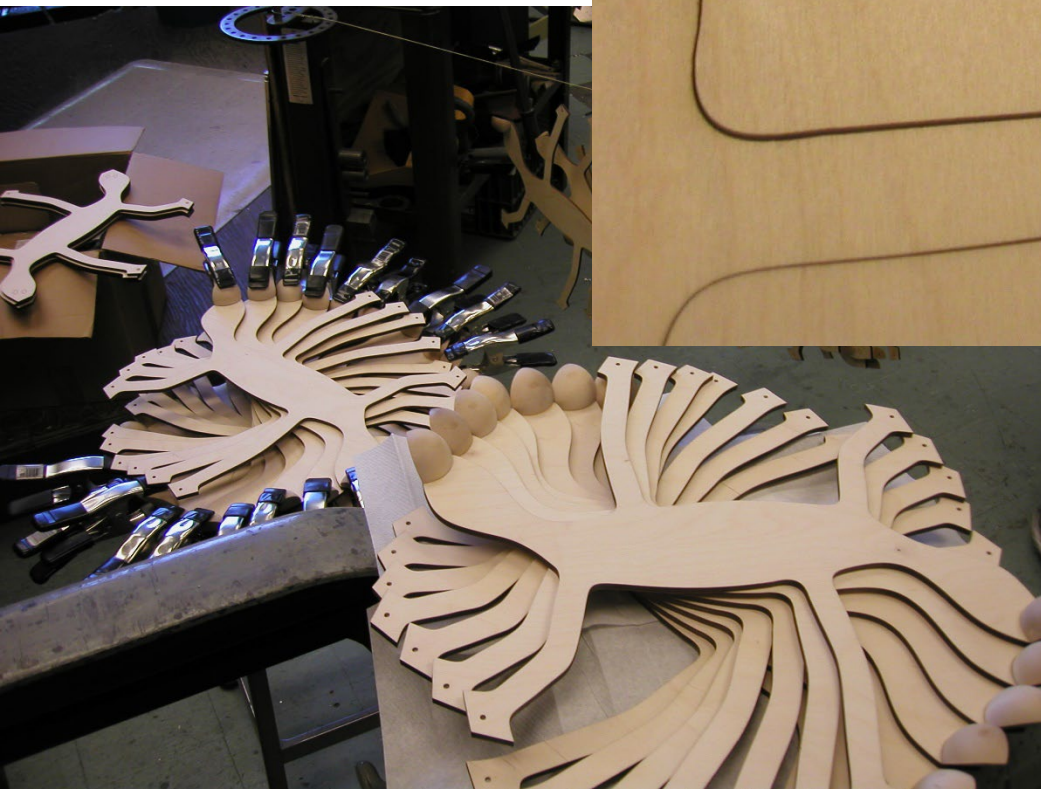
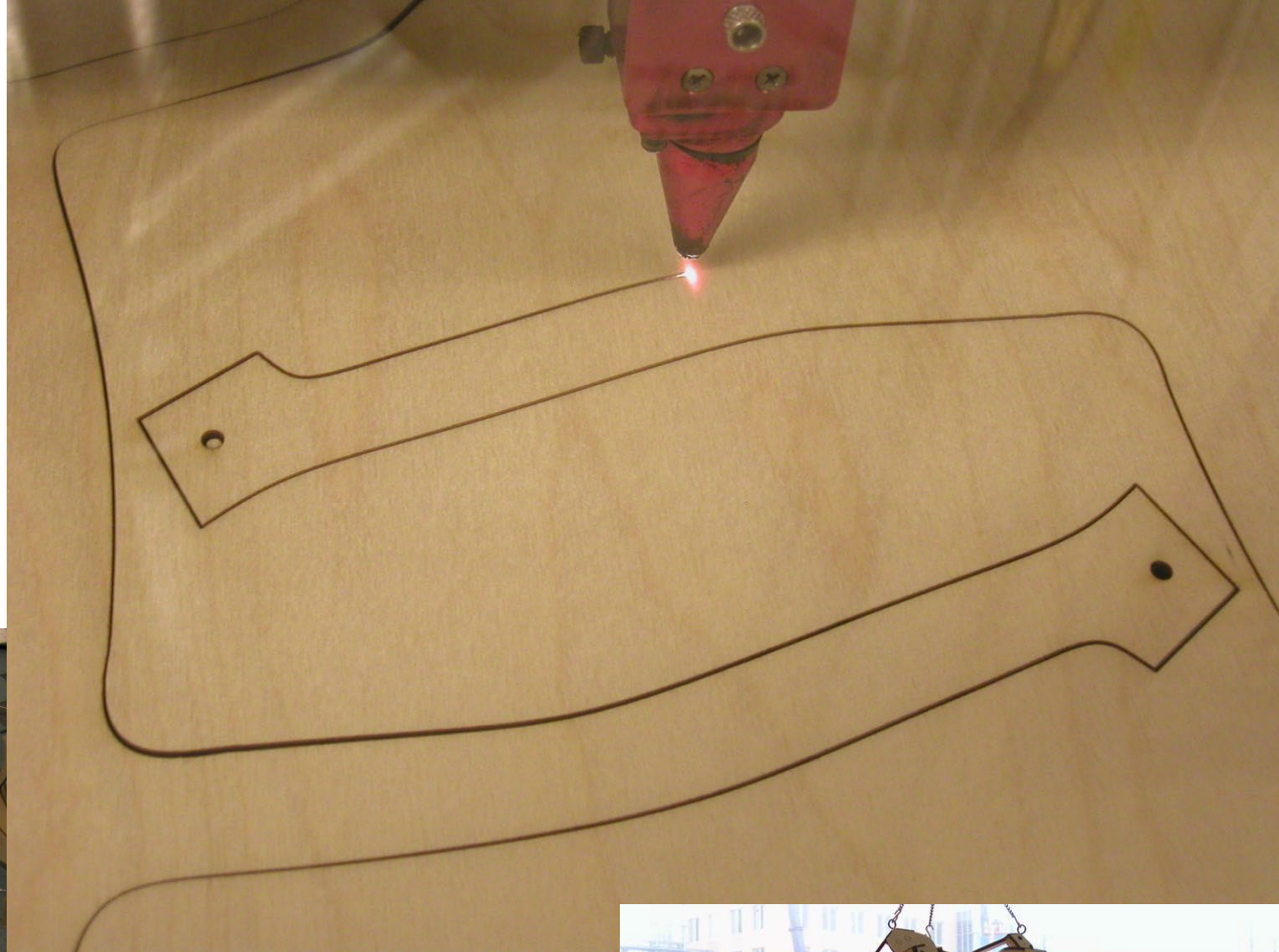
DONUT GAMES

04



Laser Cutting

George Hart at MIT,
2003

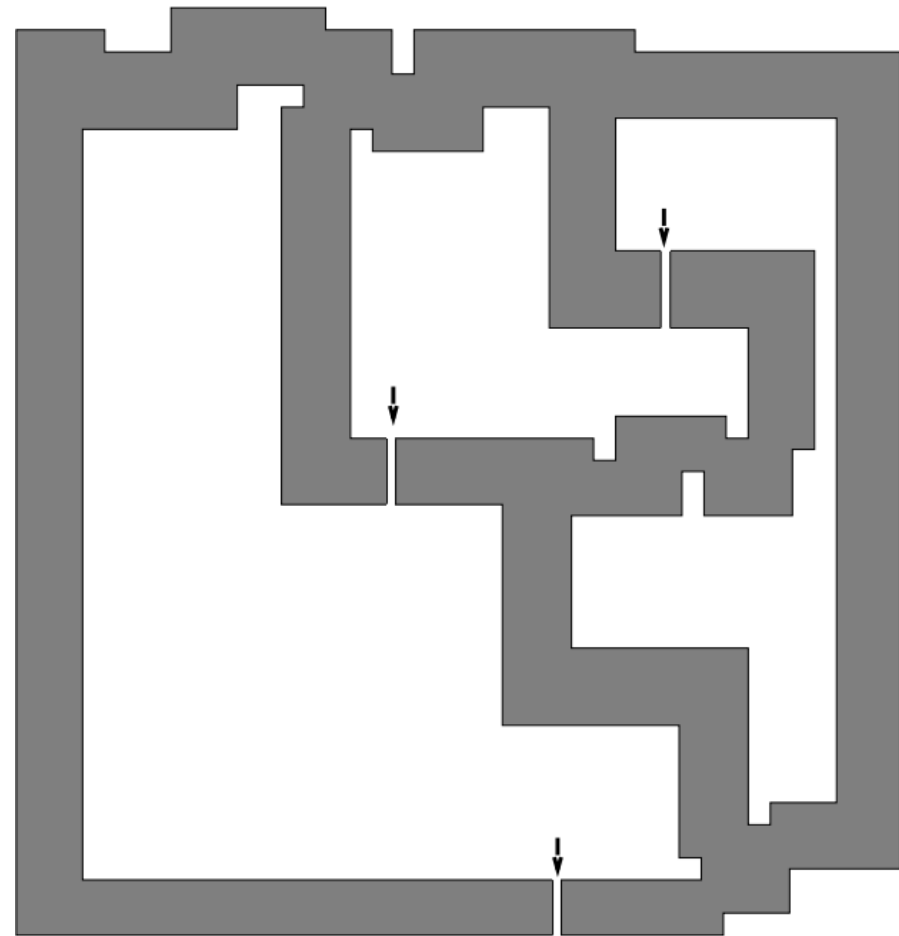
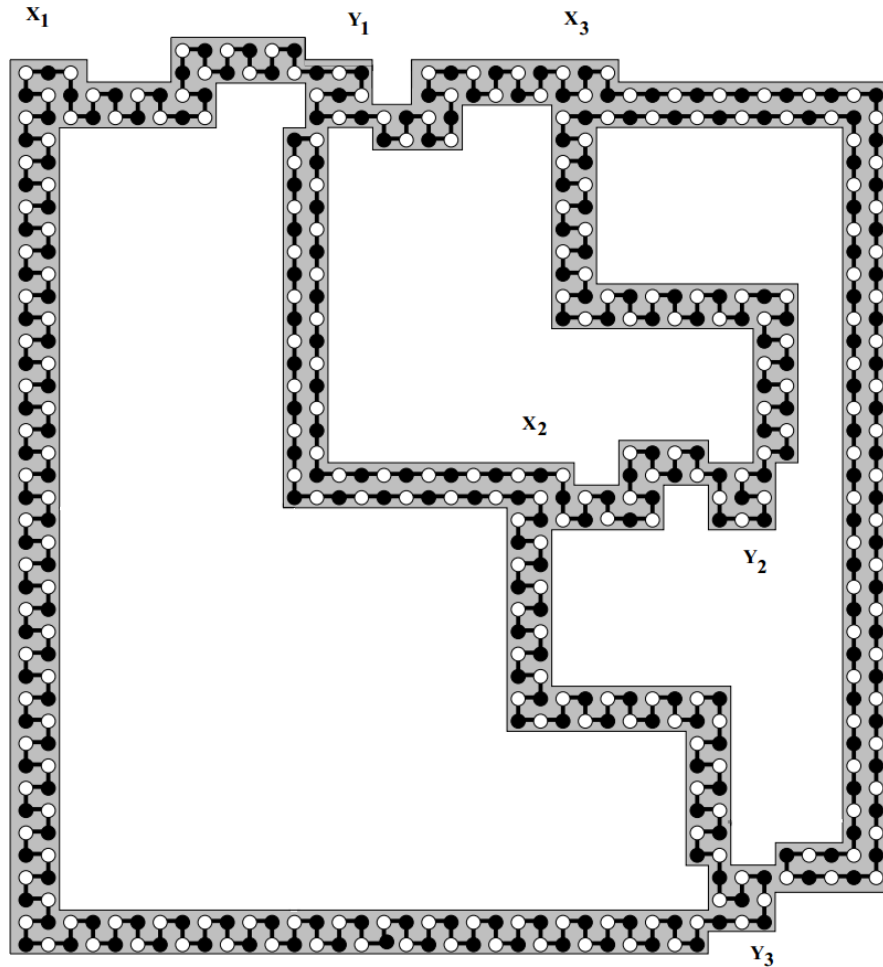


3D Printing



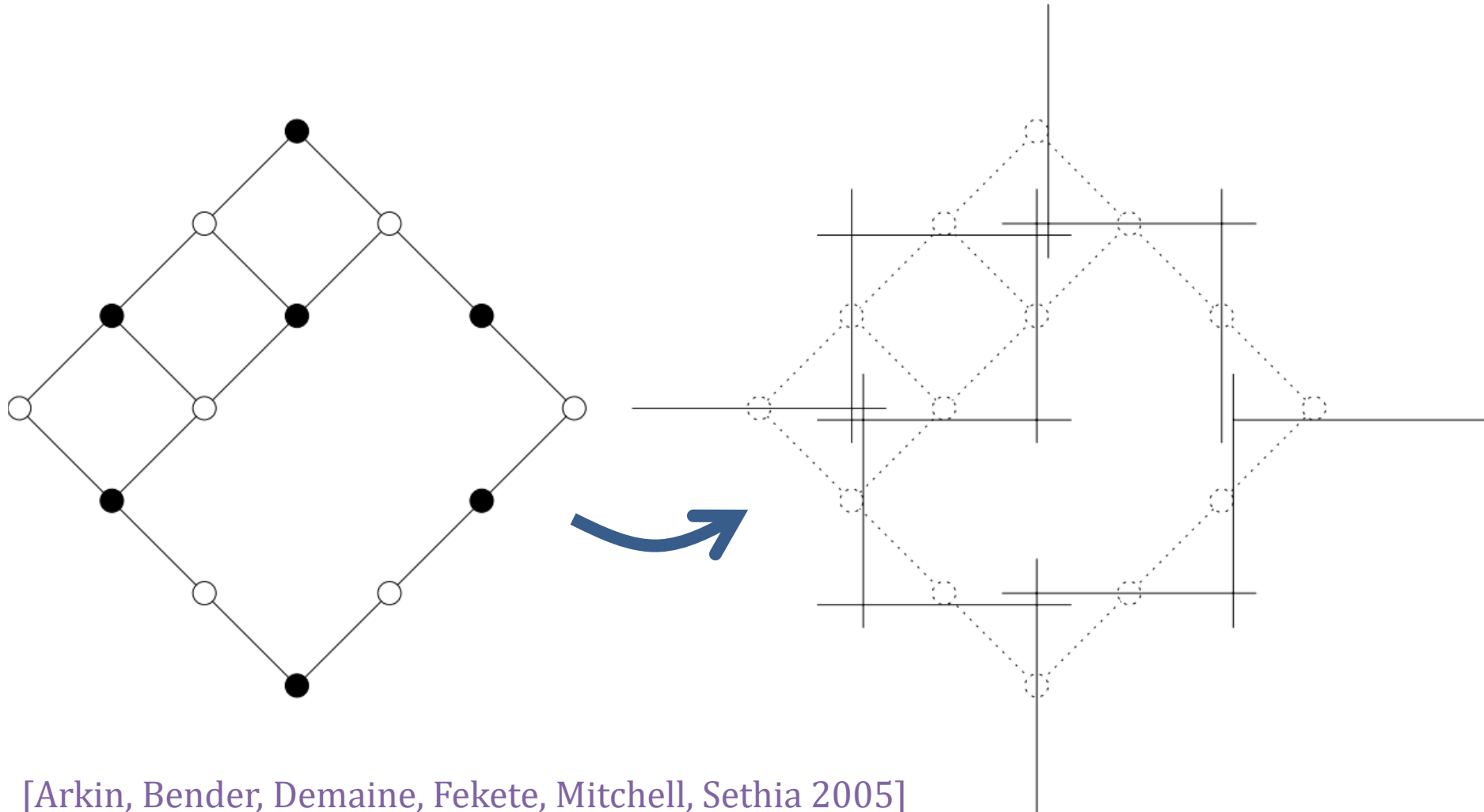
Milling & Lawn Mowing

[Arkin, Fekete, Mitchell 2000]



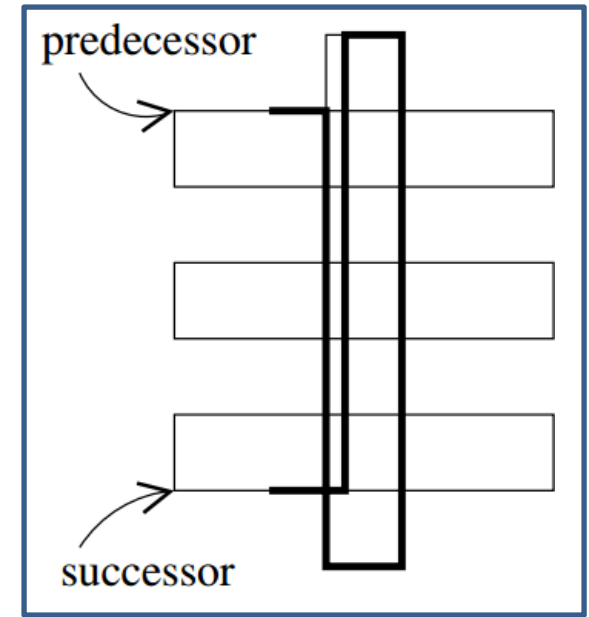
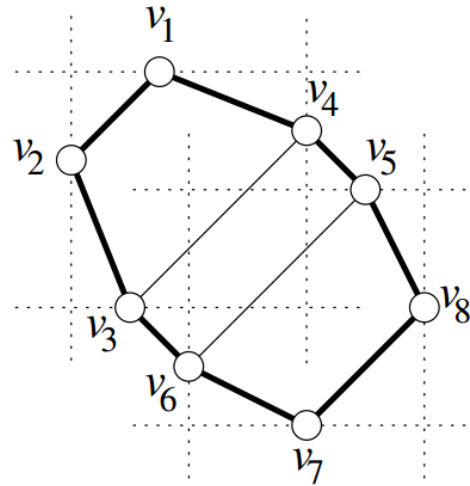
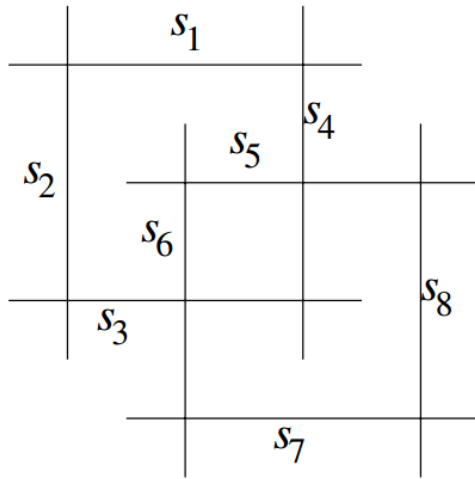


Hamiltonicity in Unit Orthogonal Segment Intersection Graphs



[Arkin, Bender, Demaine, Fekete, Mitchell, Sethia 2005]

Minimum-Turn Milling



[Arkin, Bender, Demaine, Fekete, Mitchell, Sethia 2005]

