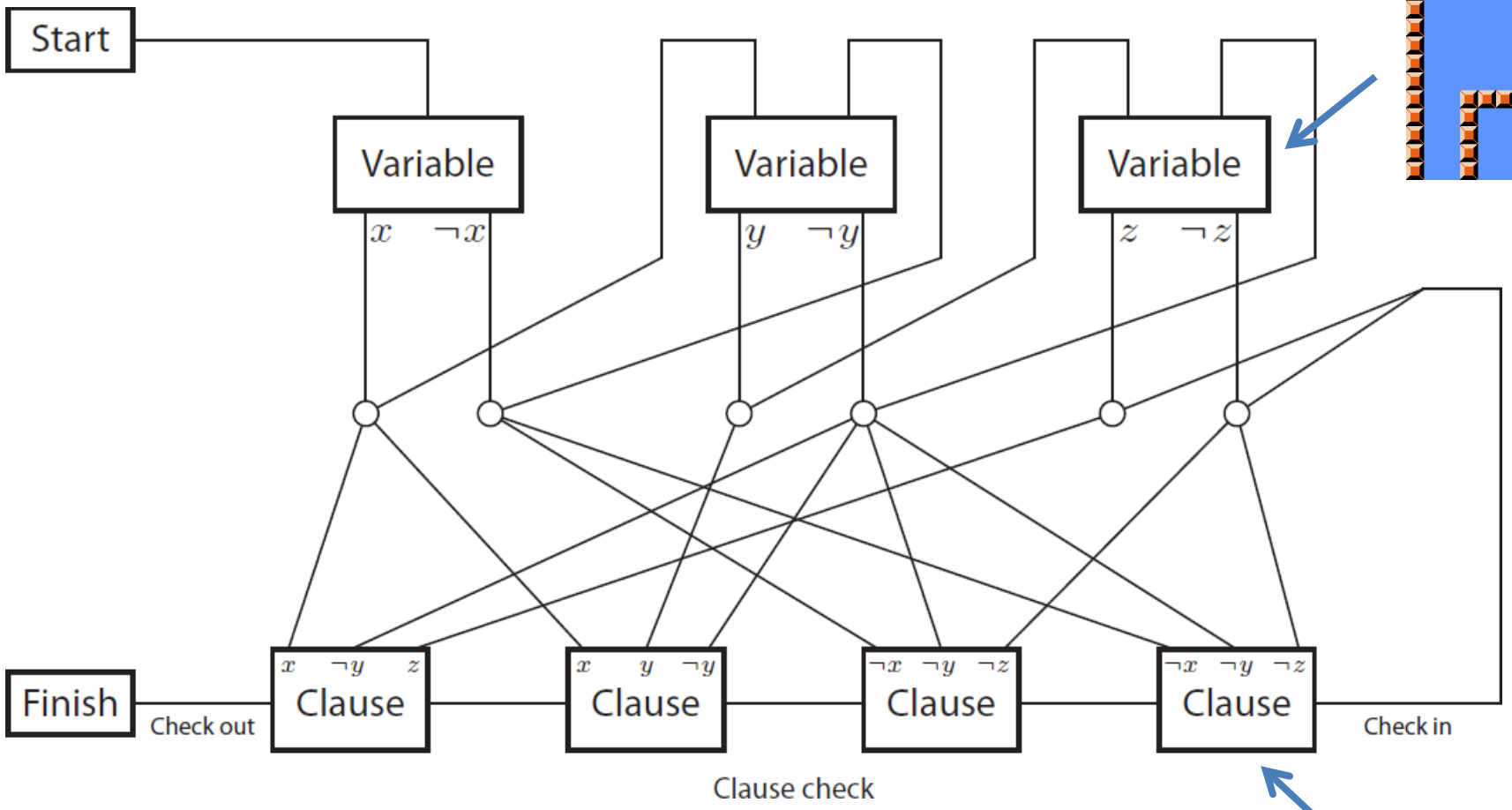
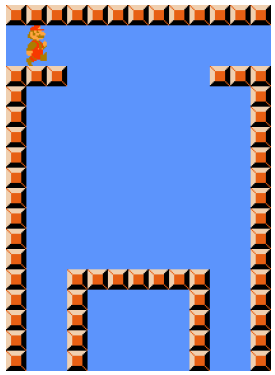
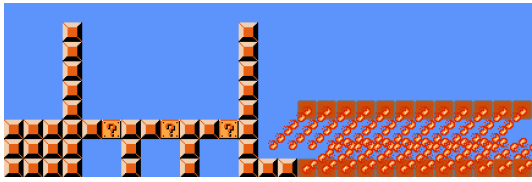
Super Mario Bros. is NP-Hard

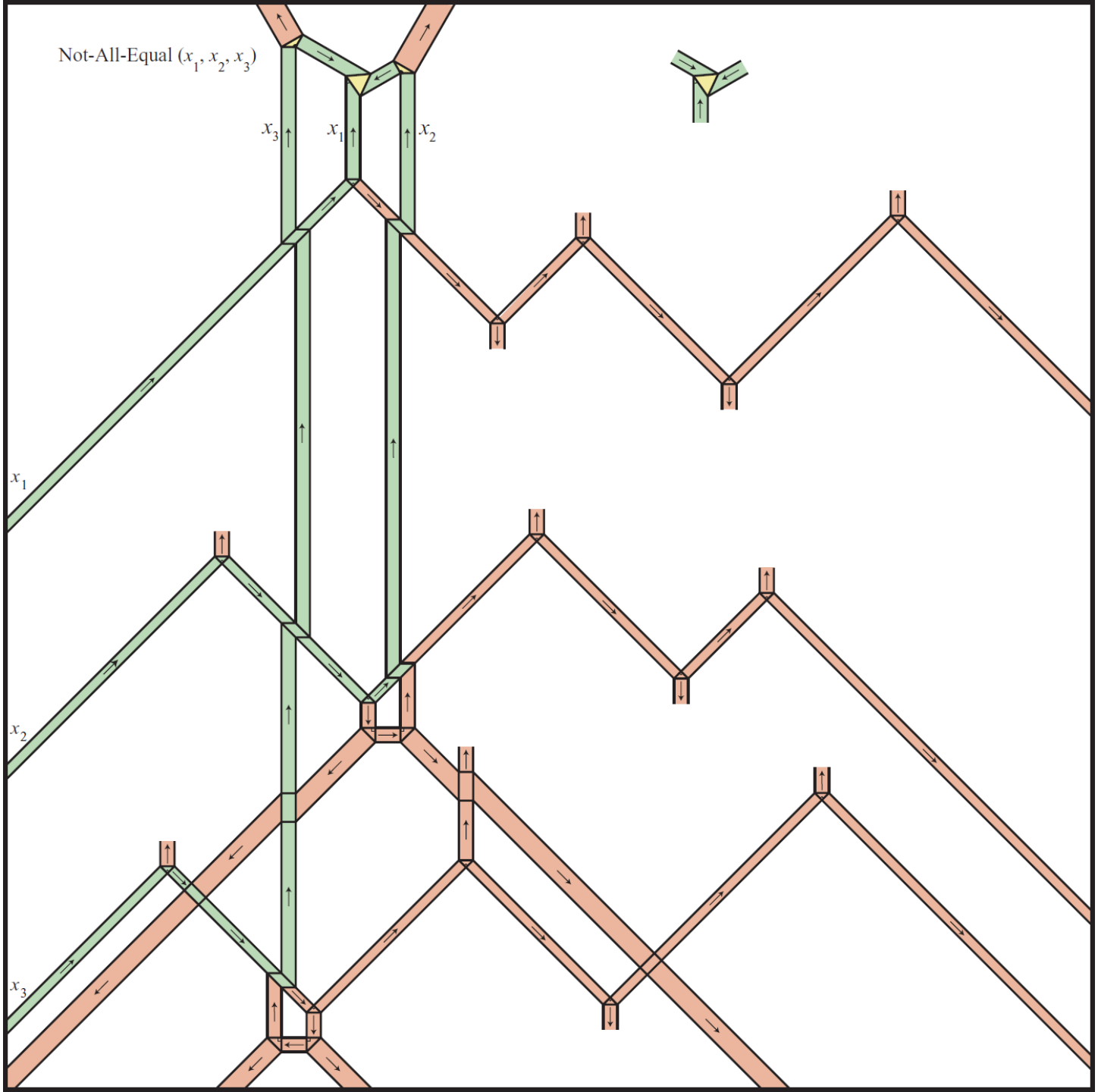
[Aloupis, Demaine, Guo, Viglietta 2014]





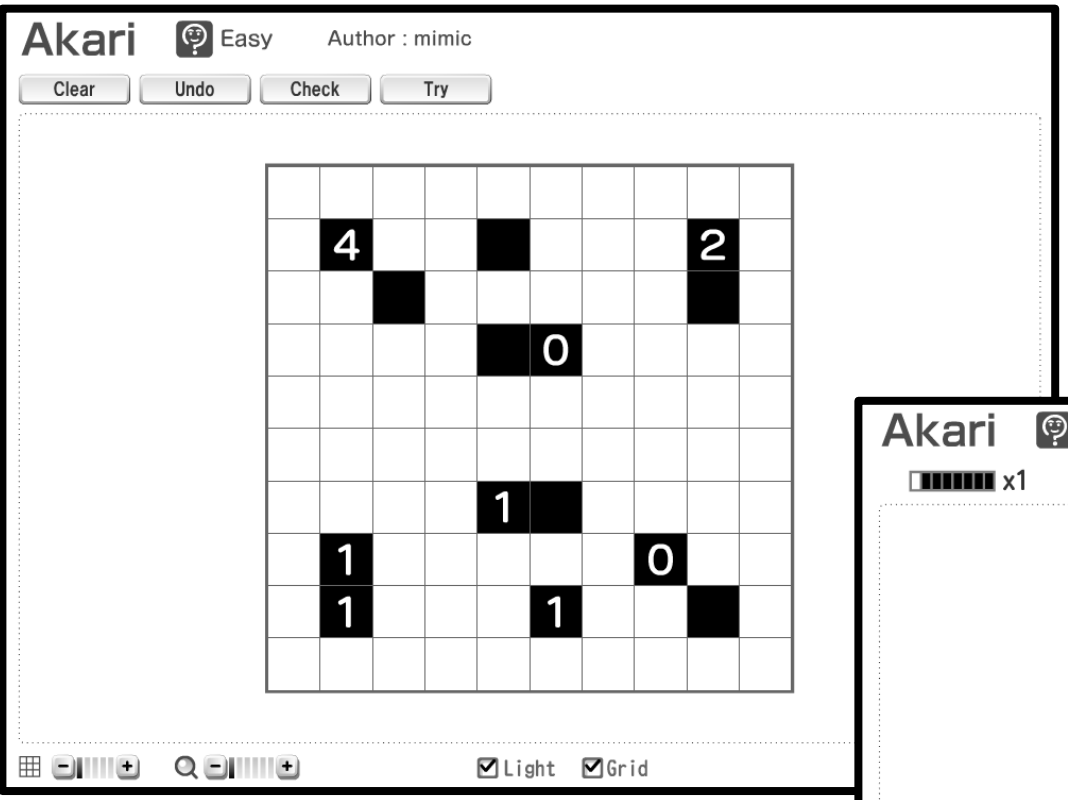


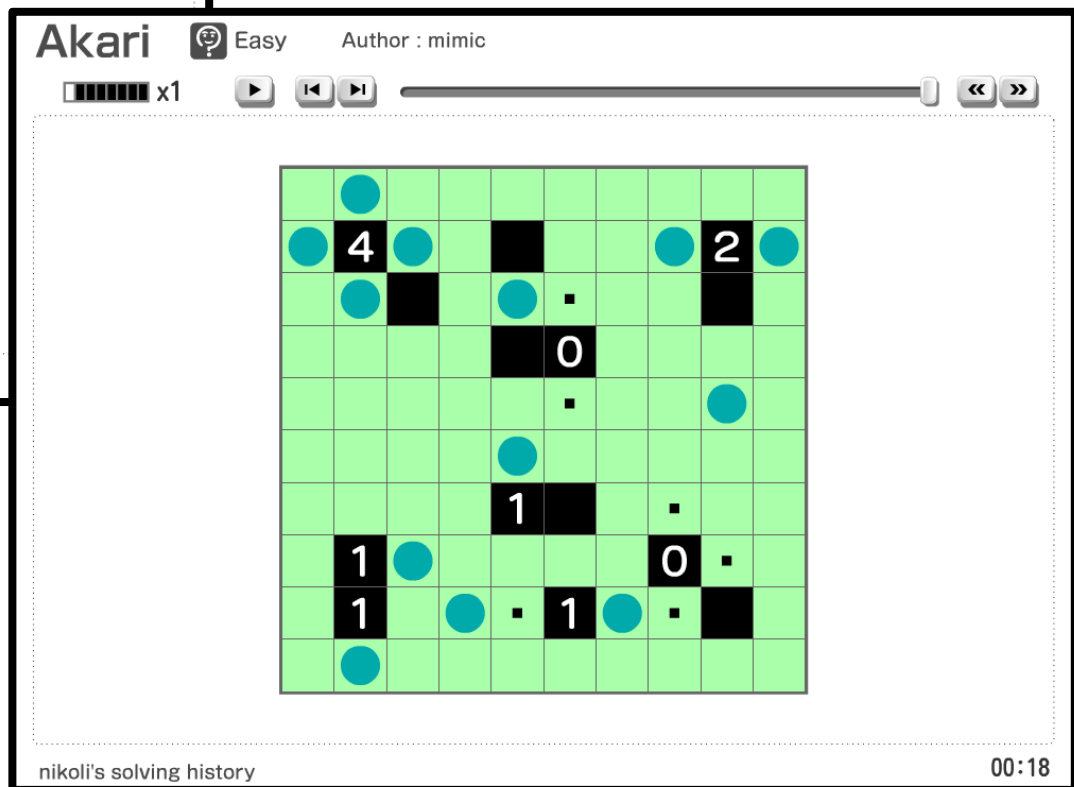
$(x \text{ OR } \neg y \text{ OR } z)$ & $(x \text{ OR } y \text{ OR } \neg y)$ &
$(\neg x \text{ OR } \neg y \text{ OR } \neg z)$ & $(\neg x \text{ OR } \neg y \text{ OR } \neg z)$



[Bern & Hayes 1996]

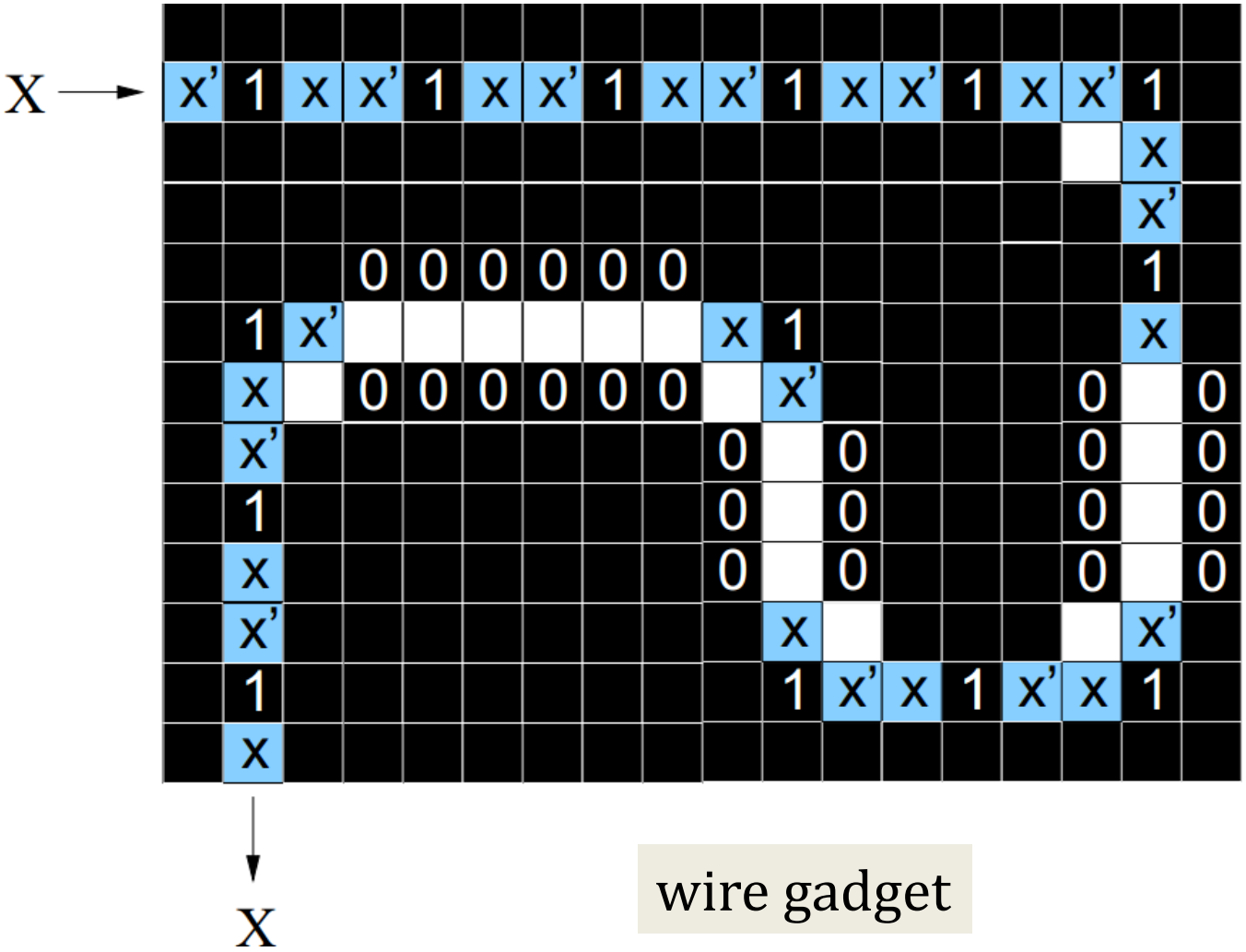
Akari / Light Up [Nikoli 2001]





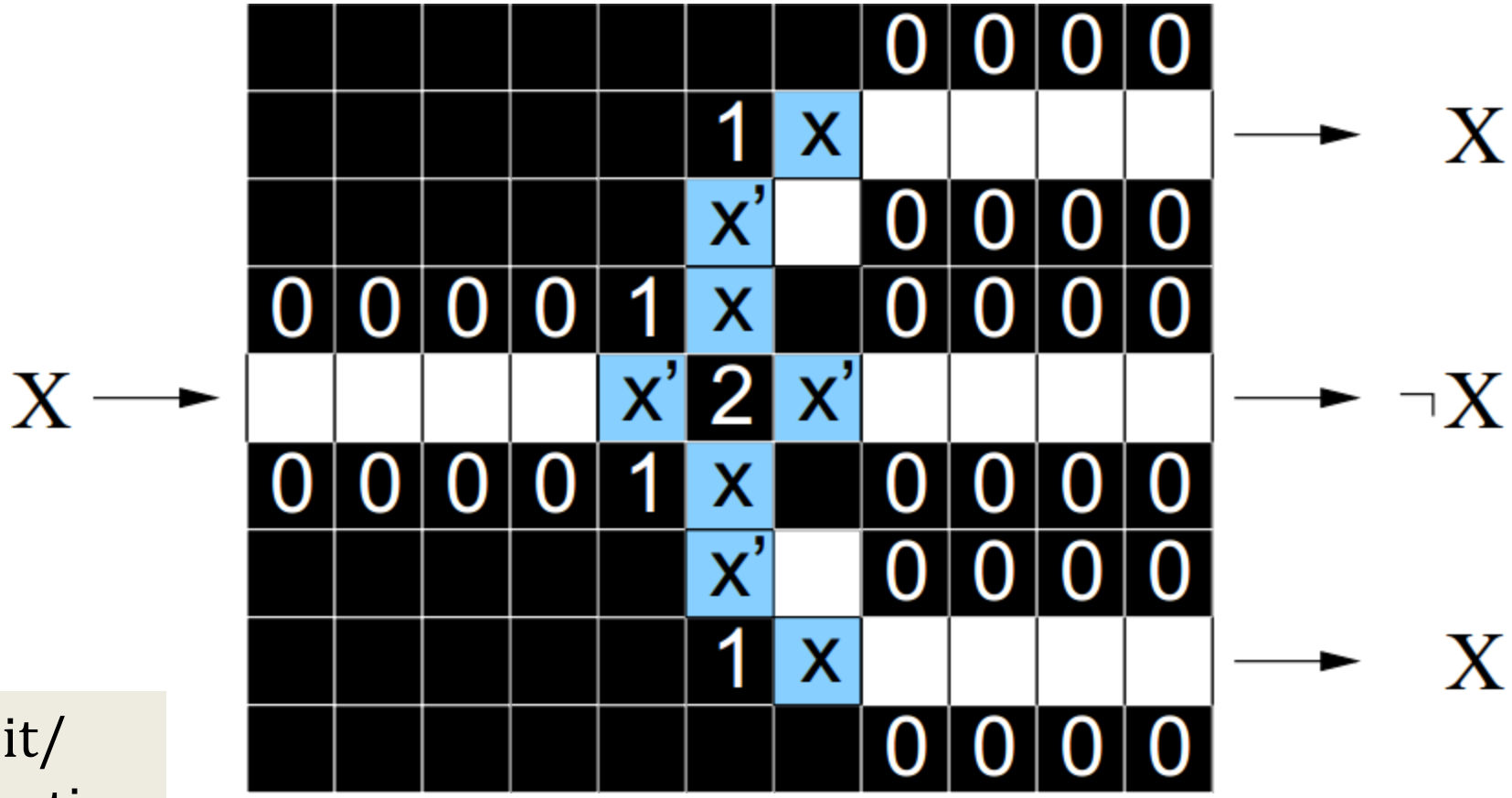
Akari / Light Up is NP-complete

[McPhail 2005]



Akari / Light Up is NP-complete

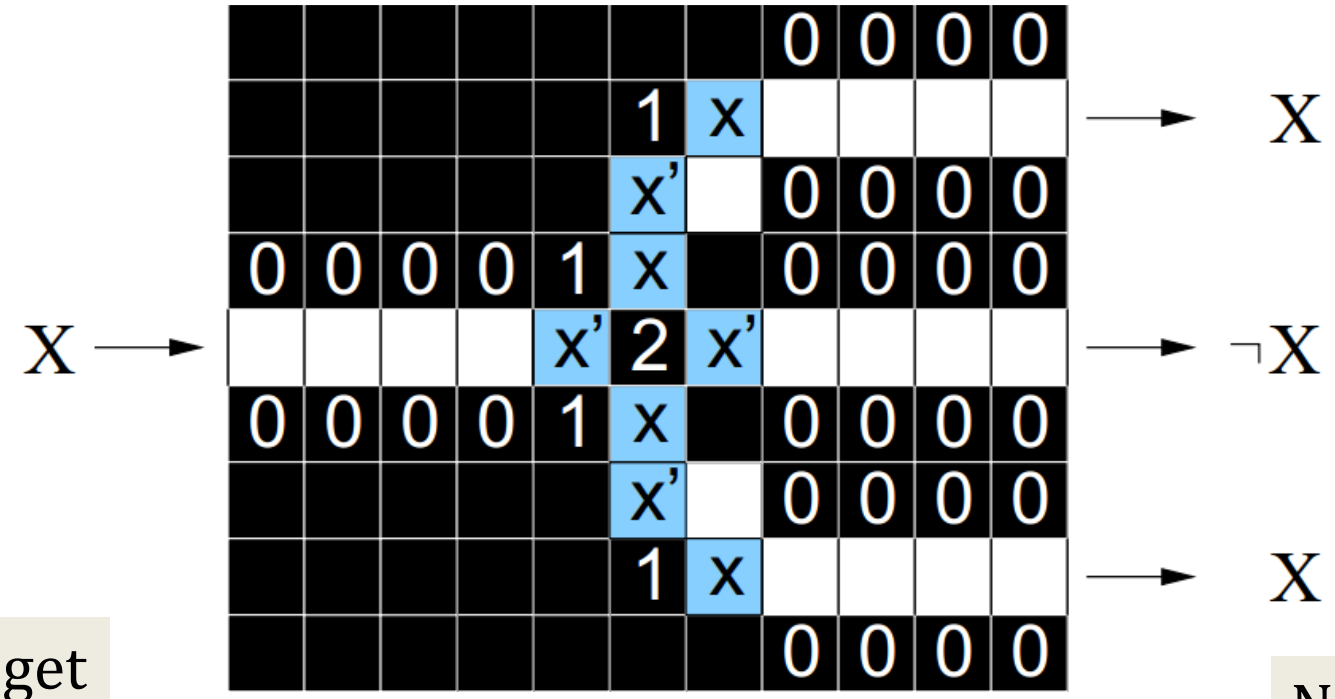
[McPhail 2005]



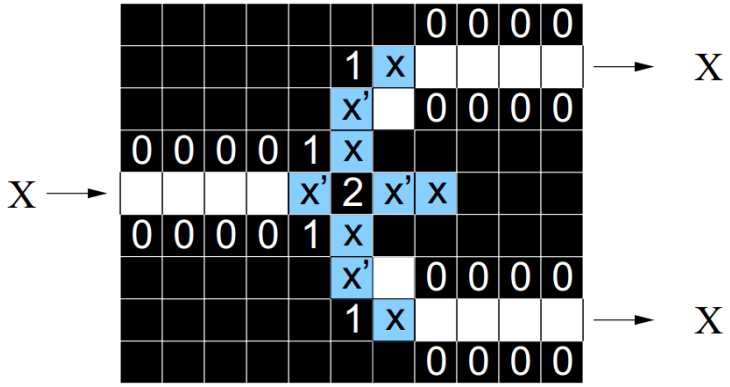
split/
negation
gadget

Akari / Light Up is NP-complete

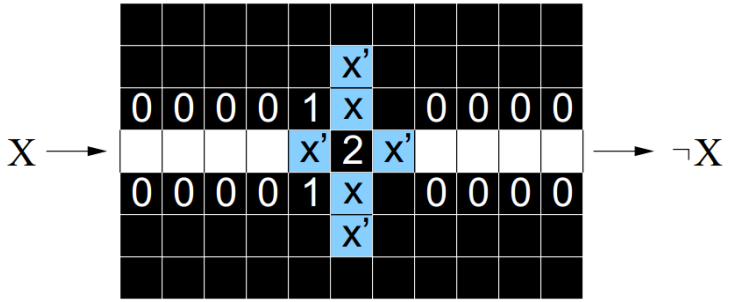
[McPhail 2005]



split gadget

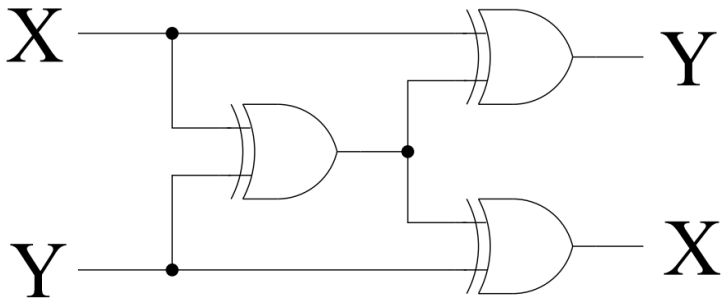


NOT gate

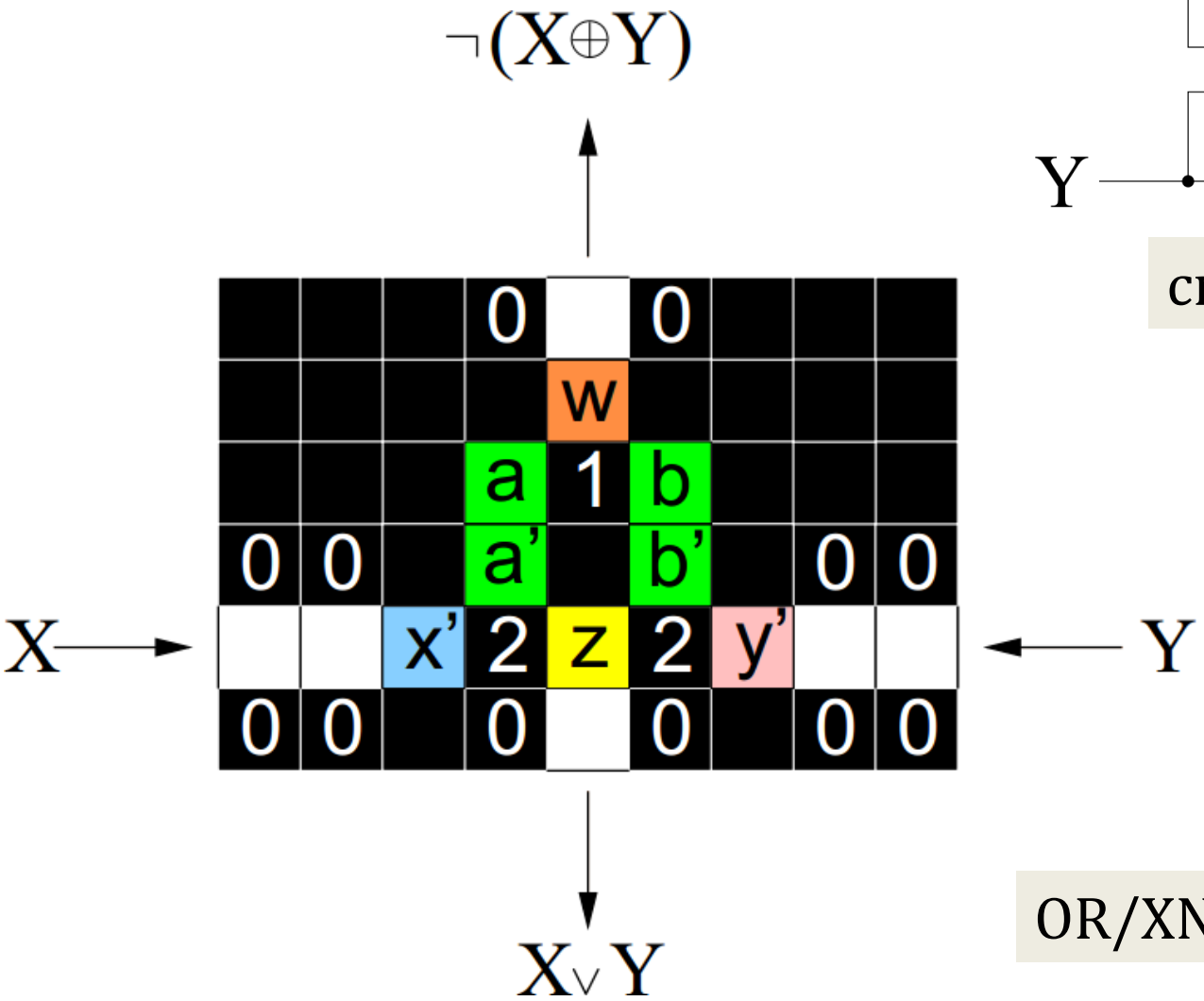


Akari / Light Up is NP-complete

[McPhail 2005]



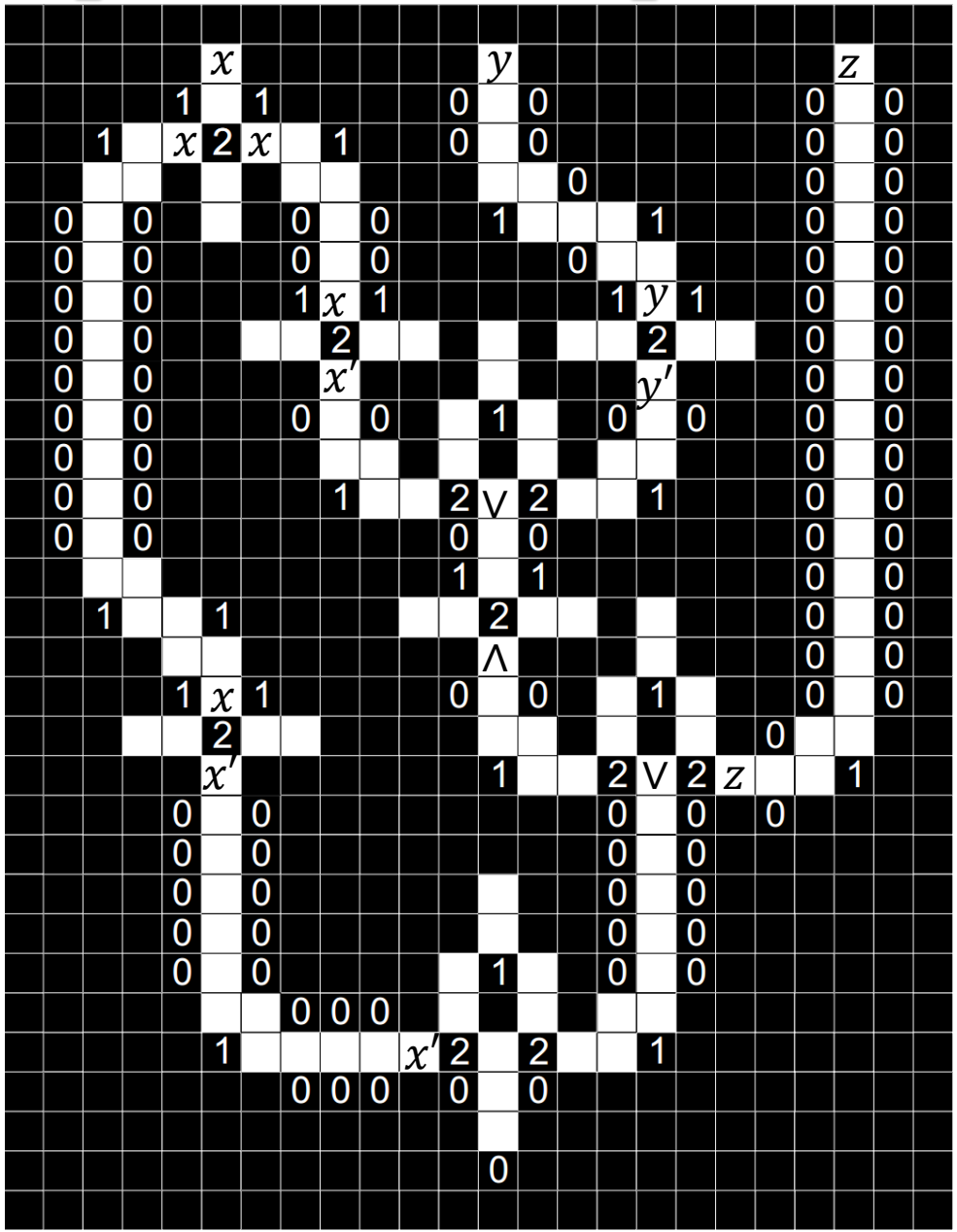
crossover gadget



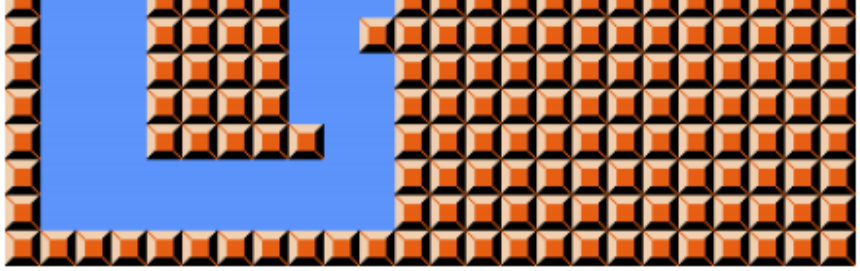
OR/XNOR gate

Akari / Light Up is NP-complete

[McPhail 2005]



$$\neg x \vee \big((x \wedge y) \vee z\big)$$



Crossover gadget for NP-hardness

AND gadget for PSPACE-hardness

Minesweeper

OR gadget for NP-hardness

Hardness Made Easy*

Learn **when to give up** the search for efficient algorithms; see **connections** between computational problems; **solve puzzles** to prove theorems, solve **open problems,** and write papers.

Topics: NP, PSPACE, EXPTIME, EXPTIME, EXPSPACE, 3SUM, approximation, fixed parameter, games & puzzles, 3SUM, key problems, gadgets, and proof styles.

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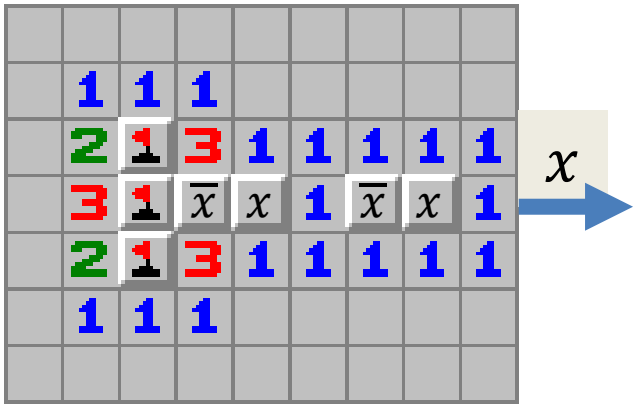
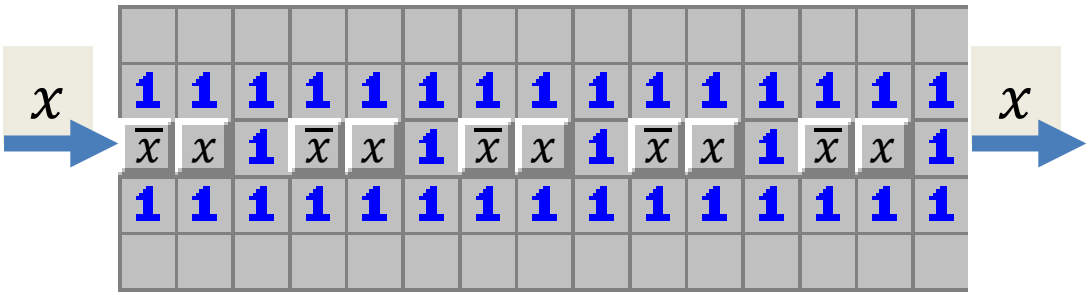
Minesweeper Consistency is NP-complete [Kaye 2000]

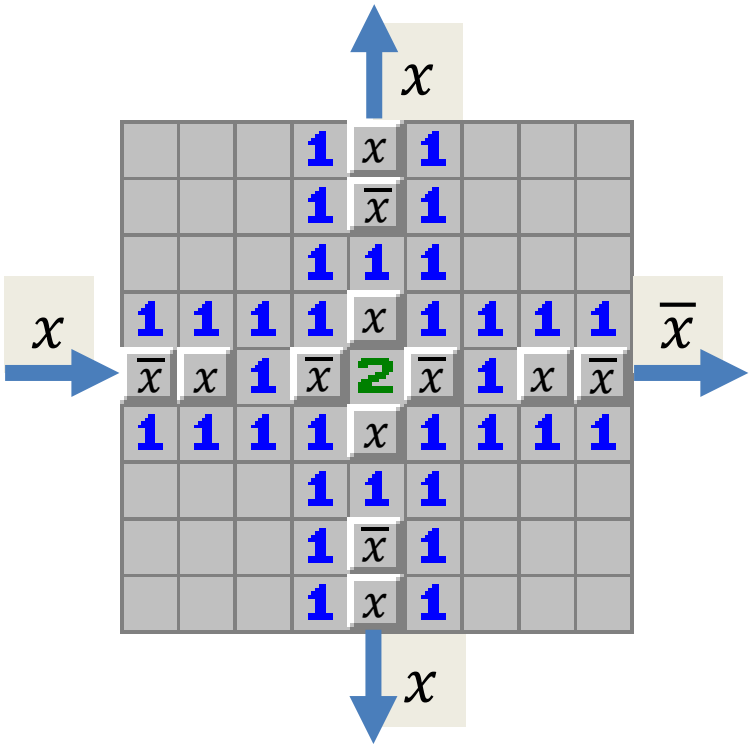


wire terminator

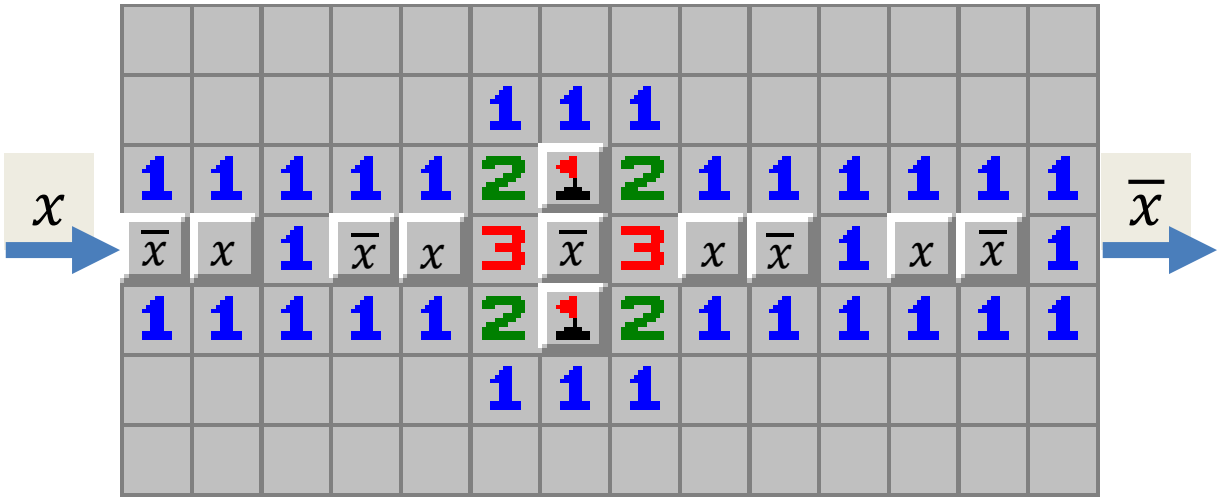
wire



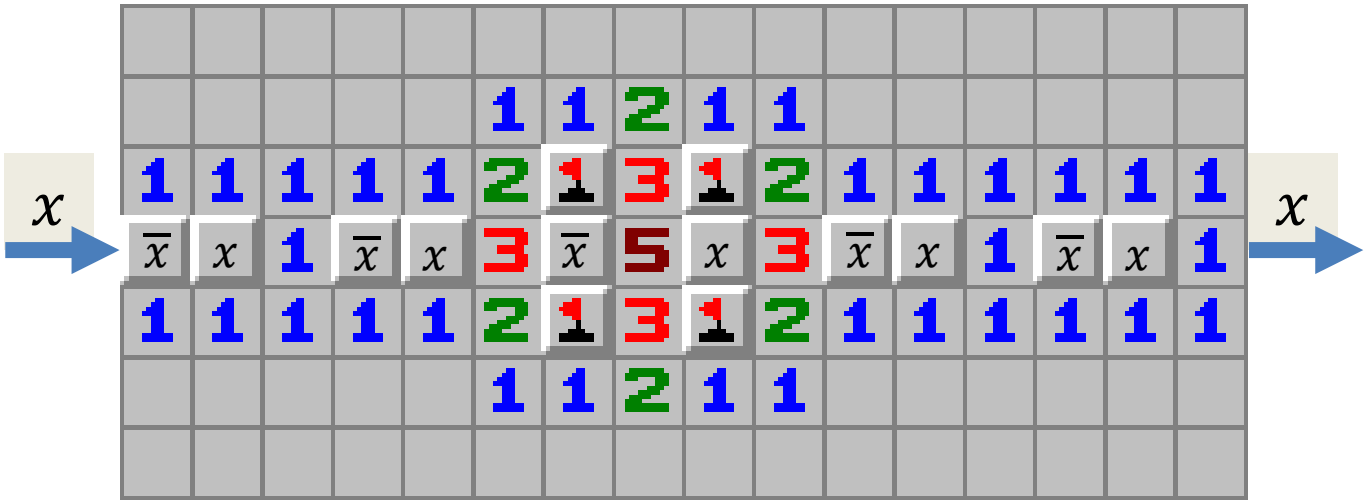
split, NOT, turn



Minesweeper Consistency is NP-complete [Kaye 2000]

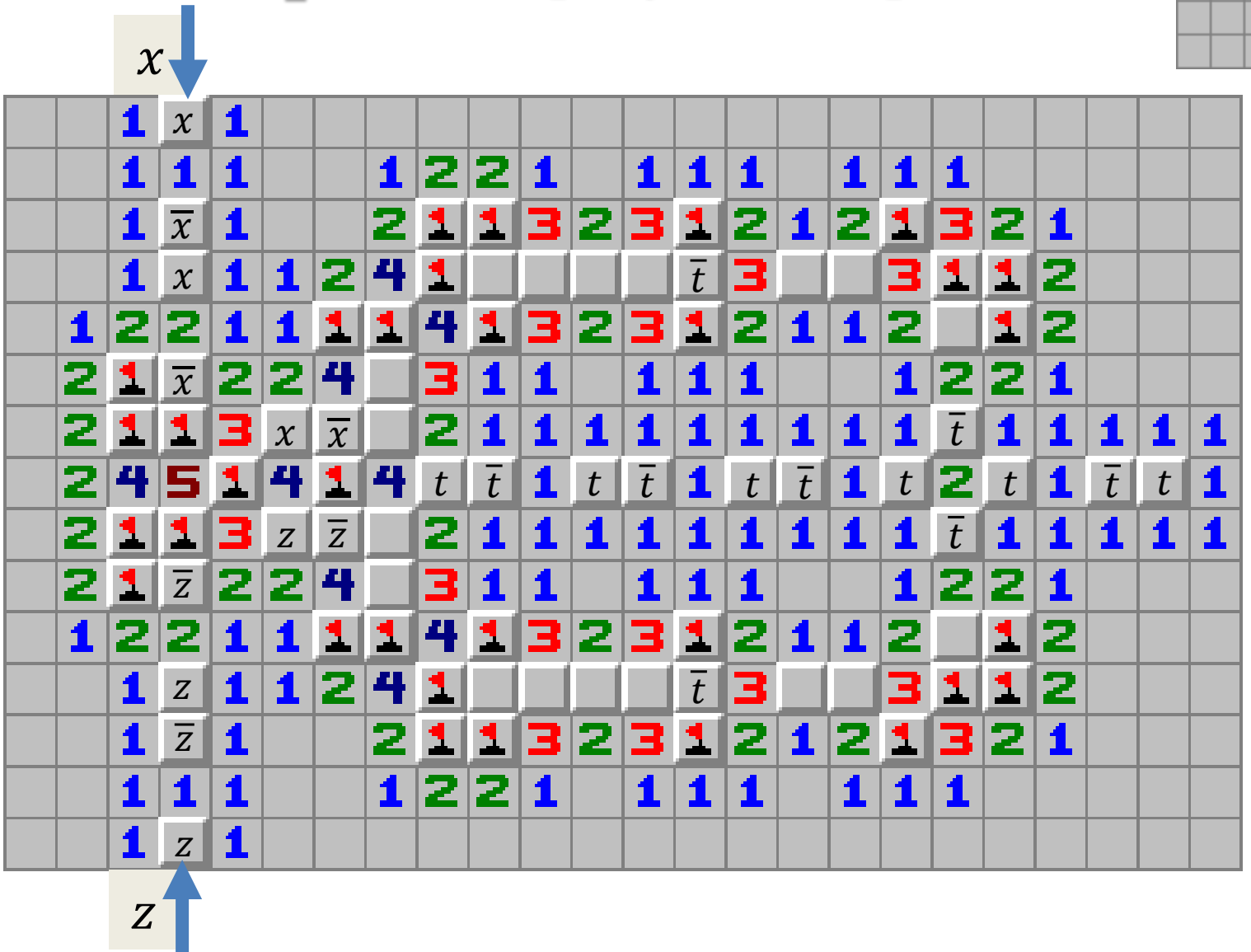


NOT



Phase changer (2 NOTs)

Minesweeper Consistency is NP-complete [Kaye 2000]



turn

AND

Minesweeper Consistency is NP-complete [Kaye 2000]

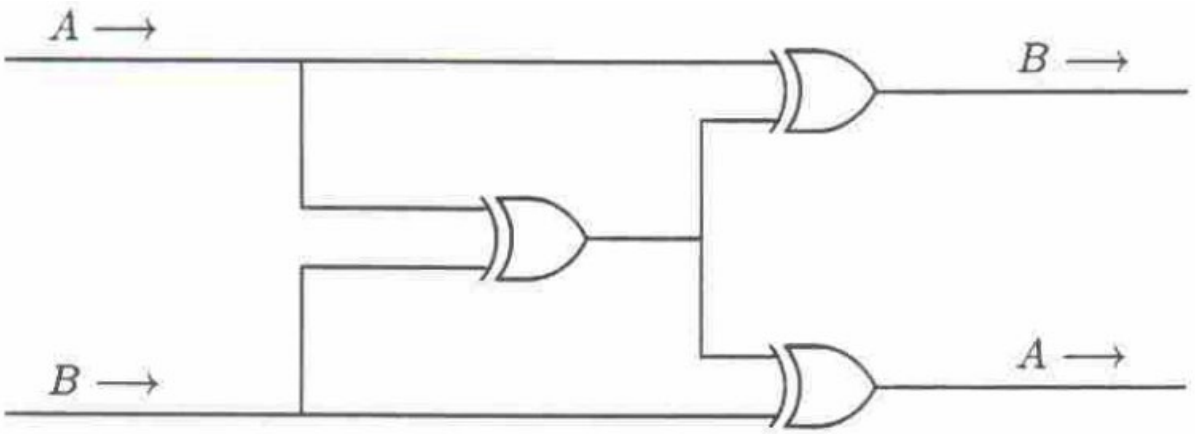


Figure 11. Crossing two wires with three XOR gates.

crossover

[Goldschlager 1977]

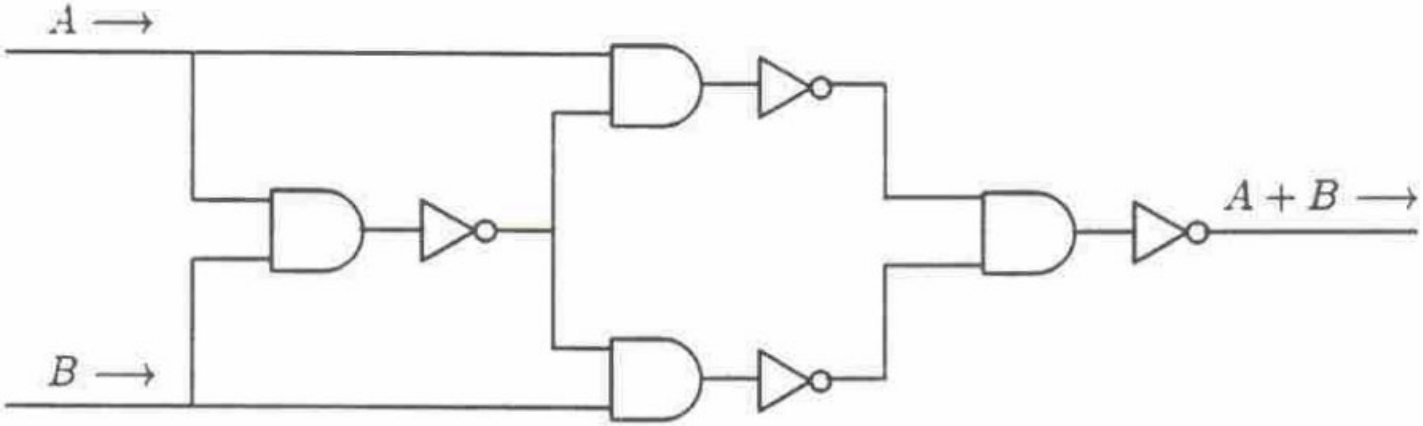
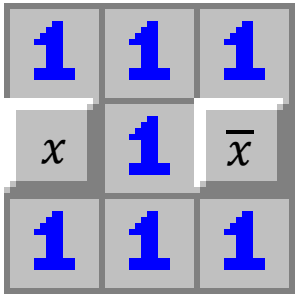


Figure 12. Making an XOR gate with AND and NOT gates.

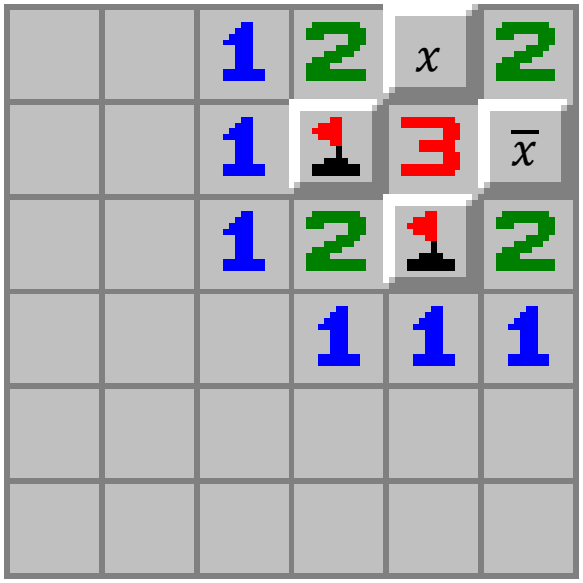
XOR

Minesweeper is CoNP-Complete

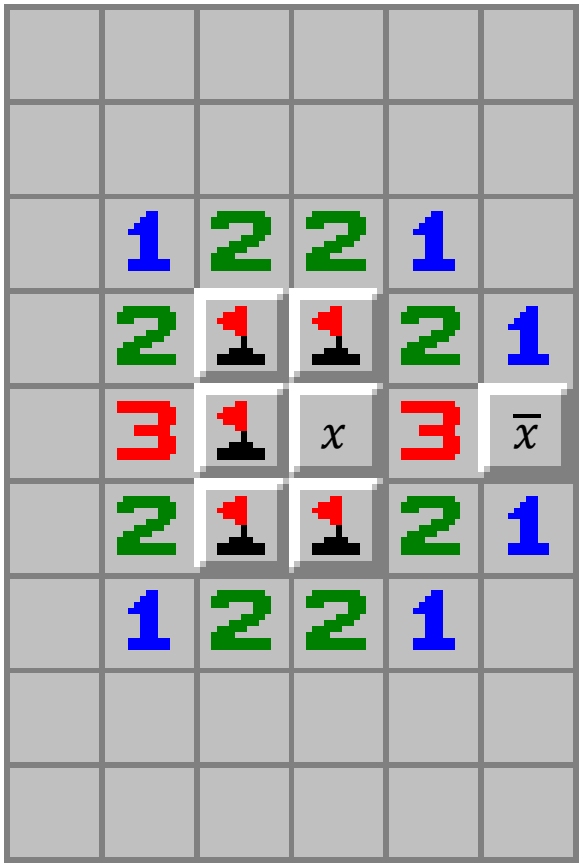
[Scott, Stege, van Rooij 2011]



wire gadget



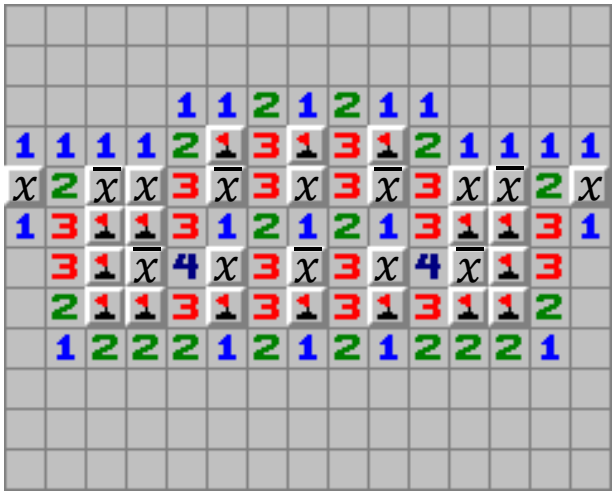
turn gadget



terminator gadget

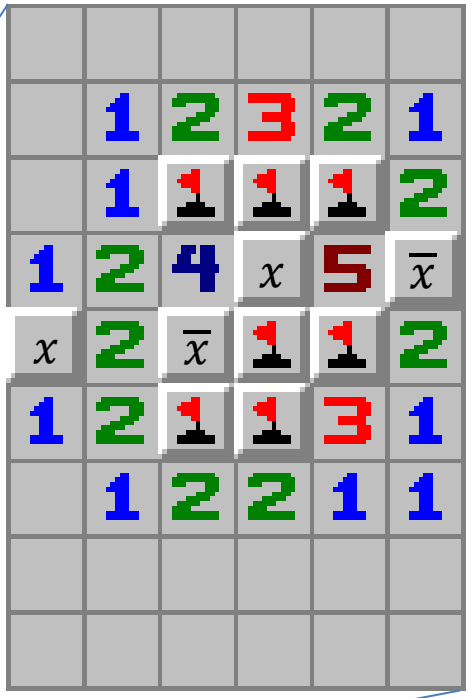
Minesweeper is CoNP-Complete

[Scott, Stege, van Rooij 2011]

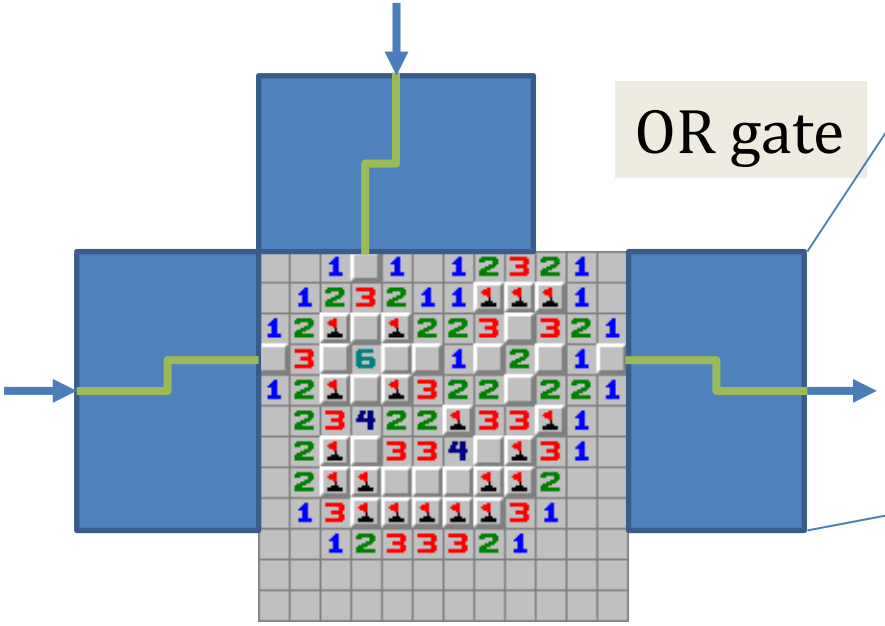


NOT gate

shifter gadget



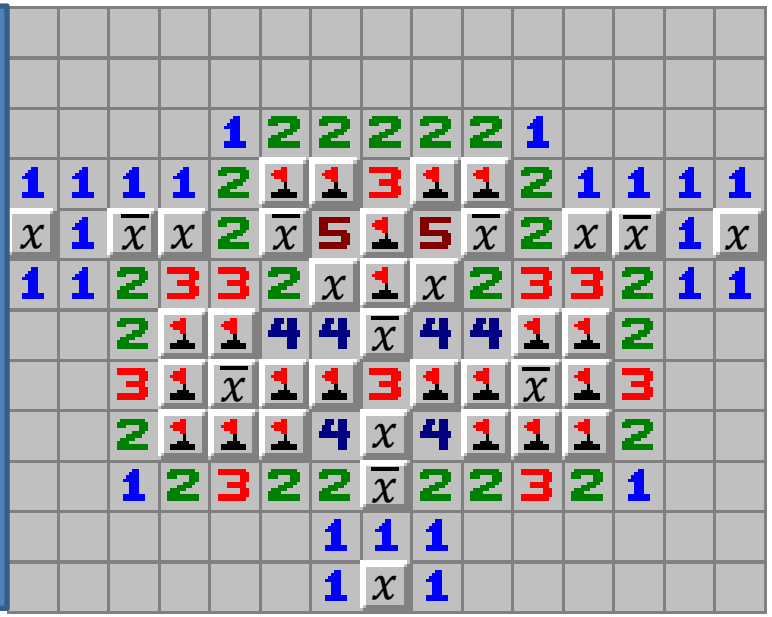
OR gate



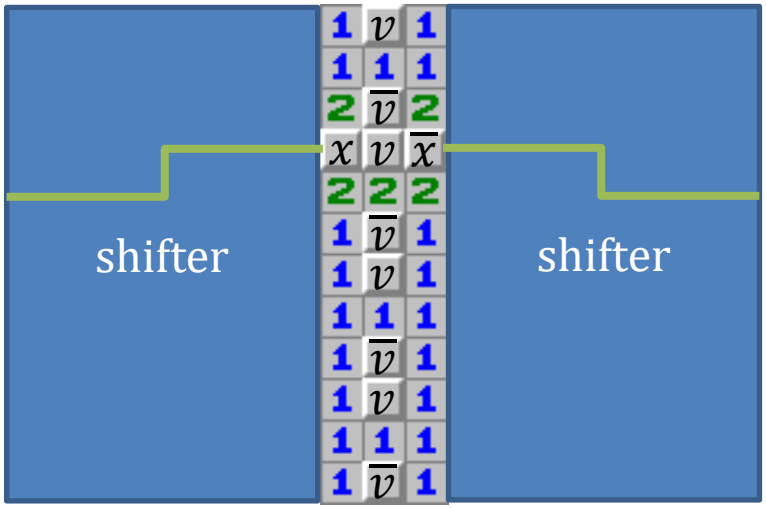
Minesweeper is CoNP-Complete

[Scott, Stege, van Rooij 2011]

NOT



split gadget



crossover gadget

Minesweeper is CoNP-Complete

[Scott, Stege, van Rooij 2011]

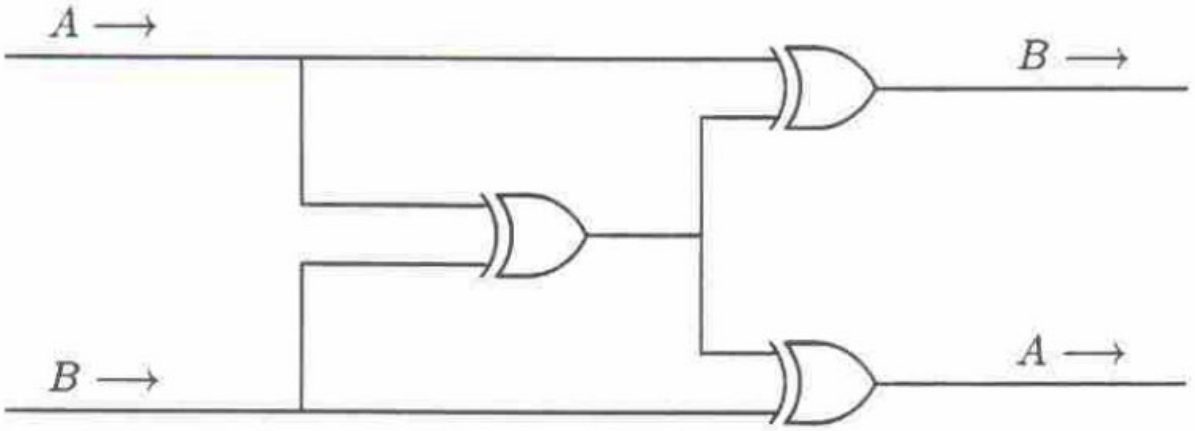


Figure 11. Crossing two wires with three XOR gates.

alternative crossover gadget

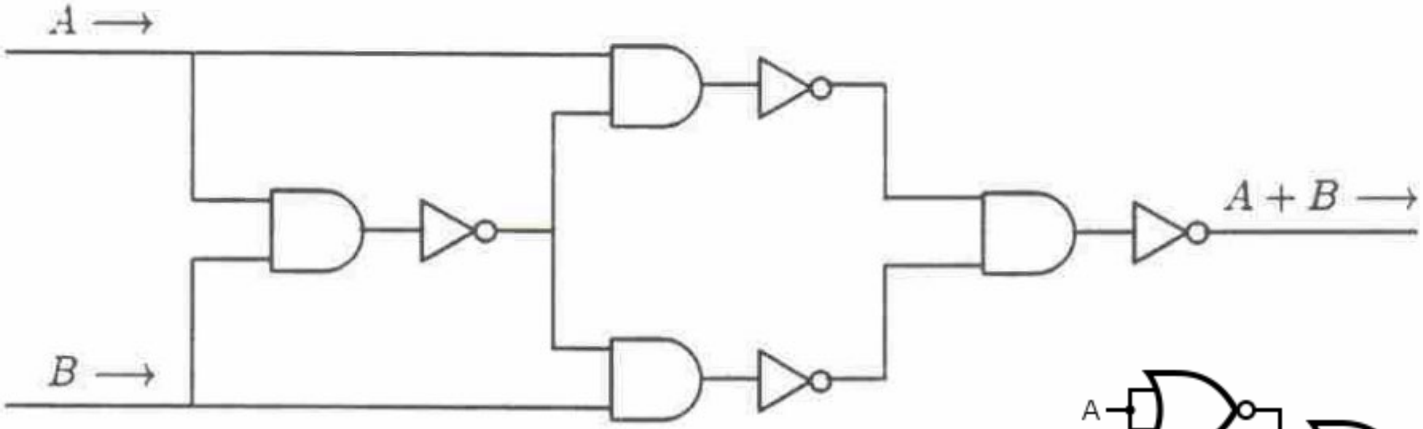
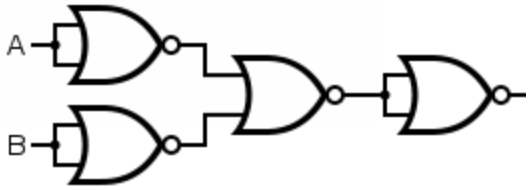


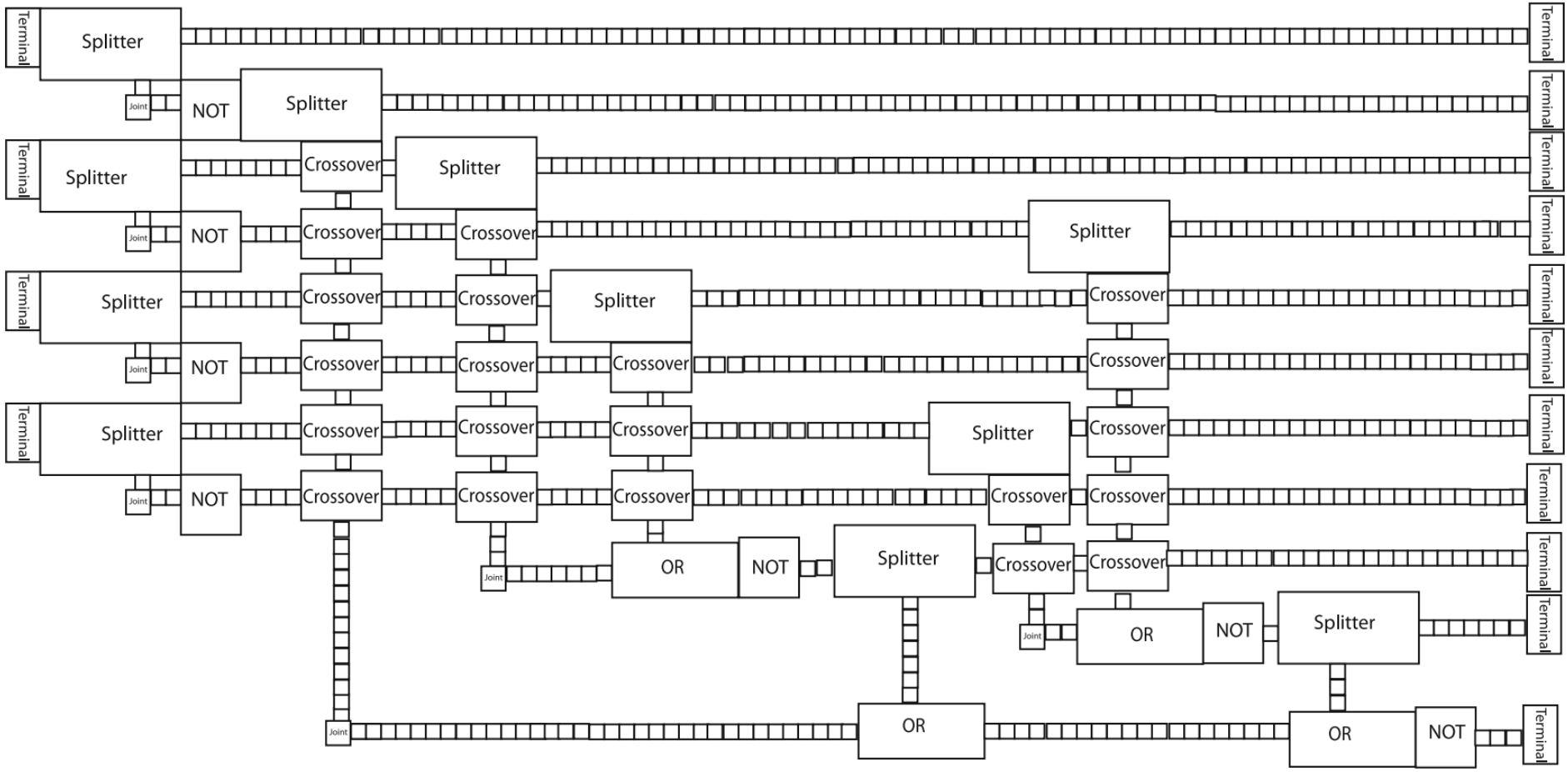
Figure 12. Making an XOR gate with AND and NOT gates.



NAND

Minesweeper is CoNP-Complete

[Scott, Stege, van Rooij 2011]



Functionally Complete Logic Gates

The following are the minimal functionally complete sets of logical connectives with arity ≤ 2:[9] [Wernick 1942]

One element

{NAND}, {NOR}.

Two elements

{∨, ¬}, {∧, ¬}, {→, ¬}, {←, ¬}, {→, ⊥}, {←, ⊥}, {→, ↚}, {←, ↚}, {→, ↛}, {→, ↚}, {←, ↛}, {←, ↚}, {↛, ¬}, {↚, ¬}, {↛, ⊤}, {↚, ⊤}, {↛, ↔}, {↚, ↔}.

Three elements

{∨, ↔, ⊥}, {∨, ↔, ↚}, {∨, ↚, ⊤}, {∧, ↔, ⊥}, {∧, ↔, ↚}, {∧, ↚, ⊤}.

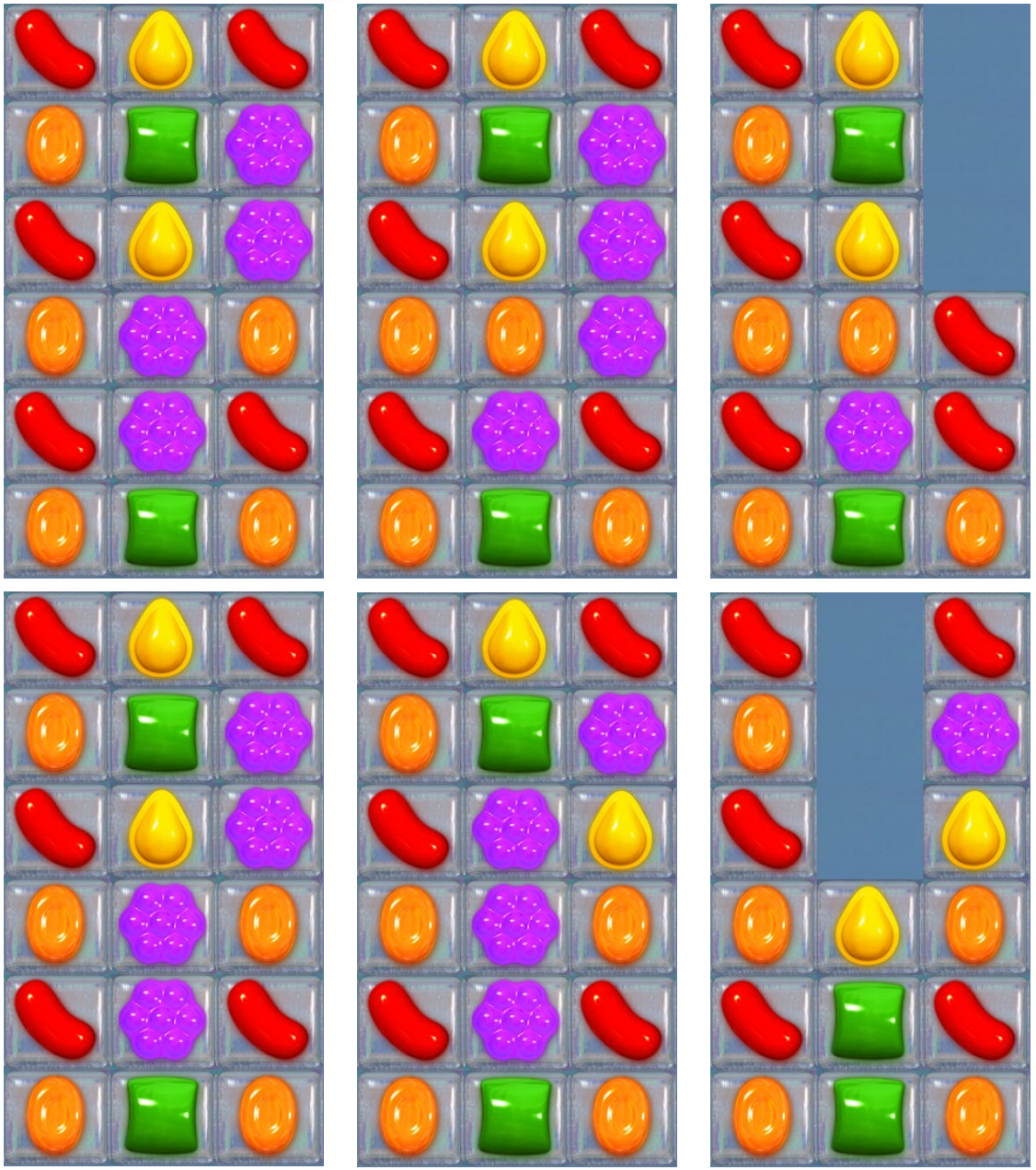
Candy Crush is NP-complete

[Walsh 2014]

true

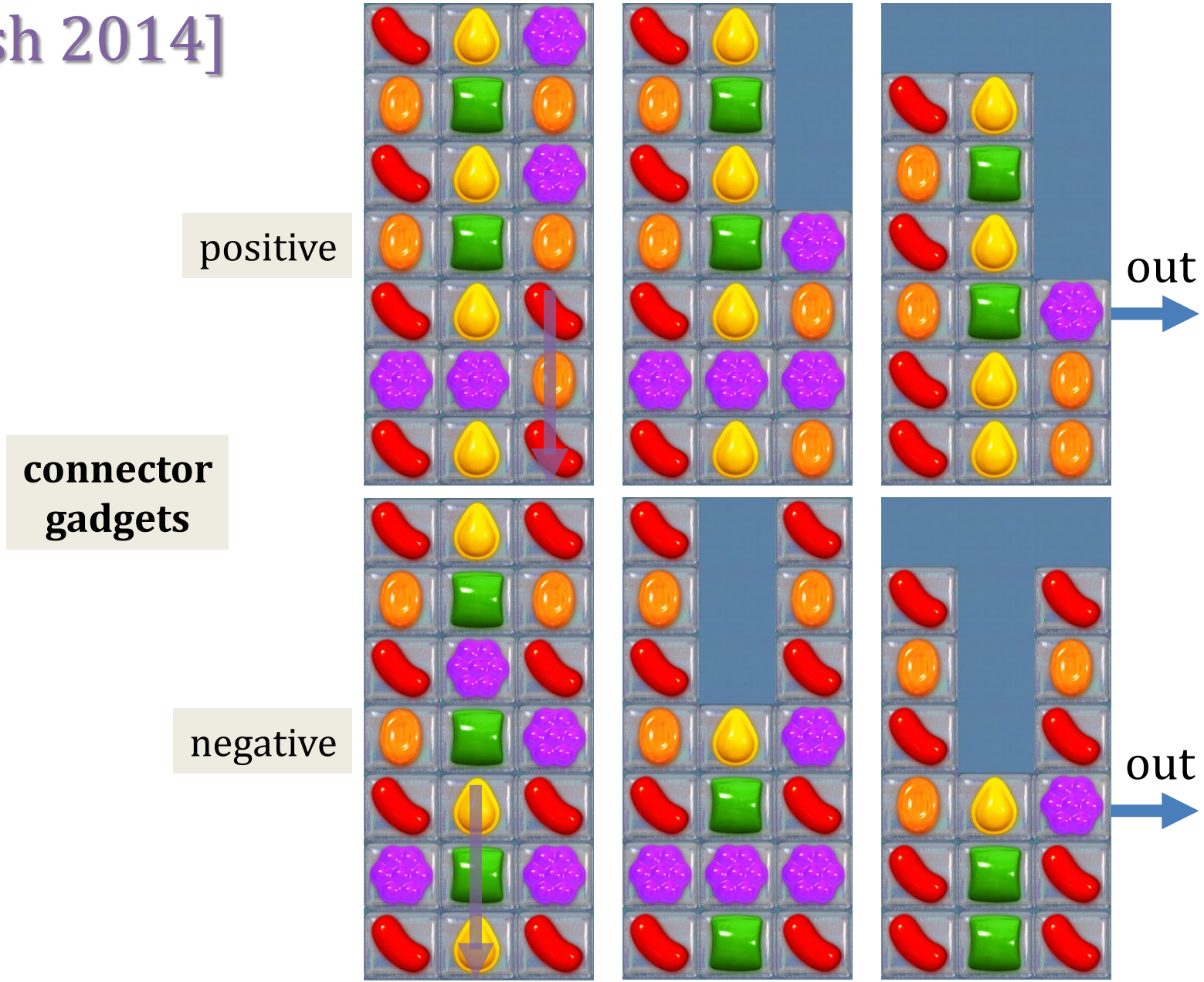
variable gadget

false



Candy Crush is NP-complete

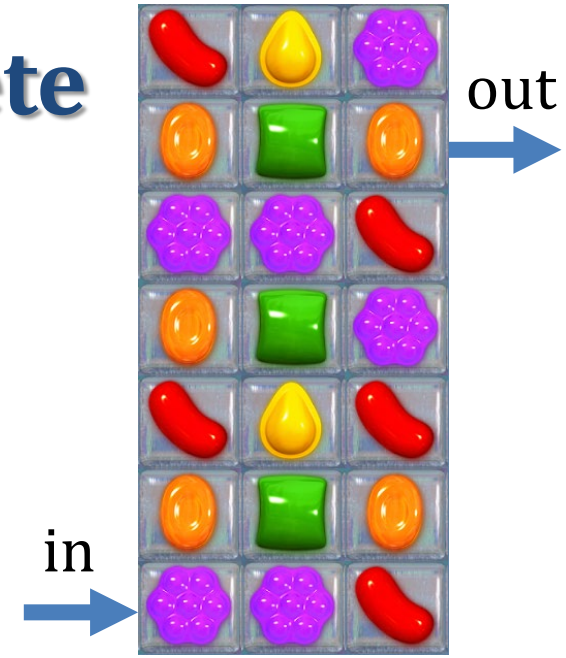
[Walsh 2014]

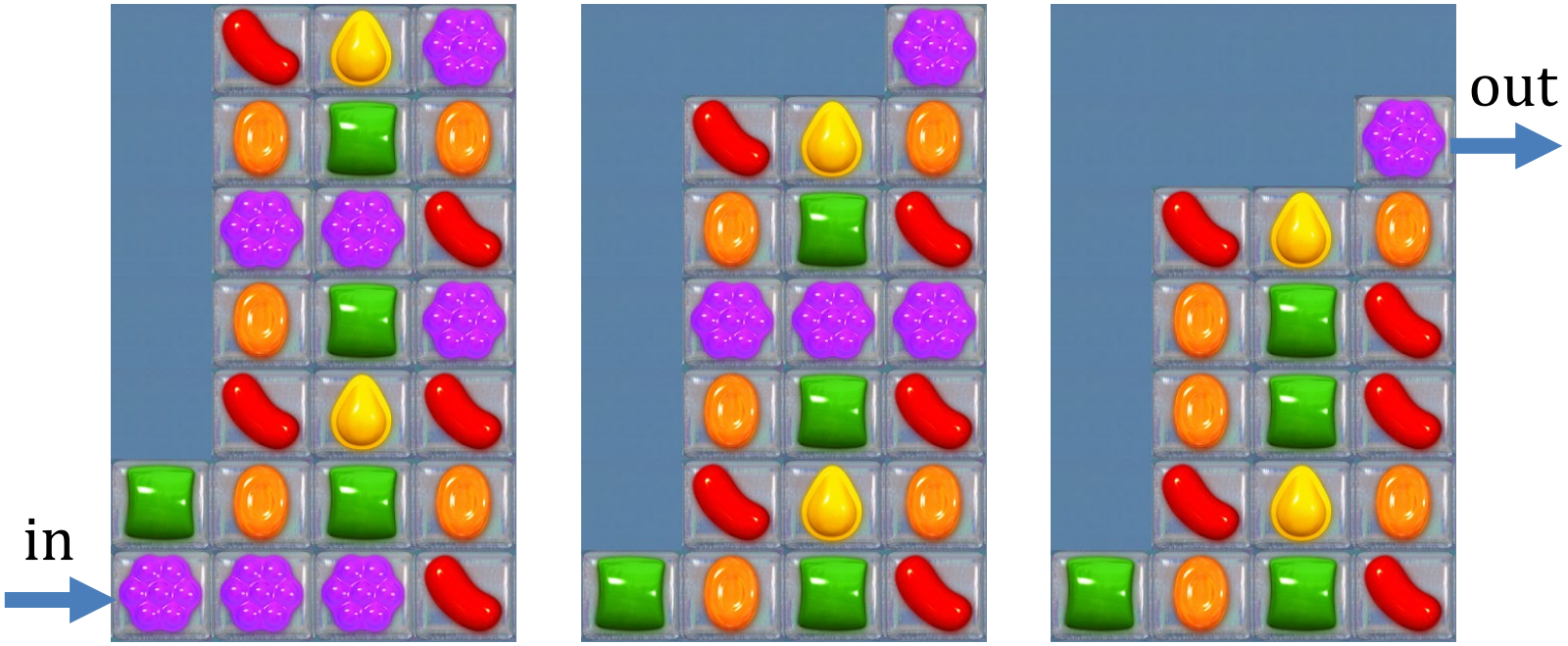


Candy Crush is NP-complete

[Walsh 2014]

wire gadget

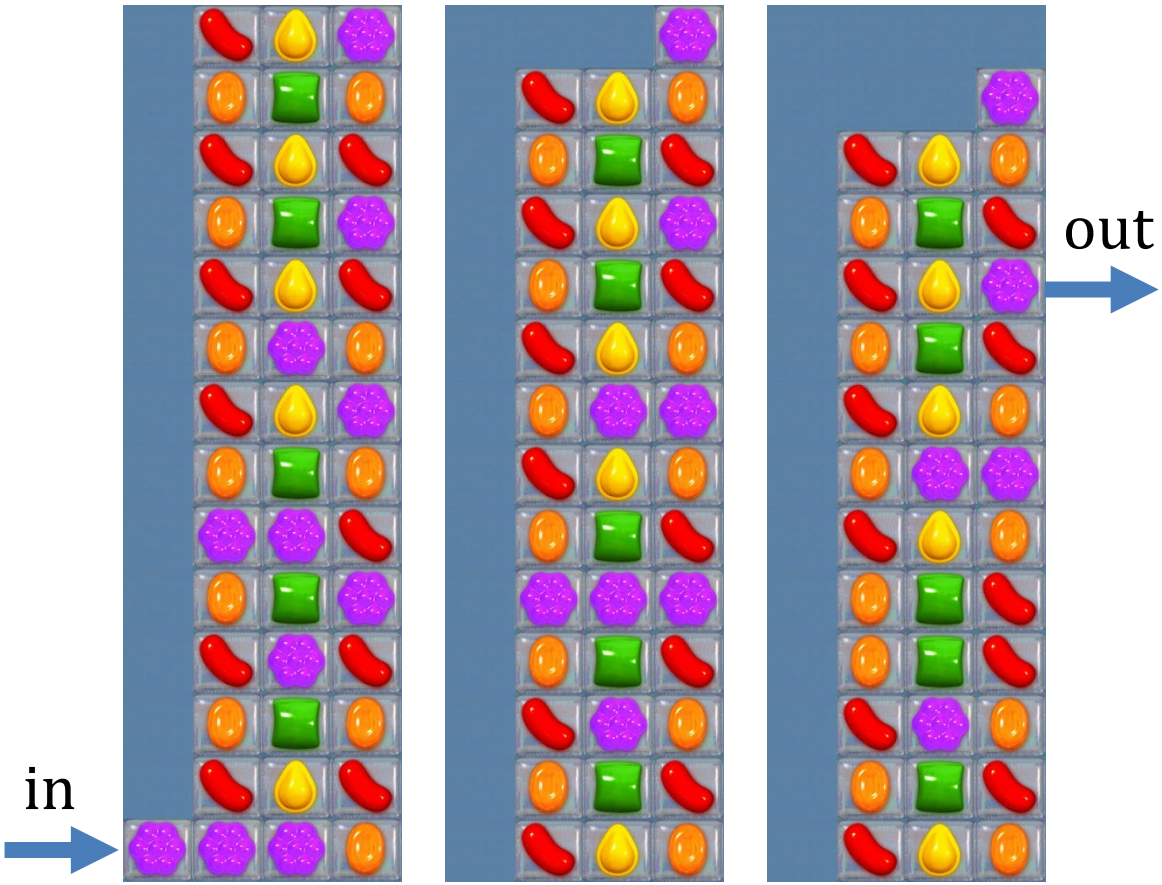


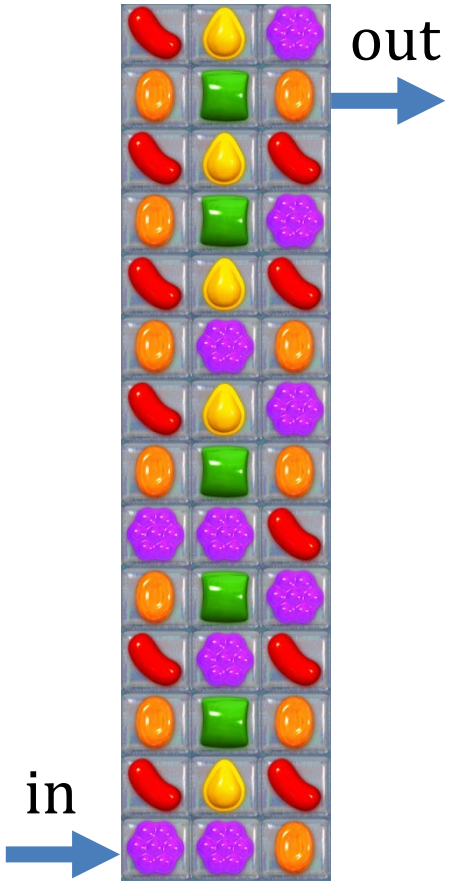


Candy Crush is NP-complete

[Walsh 2014]

modified wire gadget

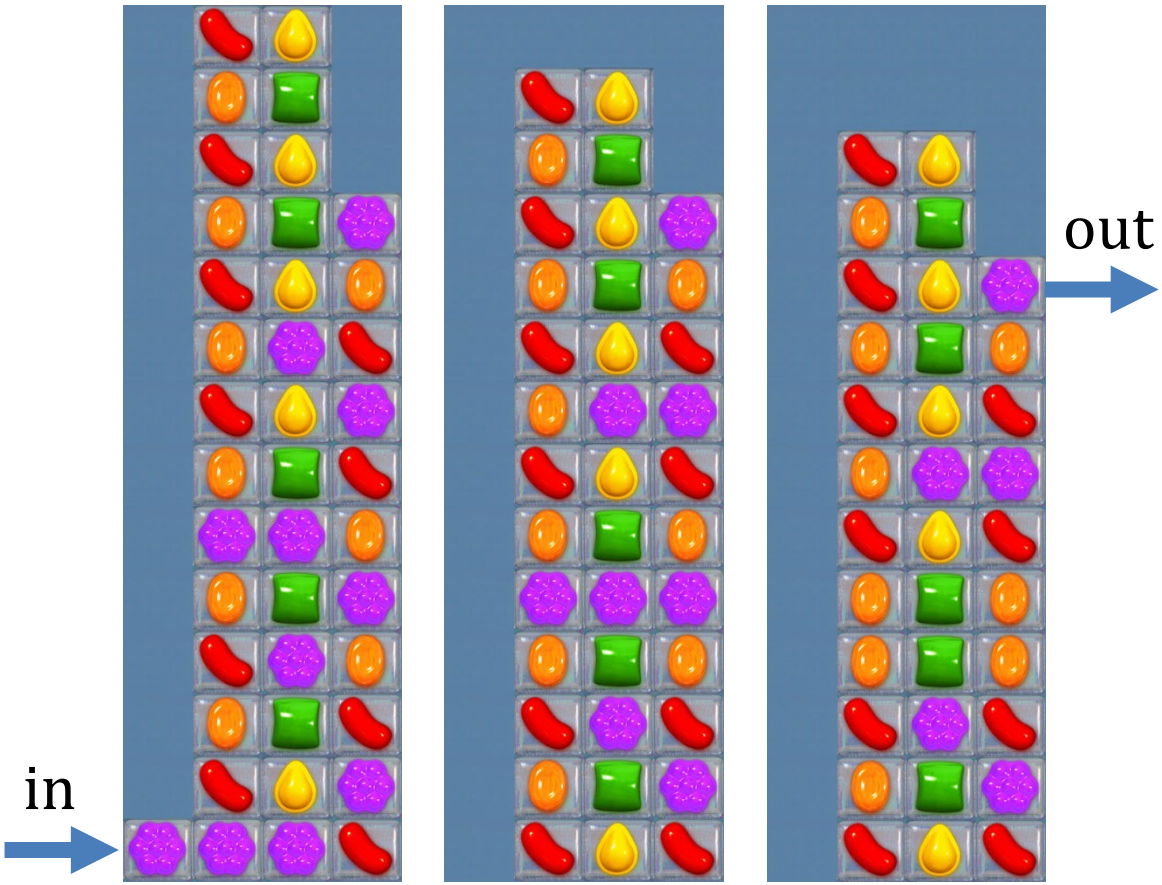


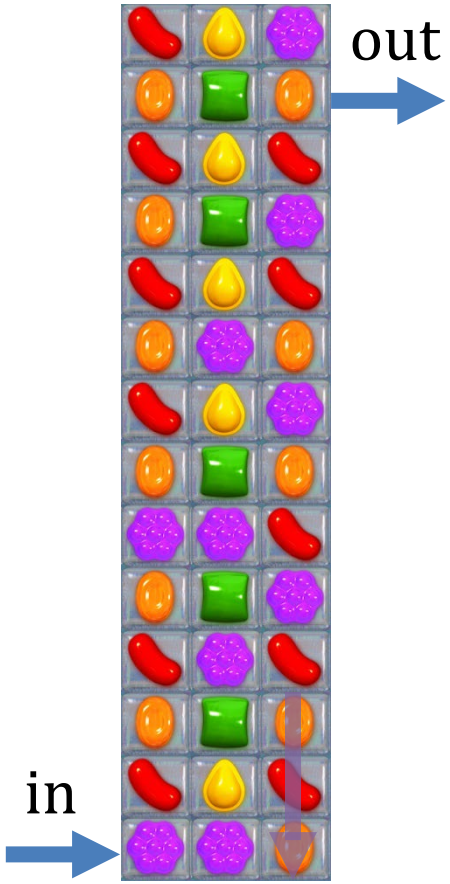


Candy Crush is NP-complete

[Walsh 2014]

modified wire gadget

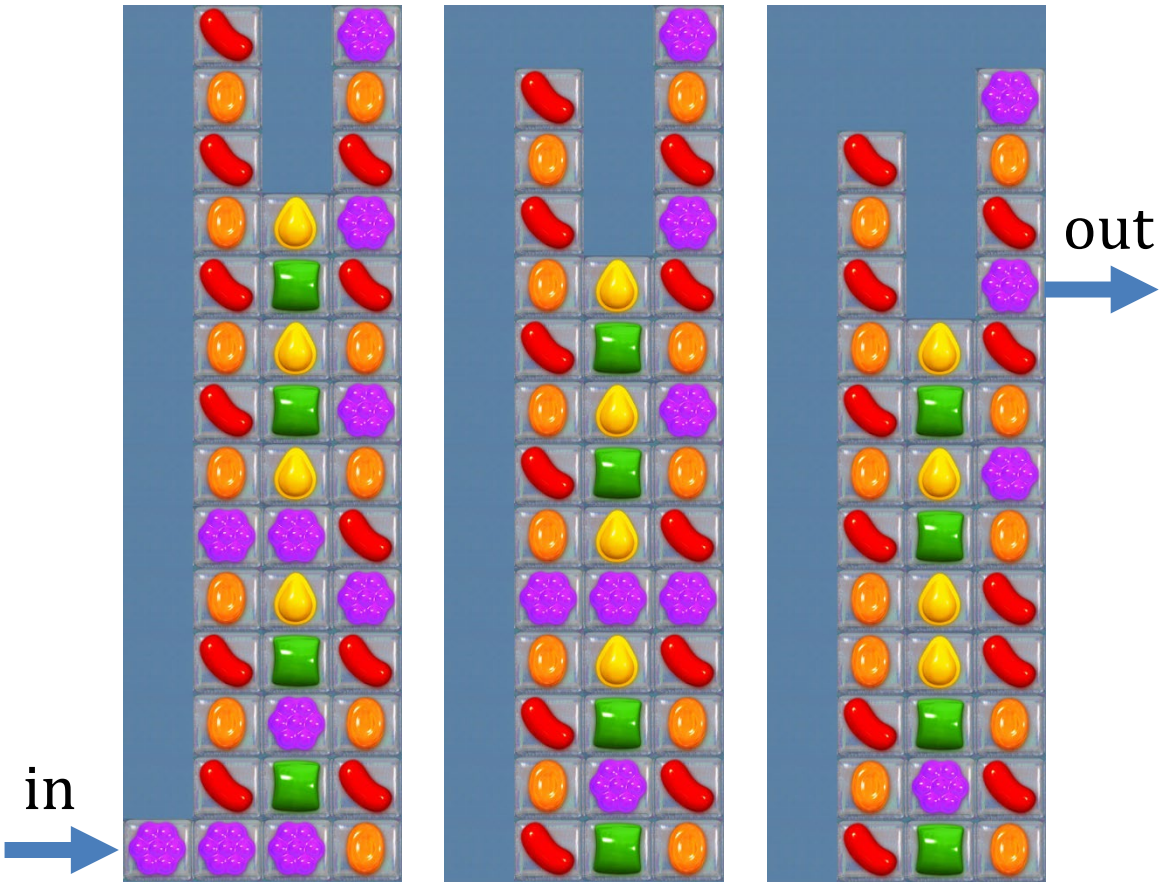


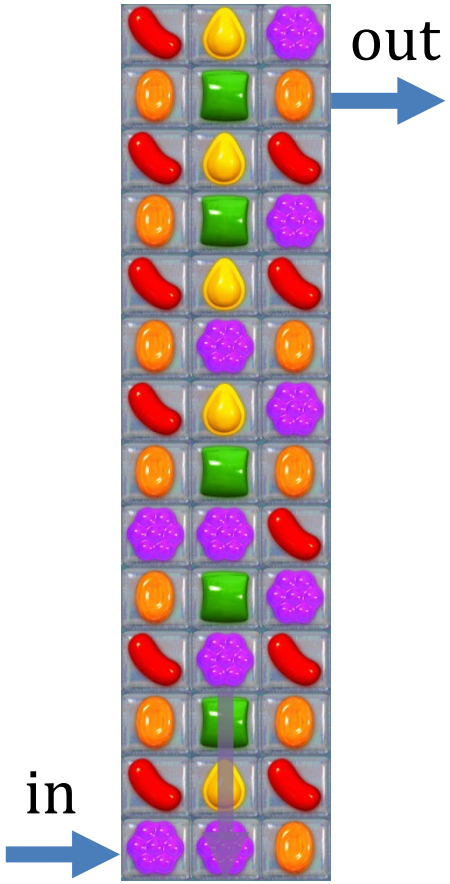


Candy Crush is NP-complete

[Walsh 2014]

modified wire gadget



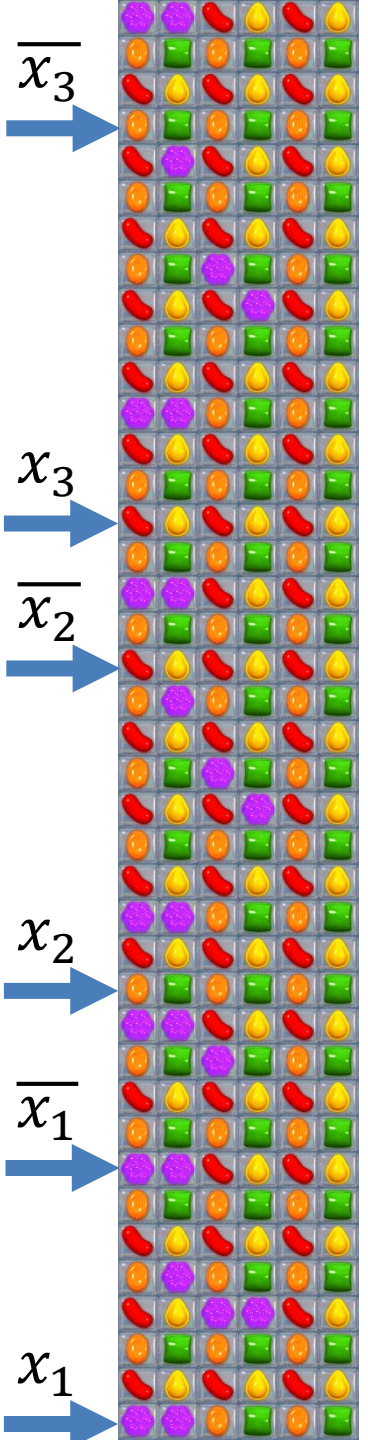


Candy Crush is NP-complete

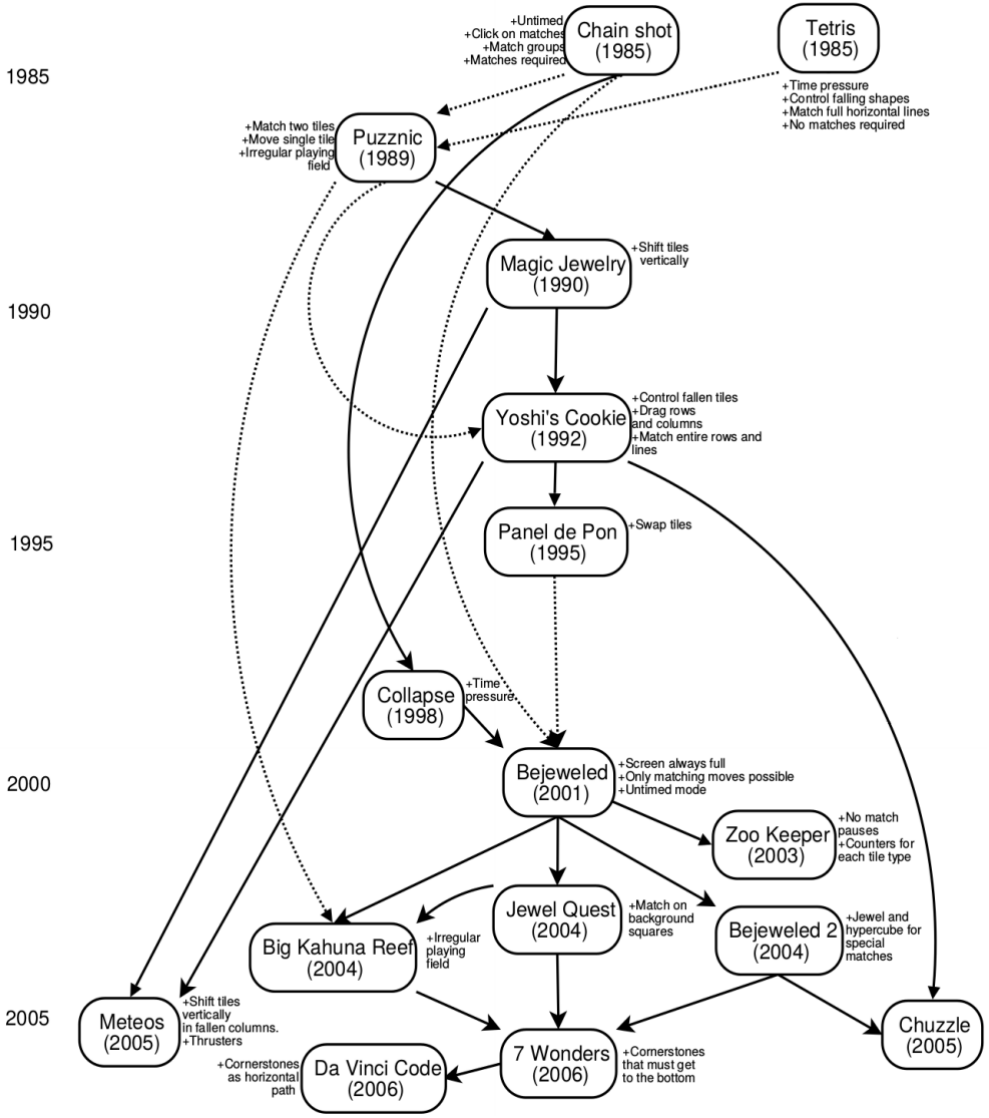
[Walsh 2014]

reward gadget

clause gadget



Bejeweled, Candy Crush, … are NP-Complete [Guala, Leucci, Natale 2014]



Bejeweled, Candy Crush, … are NP-Complete [Guala, Leucci, Natale 2014]