

KOTAKU™

TOP STORIES

Classic Nintendo Games are (NP-)Hard

Greg Aloupis*

Erik D. Demaine†

Alan Guo††

March 26, 2012

NEW YORK

TOTAL RECALL

Mon. - Fri.
11PM - Mid.
(EASTERN)



NINTENDO

Science Proves Old Video Games Were Super Hard

BY LUKE PLUNKETT +

MAR 12, 2012 11:00 PM

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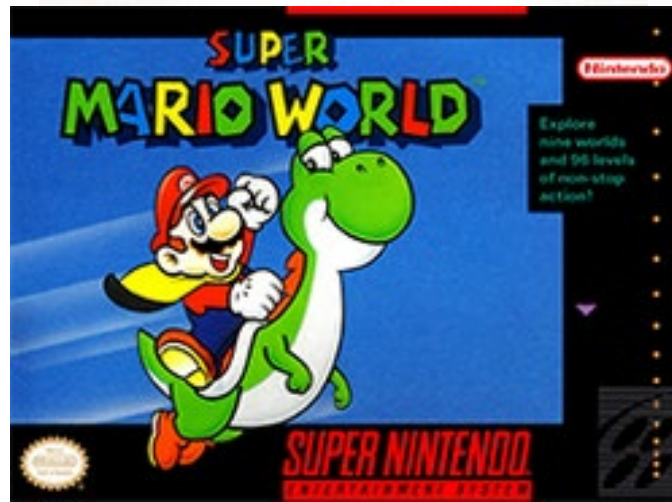
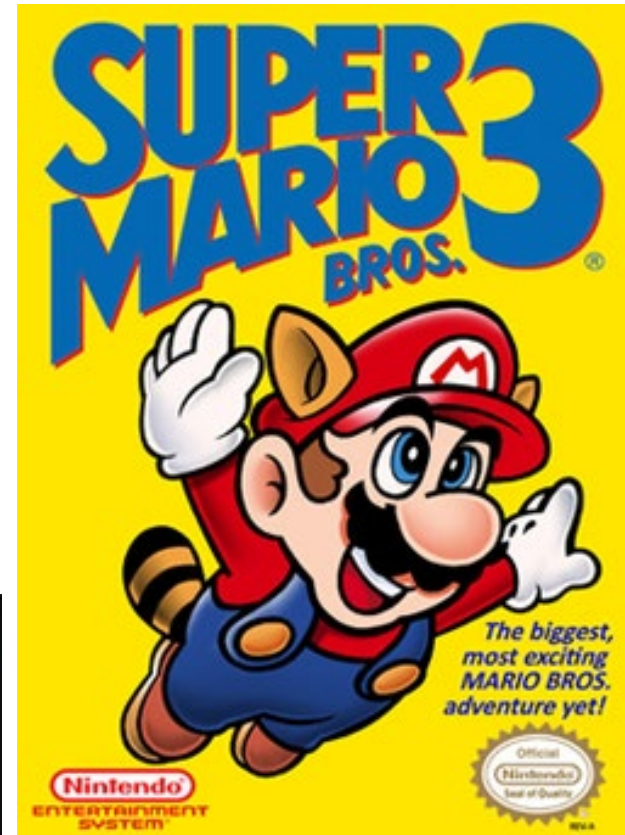
411

33,867 232

Super Mario Bros. is NP-Hard

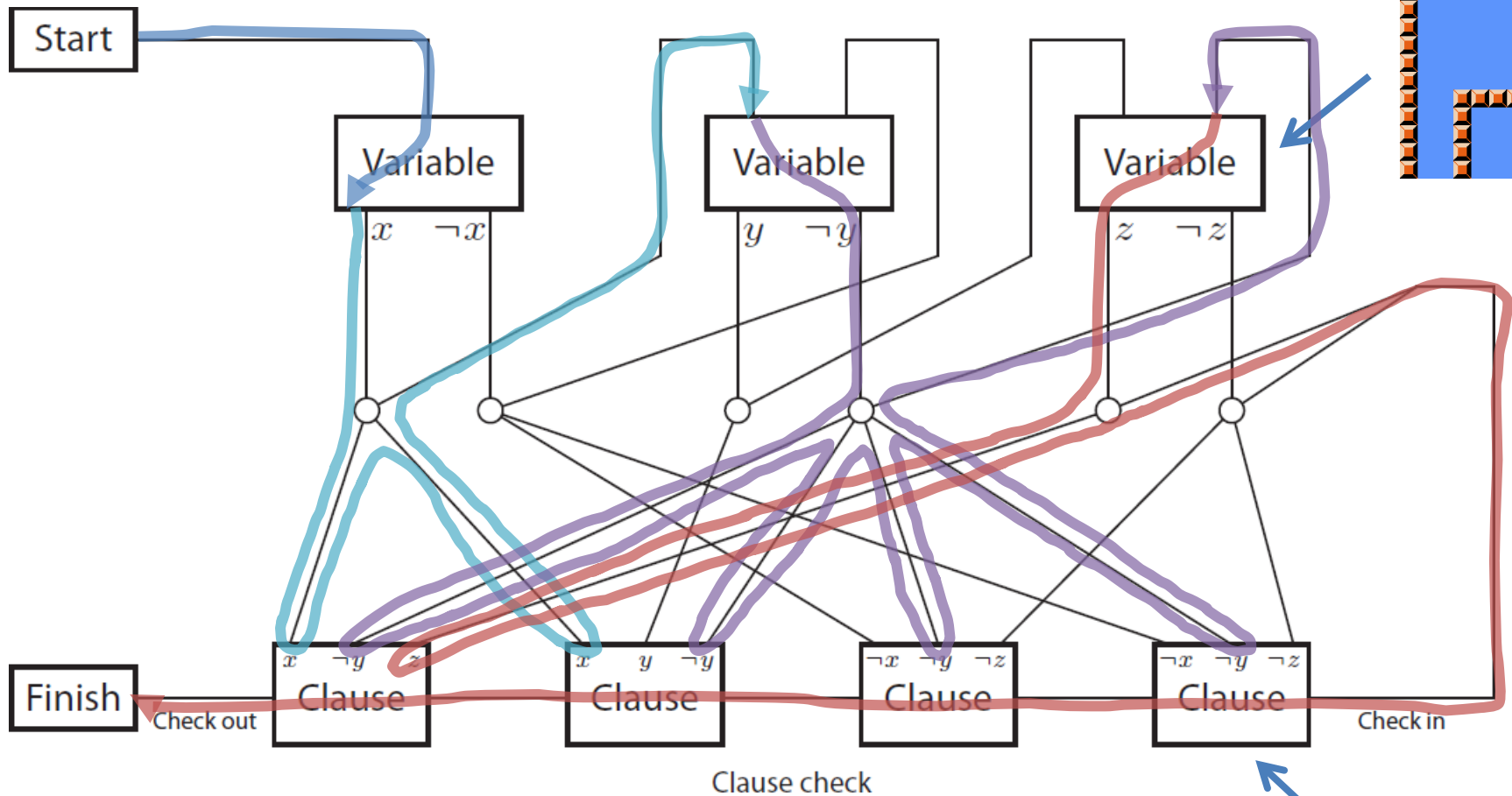
[Aloupis, Demaine, Guo 2012]

The Lost Levels

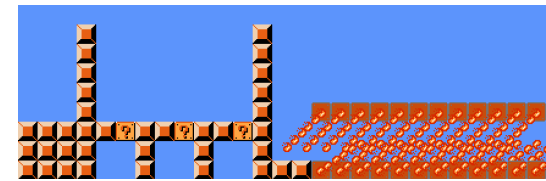


Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]



$(x \text{ OR } \neg y \text{ OR } z) \ \& \ (x \text{ OR } y \text{ OR } \neg y) \ \& \ (\neg x \text{ OR } \neg y \text{ OR } \neg z) \ \& \ (\neg x \text{ OR } \neg y \text{ OR } \neg z)$

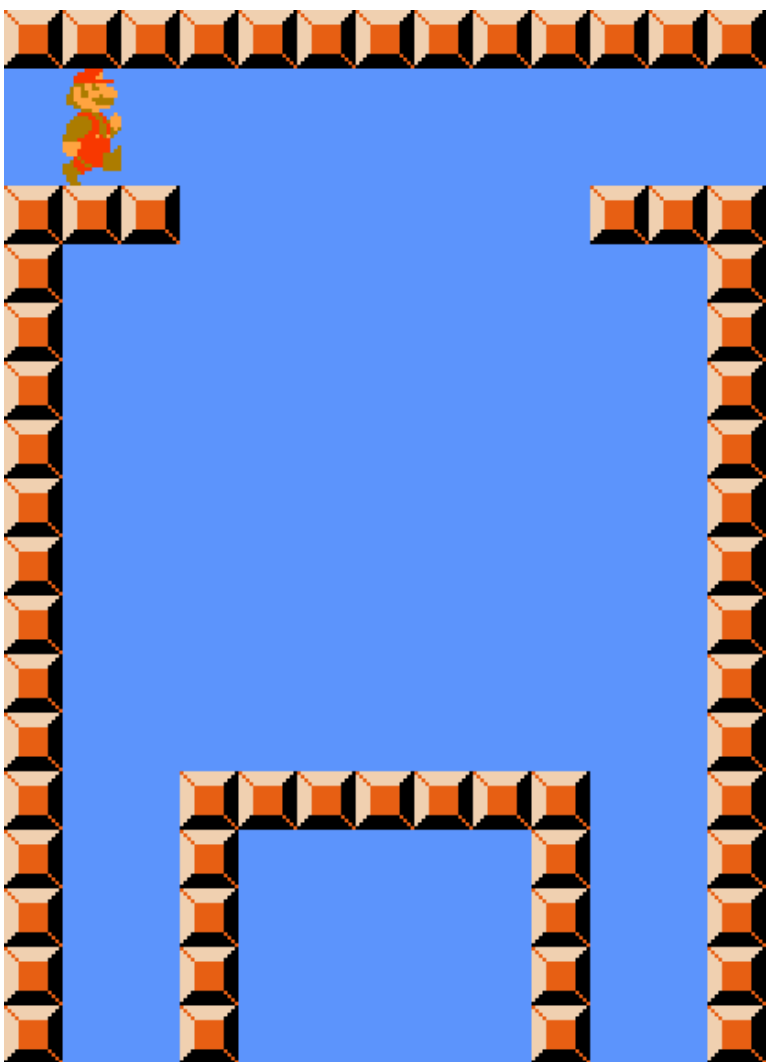




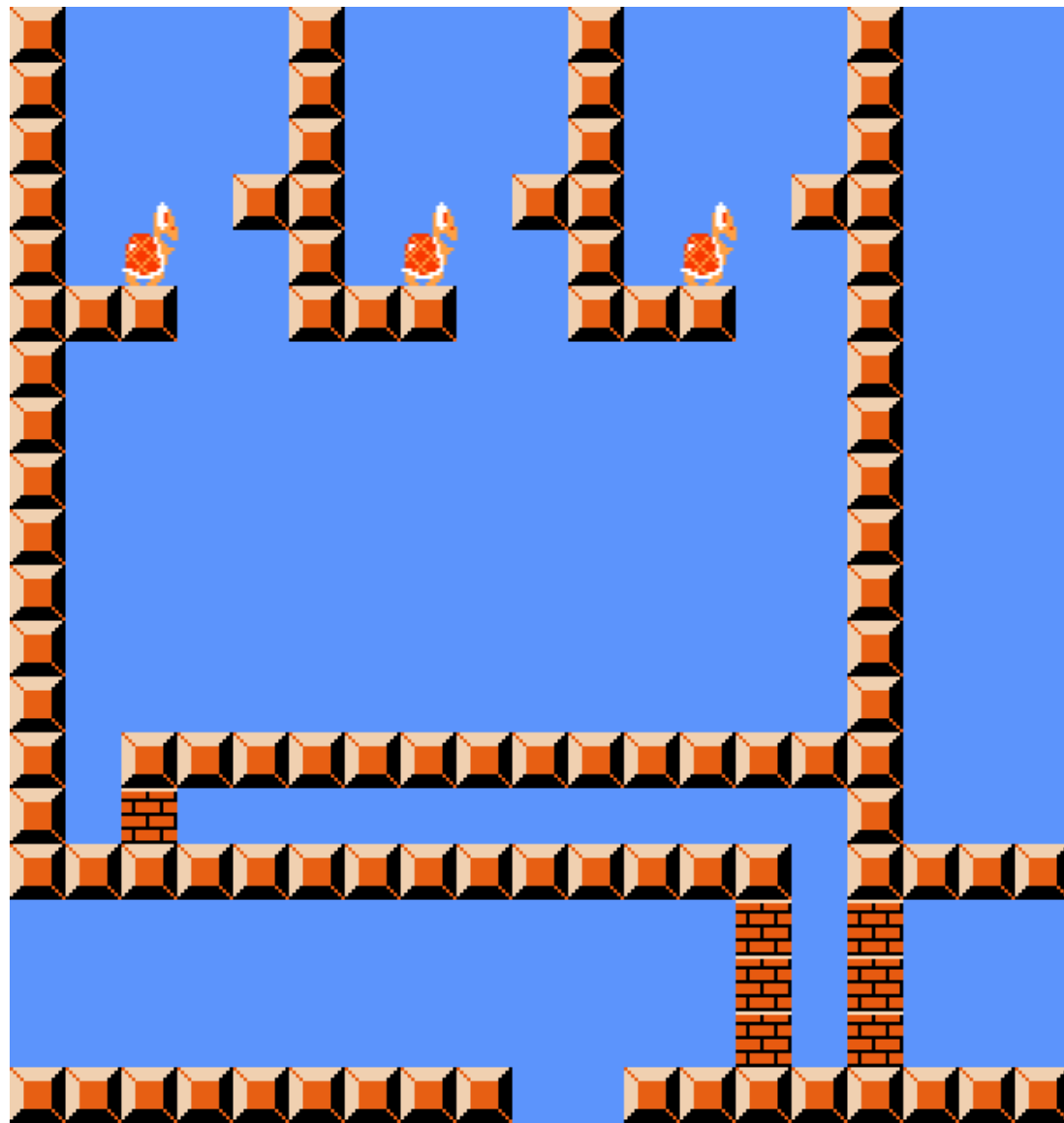
Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo 2012]

clause



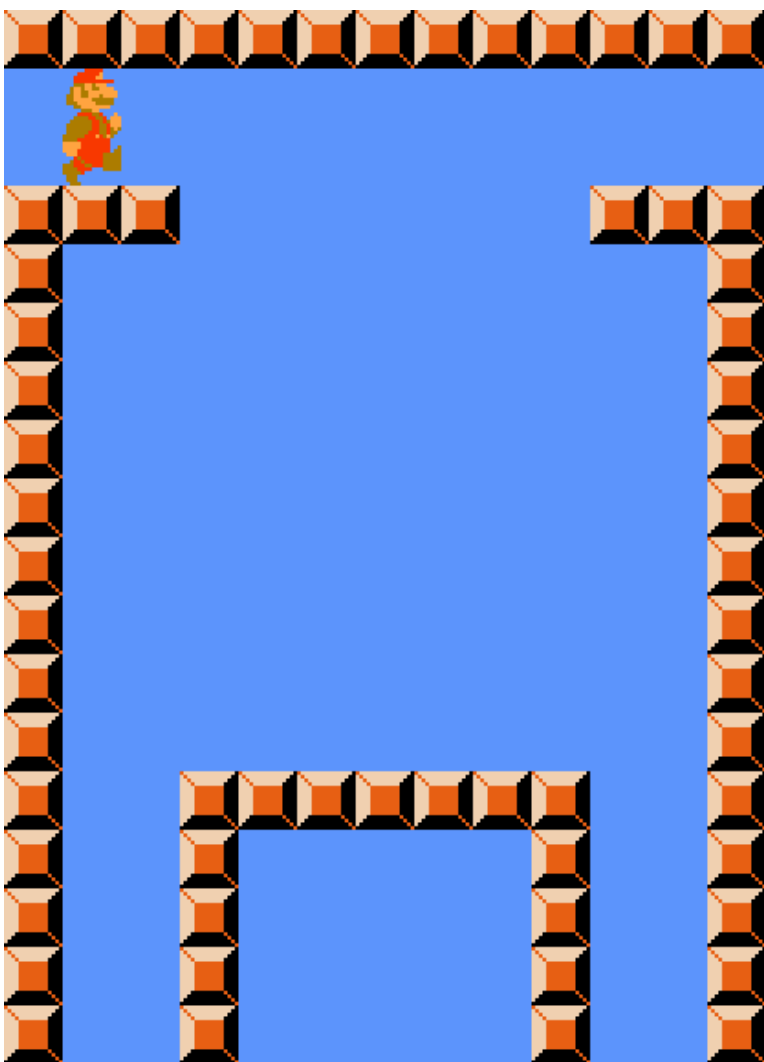
variable



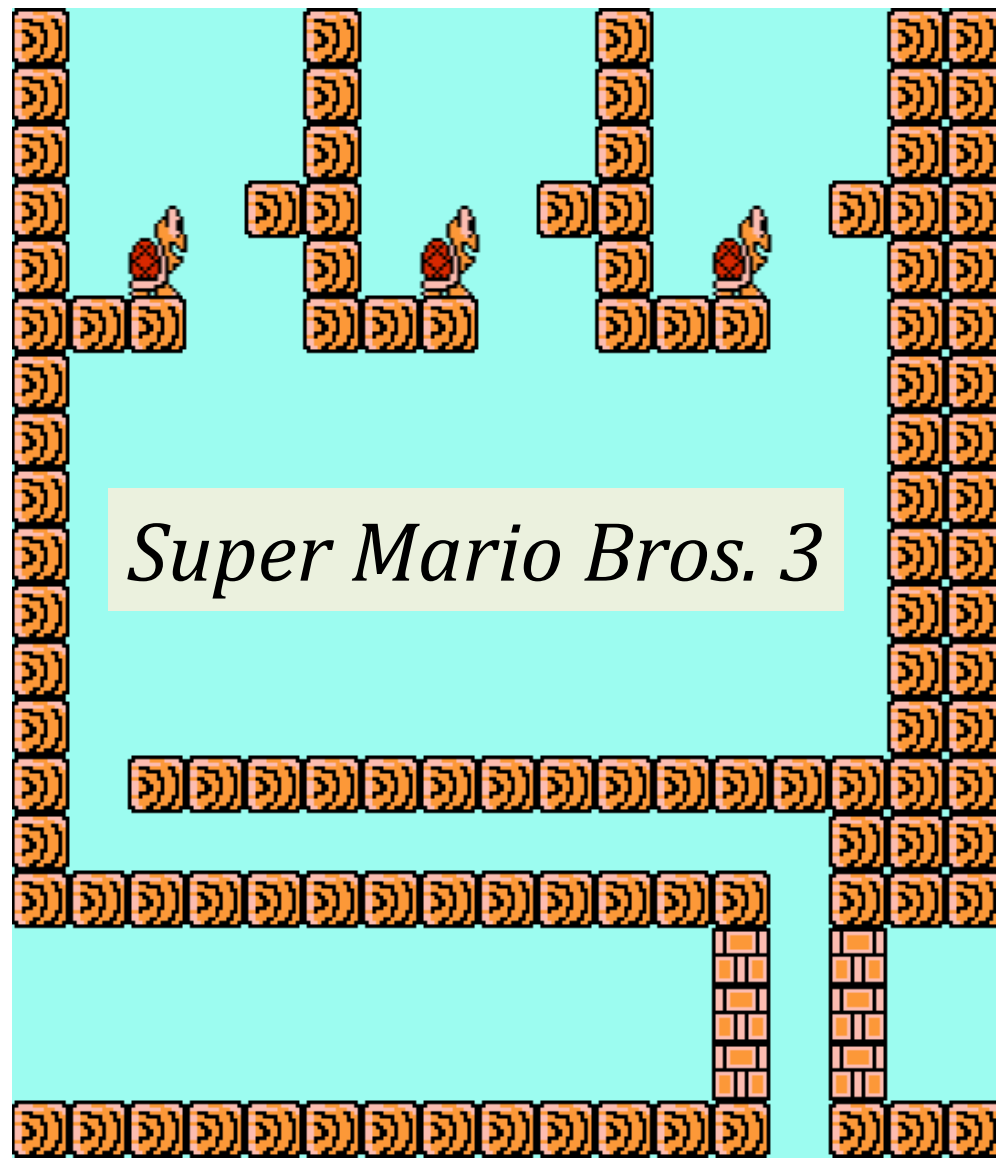
Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo 2012]

clause



variable



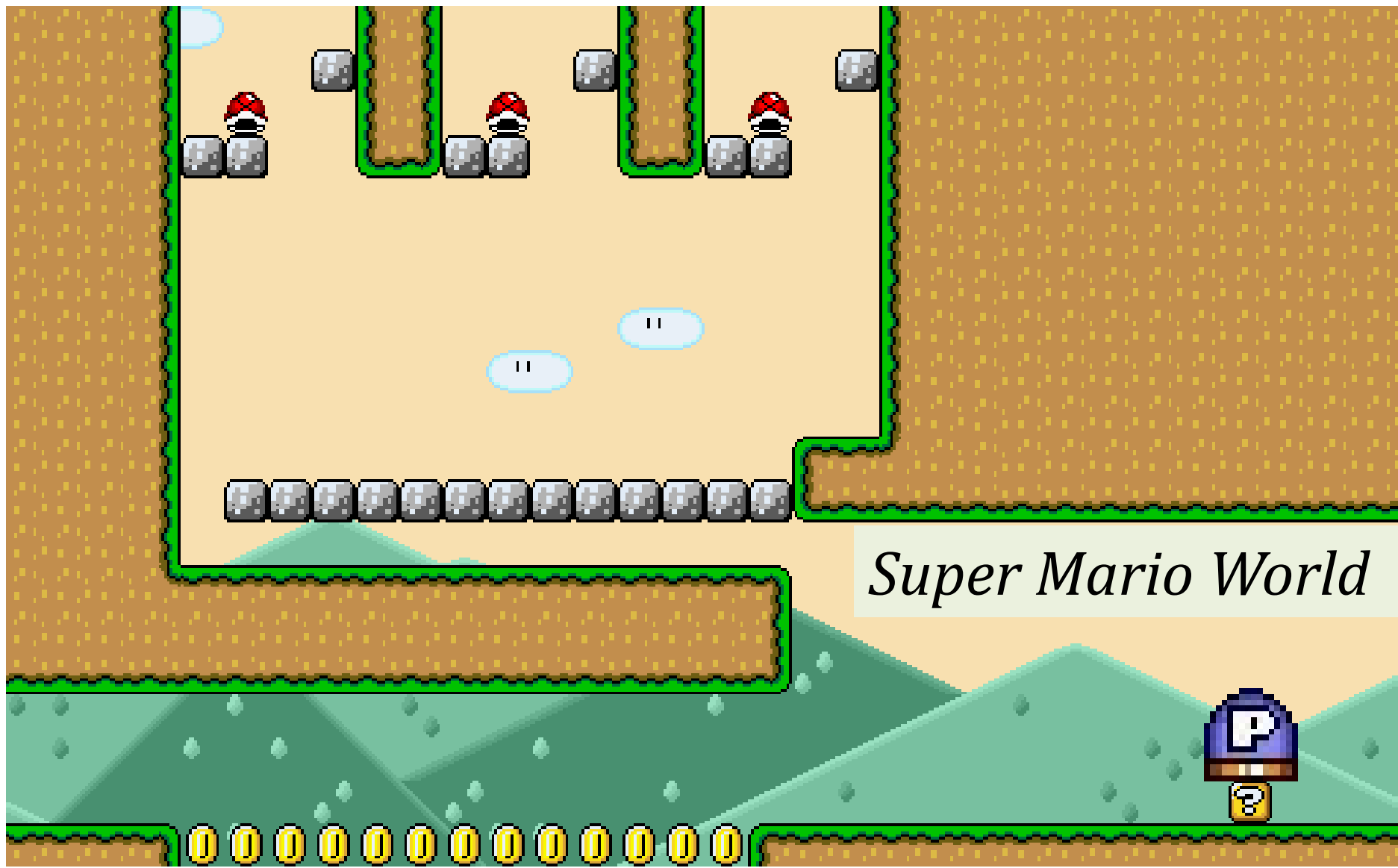
Super Mario Bros. 3



Super Mario World is NP-Hard

[Aloupis, Demaine, Guo 2012]

clause

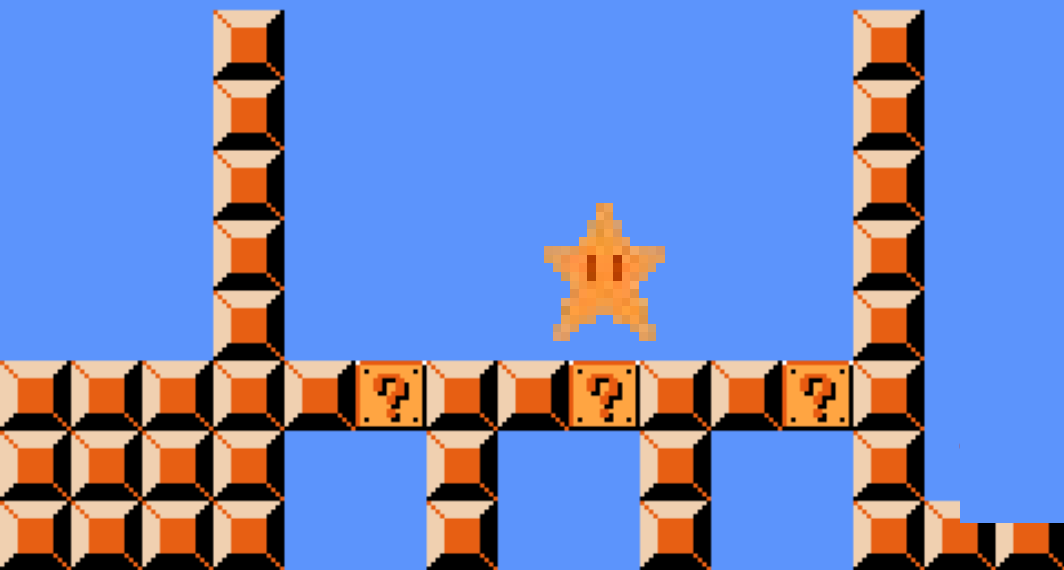




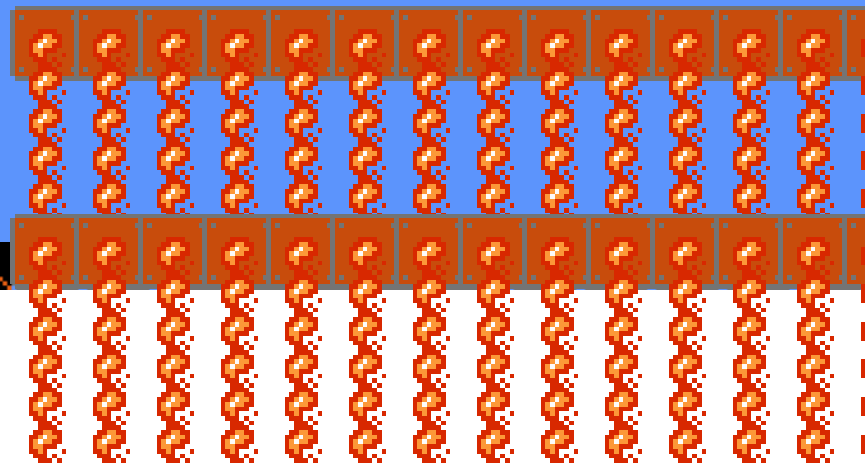
Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

clause



Super Mario Bros.





Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

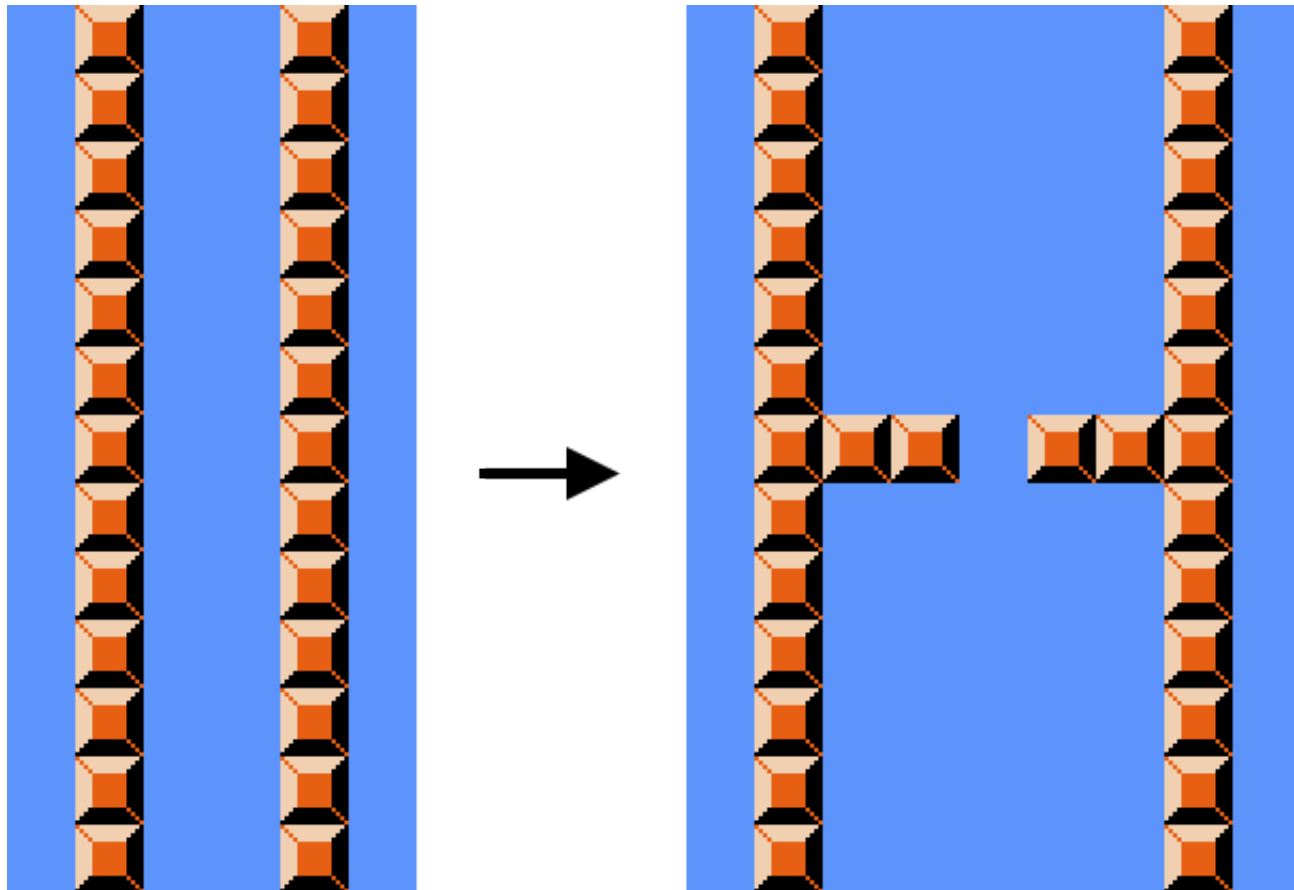
- **Glitch:** Wall jump



Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

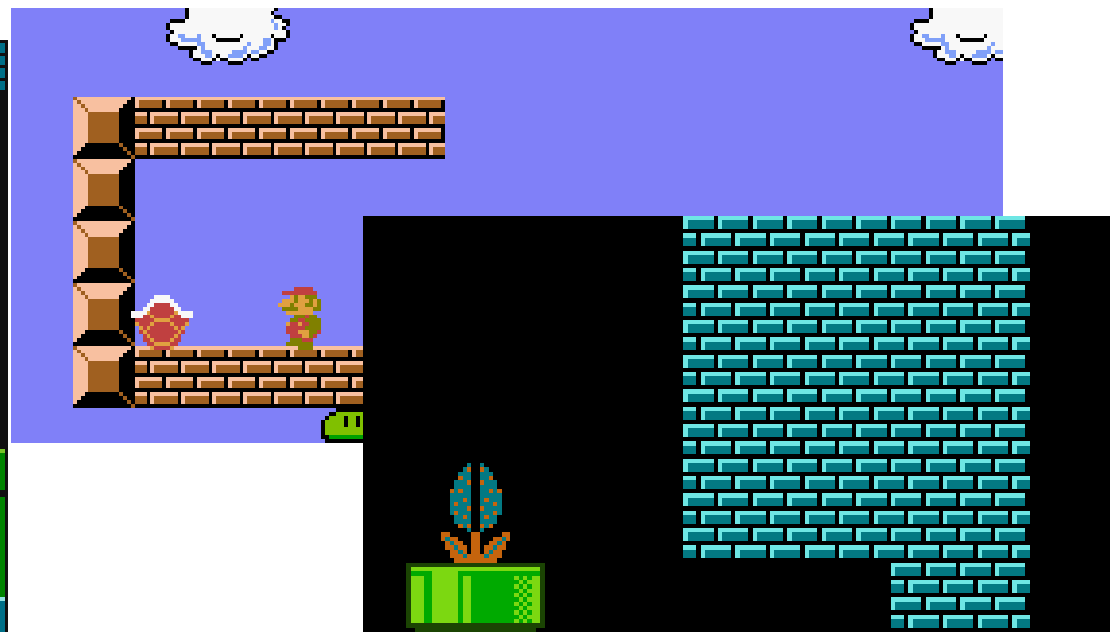
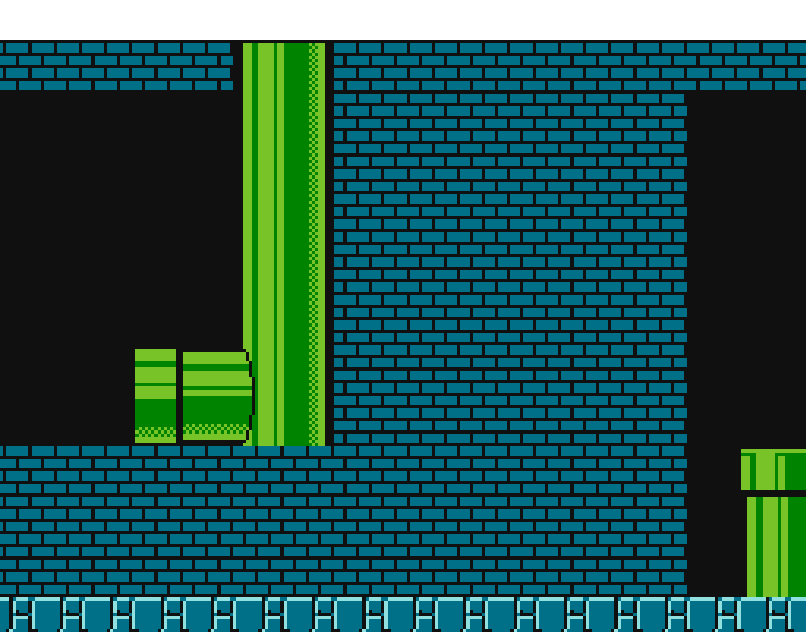
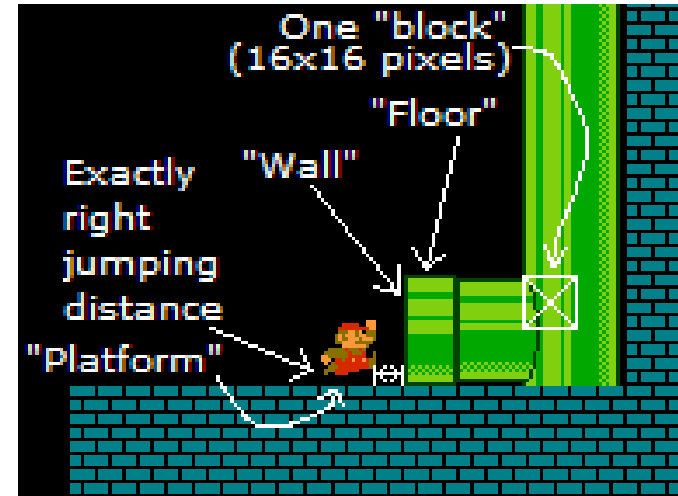
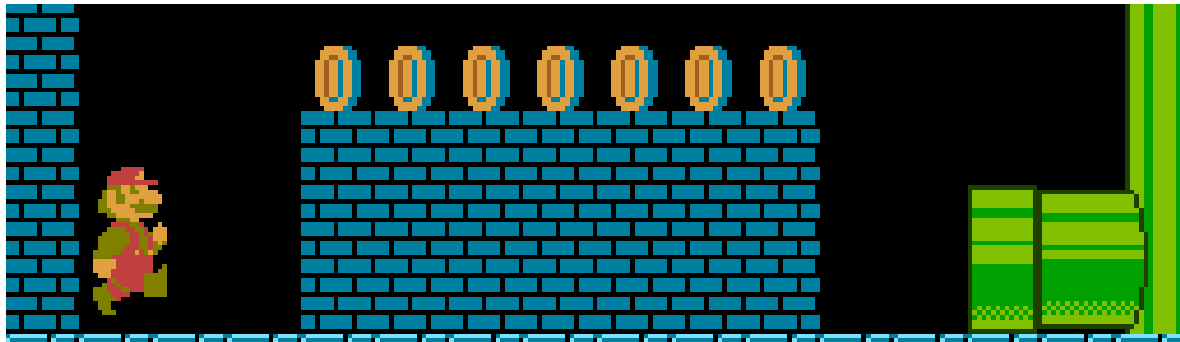
- **Glitch:** Wall jump



Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

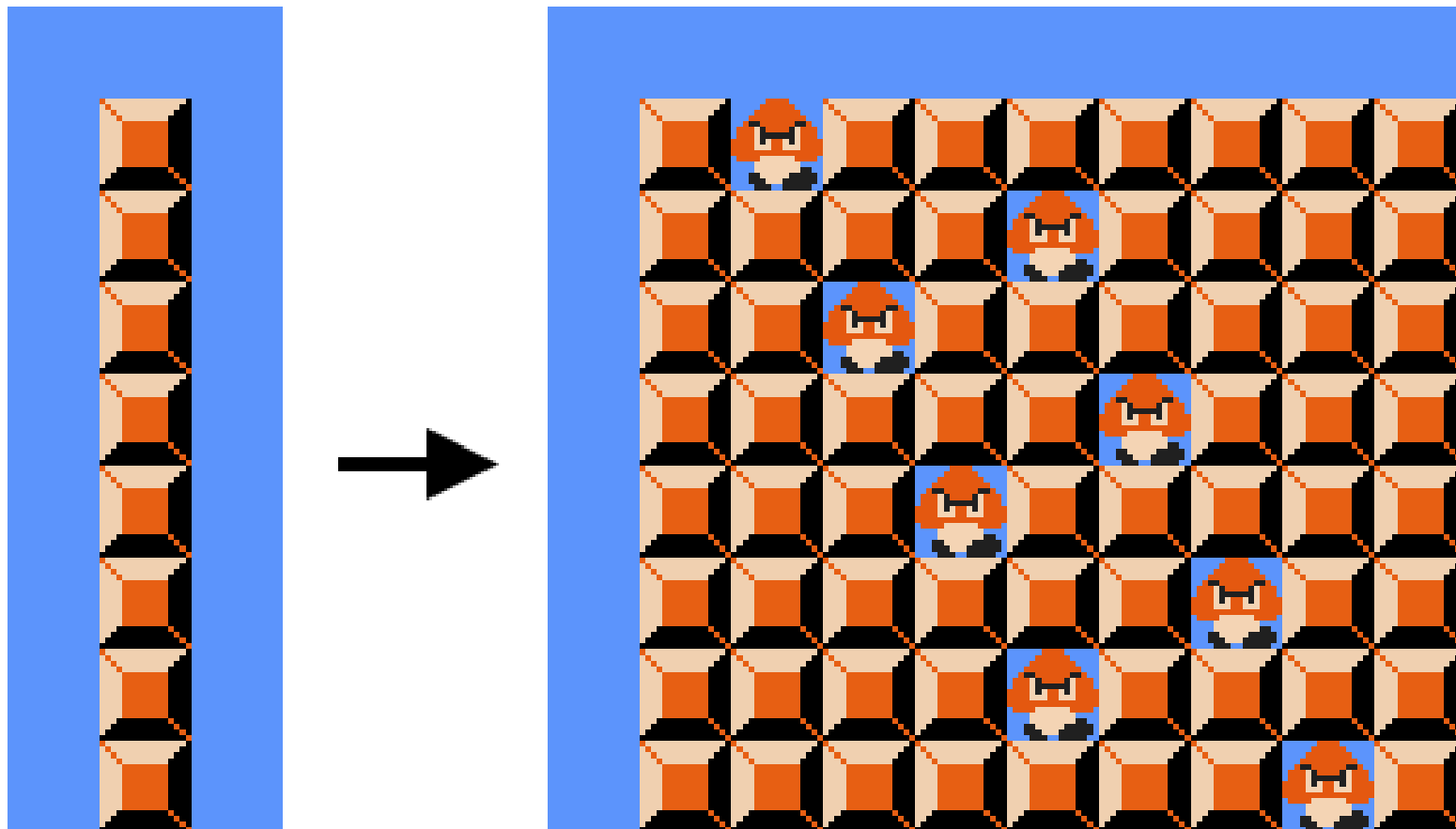
- **Glitch:** Jump through walls



Super Mario Bros. is NP-Hard

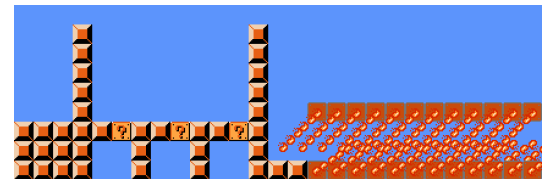
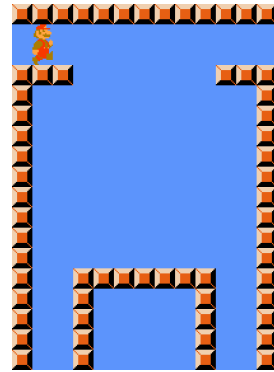
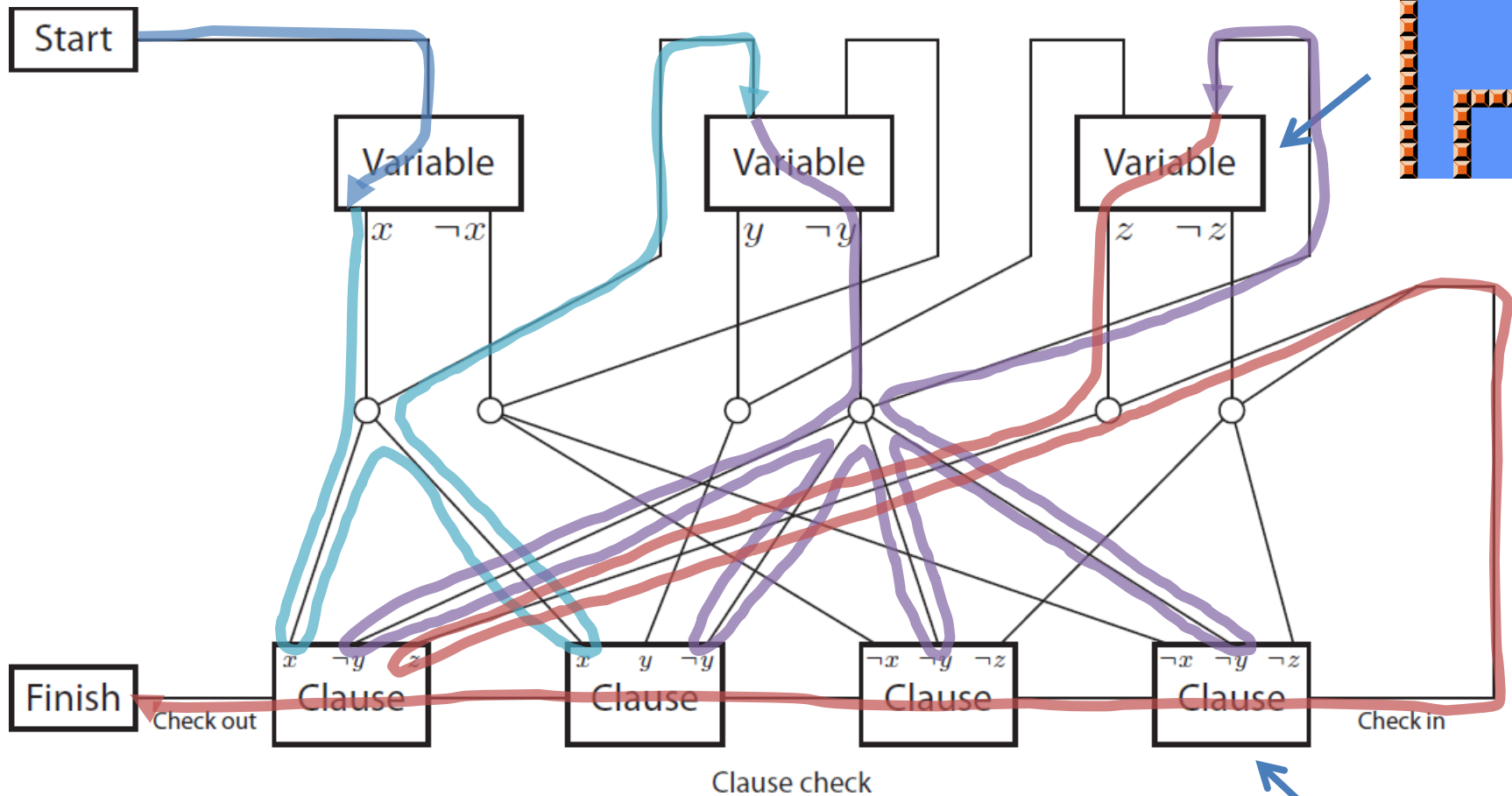
[Aloupis, Demaine, Guo, Viglietta 2014]

- **Glitch:** Jump through walls



Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]



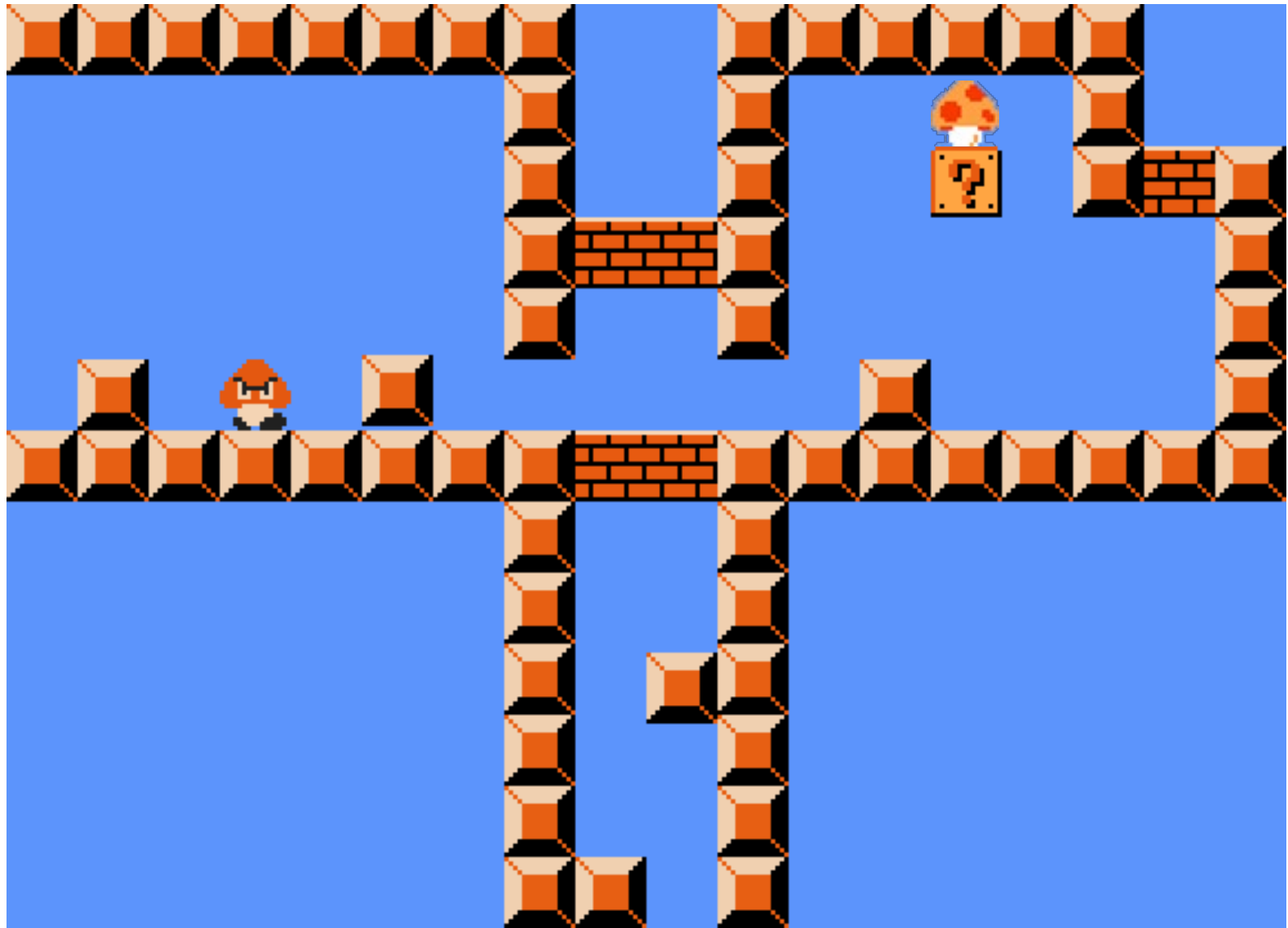
$(x \text{ OR } \neg y \text{ OR } z) \& (x \text{ OR } y \text{ OR } \neg y) \&$
 $(\neg x \text{ OR } \neg y \text{ OR } \neg z) \& (\neg x \text{ OR } \neg y \text{ OR } \neg z)$



Super Mario Bros. is NP-Hard

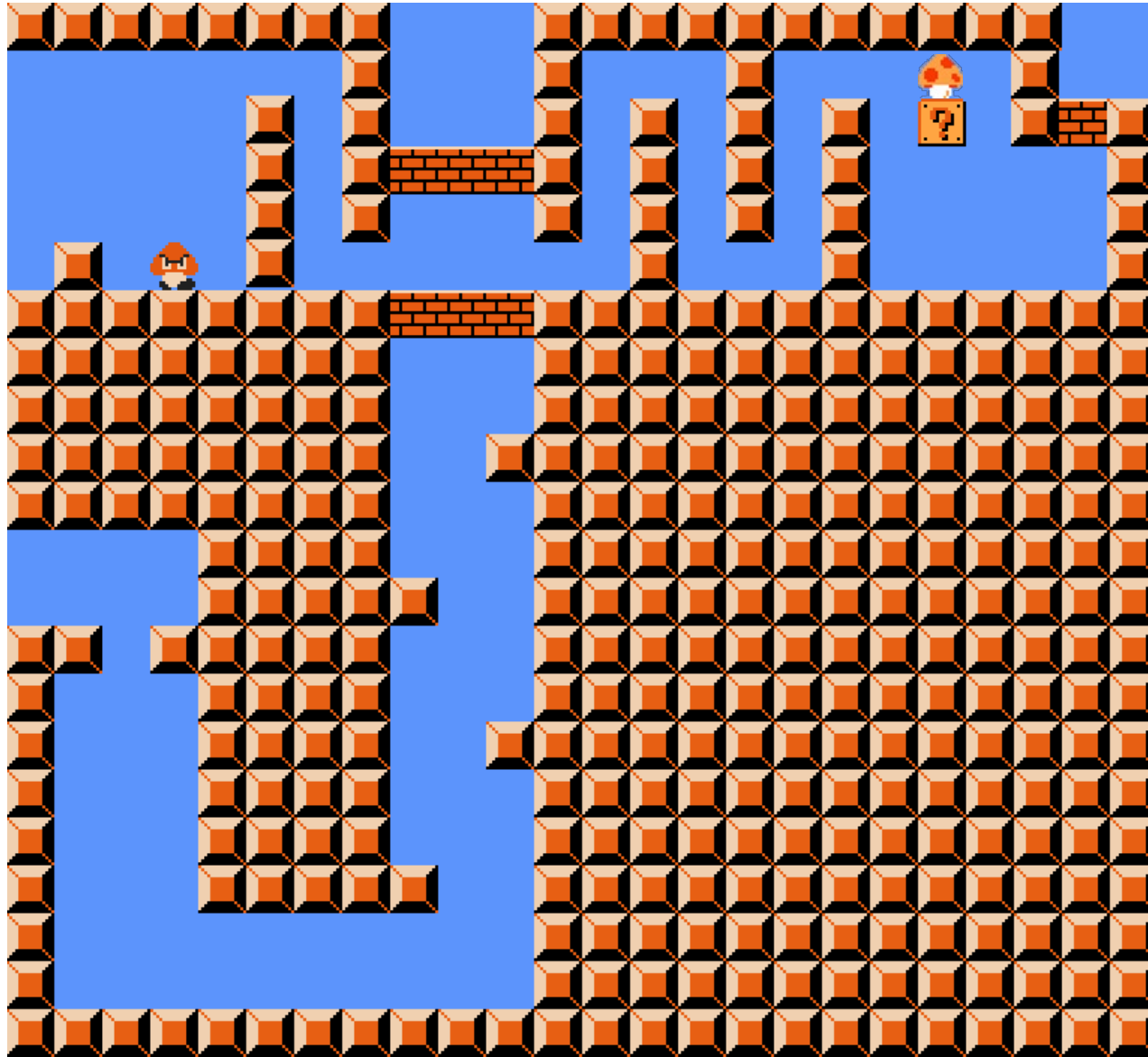
[Aloupis, Demaine, Guo 2012]

crossover v1



Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

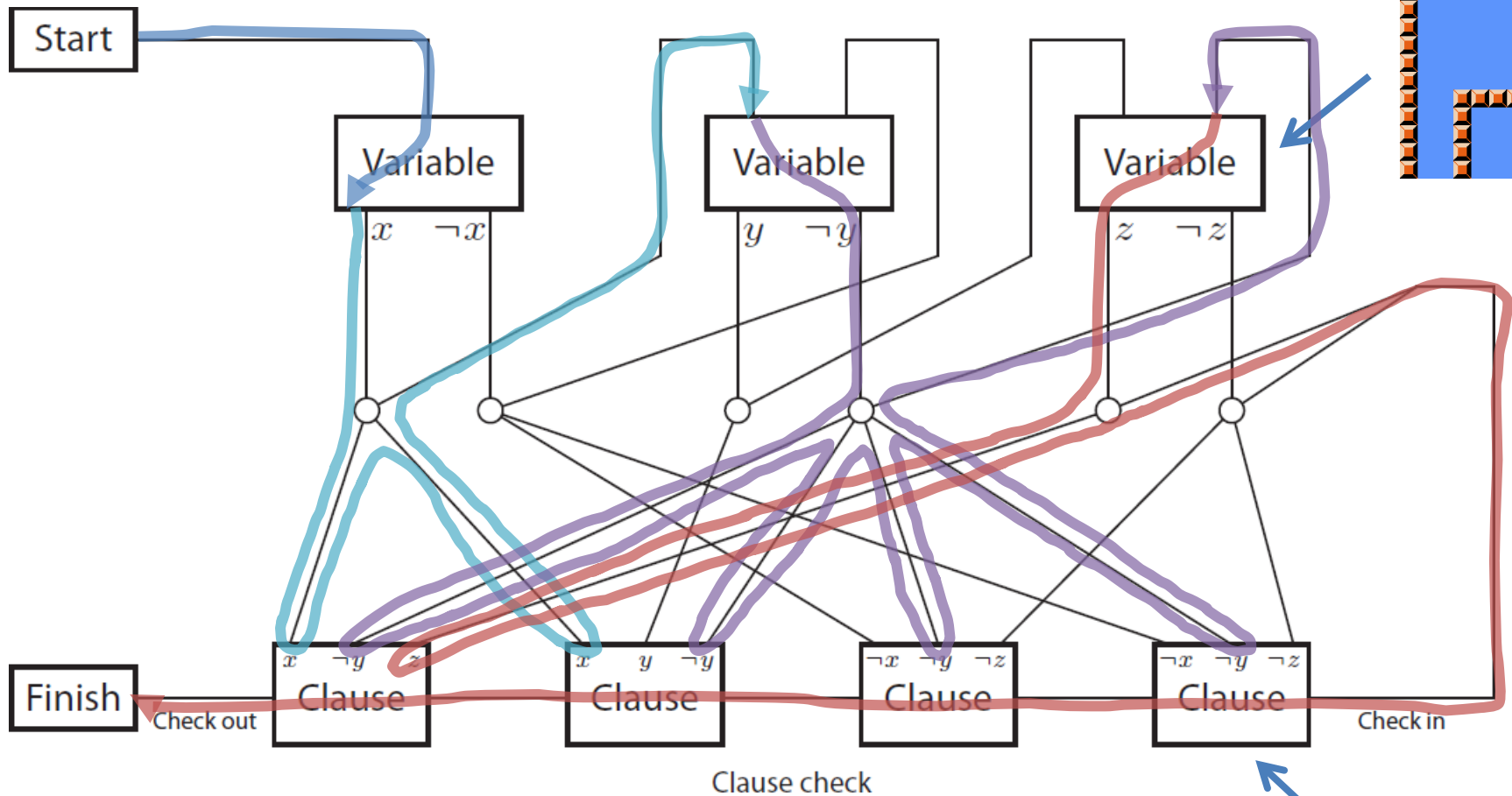


crossover v2

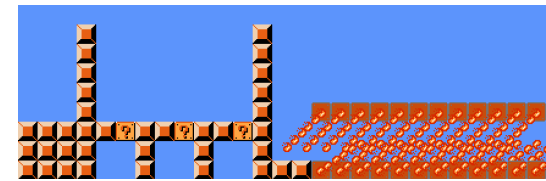
(leaks from horizontal to vertical if both traversed)

Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]



$(x \text{ OR } \neg y \text{ OR } z) \ \& \ (x \text{ OR } y \text{ OR } \neg y) \ \& \ (\neg x \text{ OR } \neg y \text{ OR } \neg z) \ \& \ (\neg x \text{ OR } \neg y \text{ OR } \neg z)$

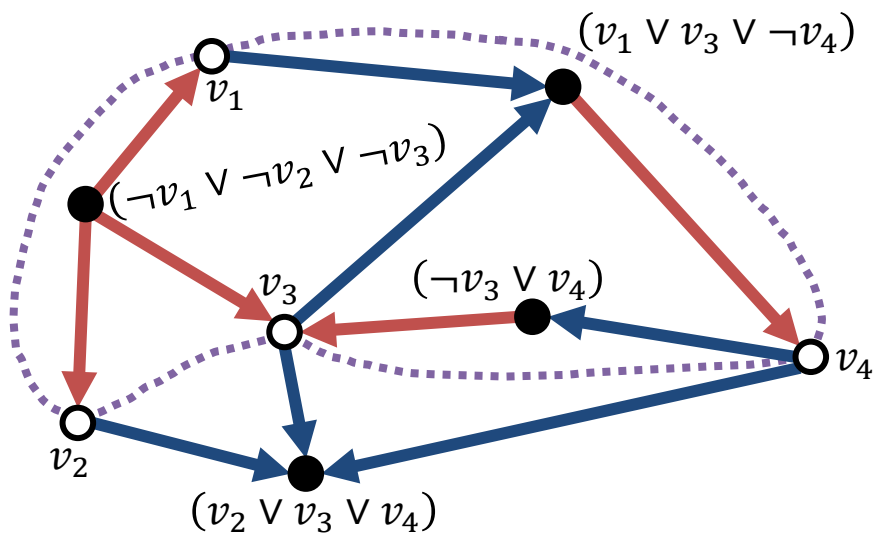


Linked Planar 3SAT

[Pilz 2018]

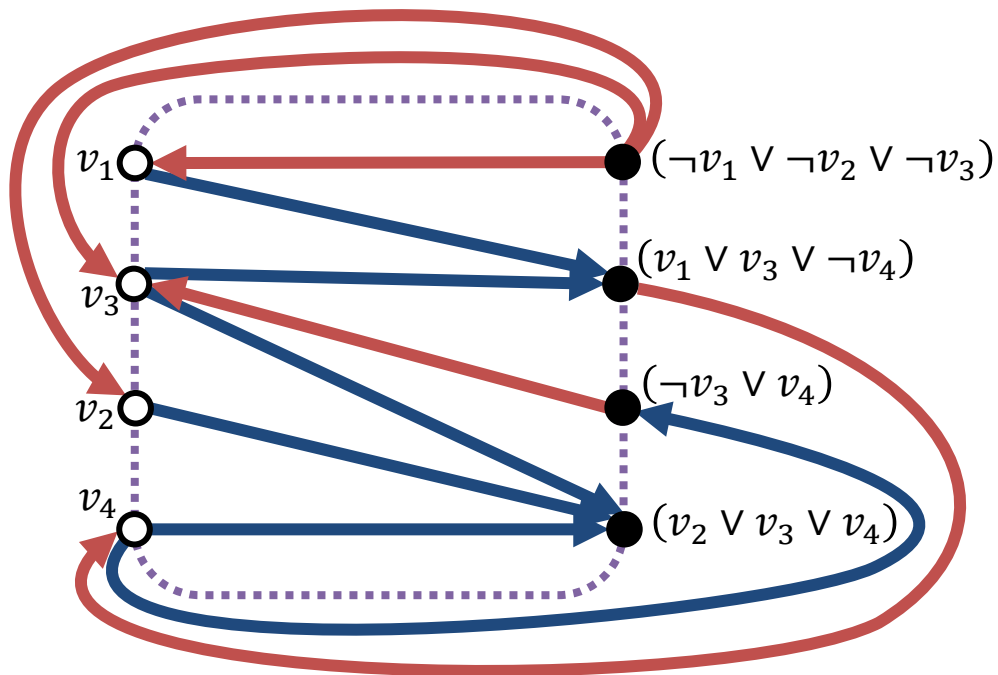
- clauses
- - ○ - - variables
- → ● positive literal
- ← ● negative literal

$$(v_1 \vee v_3 \vee \neg v_4) \wedge (\neg v_1 \vee \neg v_2 \vee \neg v_3) \\ \wedge (\neg v_3 \vee v_4) \wedge (v_2 \vee v_3 \vee v_4)$$



Planar 3SAT

$$(\neg v_1 \vee \neg v_2 \vee \neg v_3) \wedge (v_1 \vee v_3 \vee \neg v_4) \\ \wedge (\neg v_3 \vee v_4) \wedge (v_2 \vee v_3 \vee v_4)$$



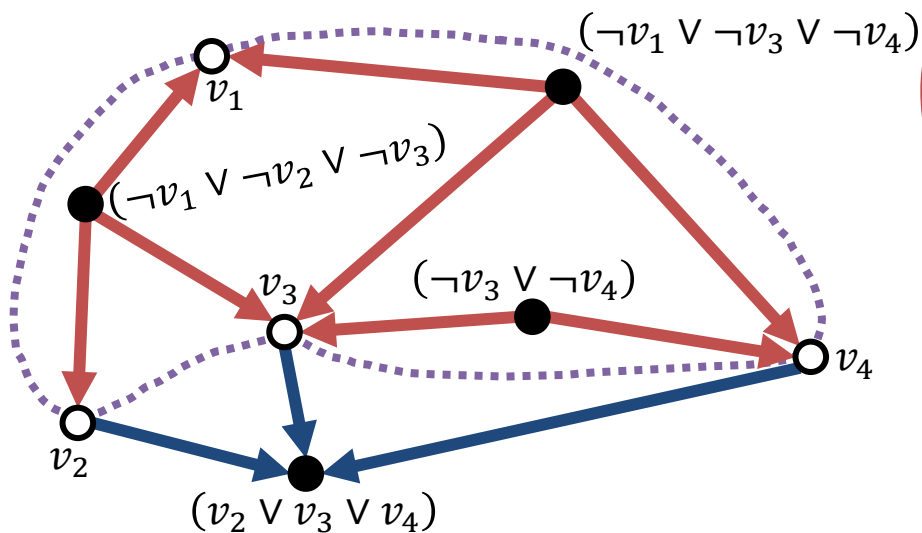
Linked Planar 3SAT

Linked Planar 3SAT

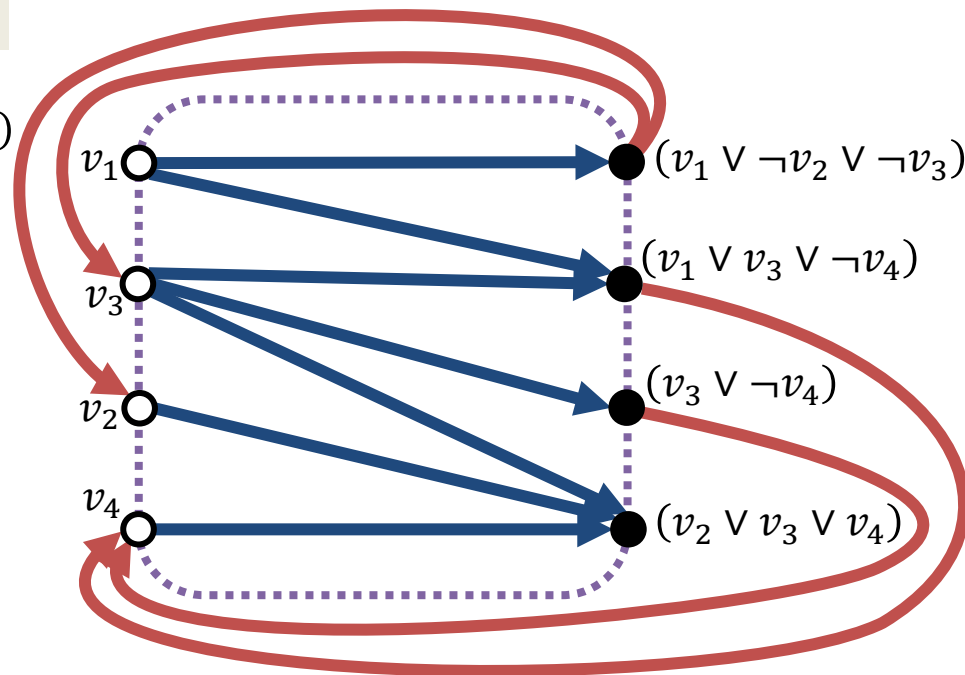
[Pilz 2018]

- clauses
- - ○ - - variables
- → ● positive literal
- ← ● negative literal

$$(\neg v_1 \vee \neg v_3 \vee \neg v_4) \wedge (\neg v_1 \vee \neg v_2 \vee \neg v_3) \wedge (\neg v_3 \vee \neg v_4) \wedge (v_2 \vee v_3 \vee v_4)$$



$$(v_1 \vee \neg v_2 \vee \neg v_3) \wedge (v_1 \vee v_3 \vee \neg v_4) \wedge (v_3 \vee \neg v_4) \wedge (v_2 \vee v_3 \vee v_4)$$

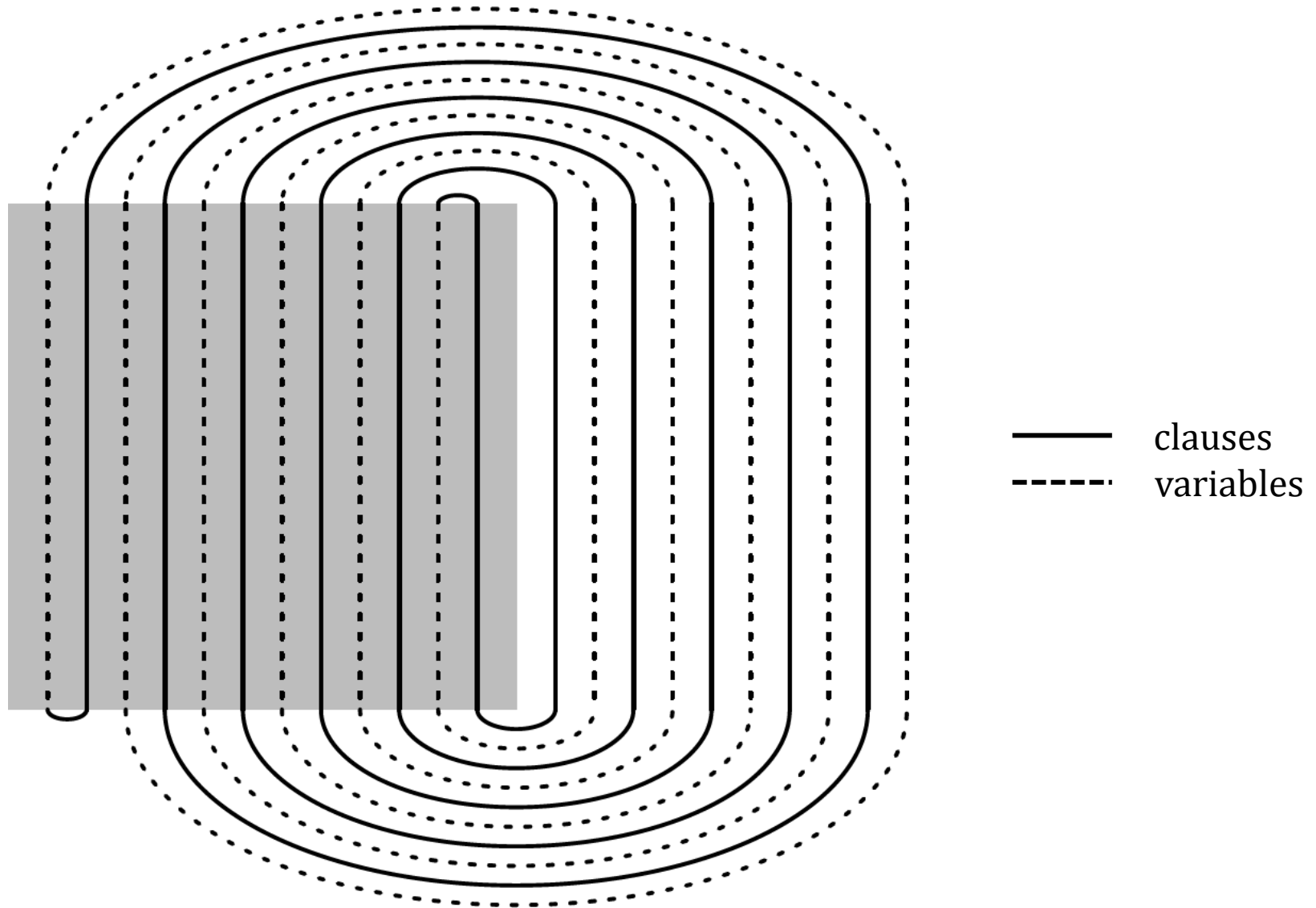


Planar Monotone 3SAT

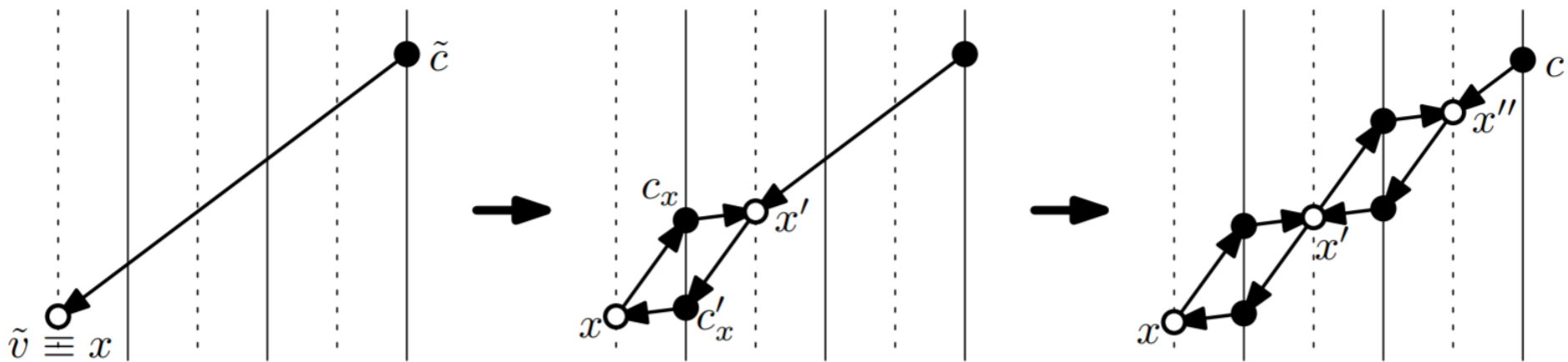
Sided Linked Planar 3SAT



Linked Planar 3SAT is Hard [Pilz 2018]



Linked Planar 3SAT is Hard [Pilz 2018]



$$c_x \equiv x \vee \neg x' \equiv x' \rightarrow x$$

$$c'_x \equiv x' \vee \neg x \equiv x \rightarrow x'$$

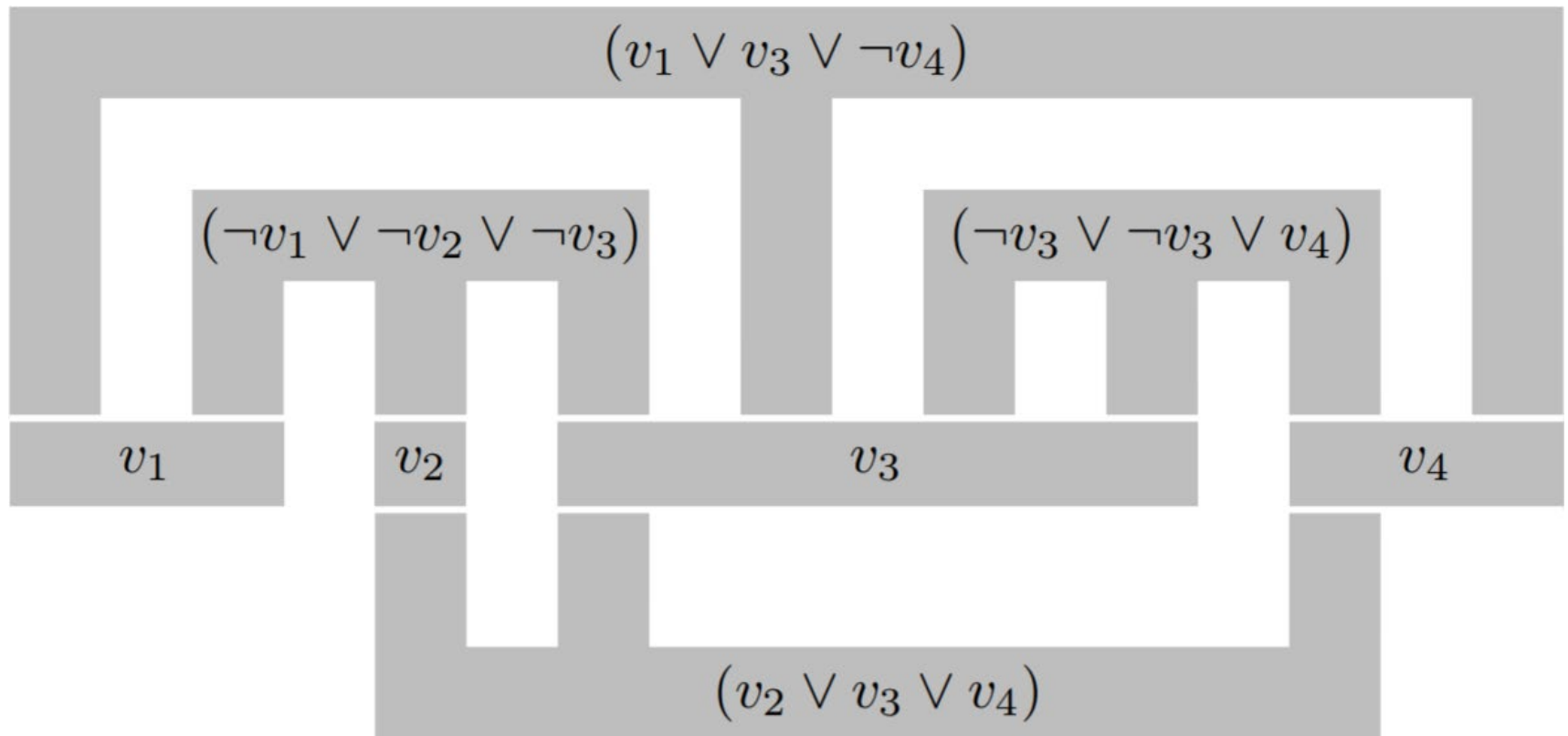
$$c_x \wedge c'_x \equiv x = x'$$

- $\text{---}\bullet\text{---}$ clauses
- $\text{---}\circ\text{---}$ variables
- $\circ \rightarrow \bullet$ positive literal
- $\circ \leftarrow \bullet$ negative literal



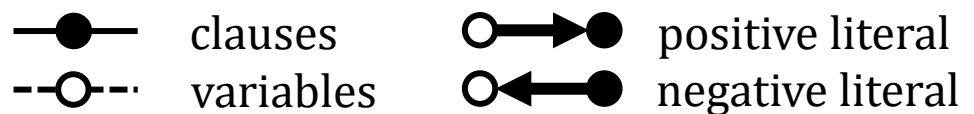
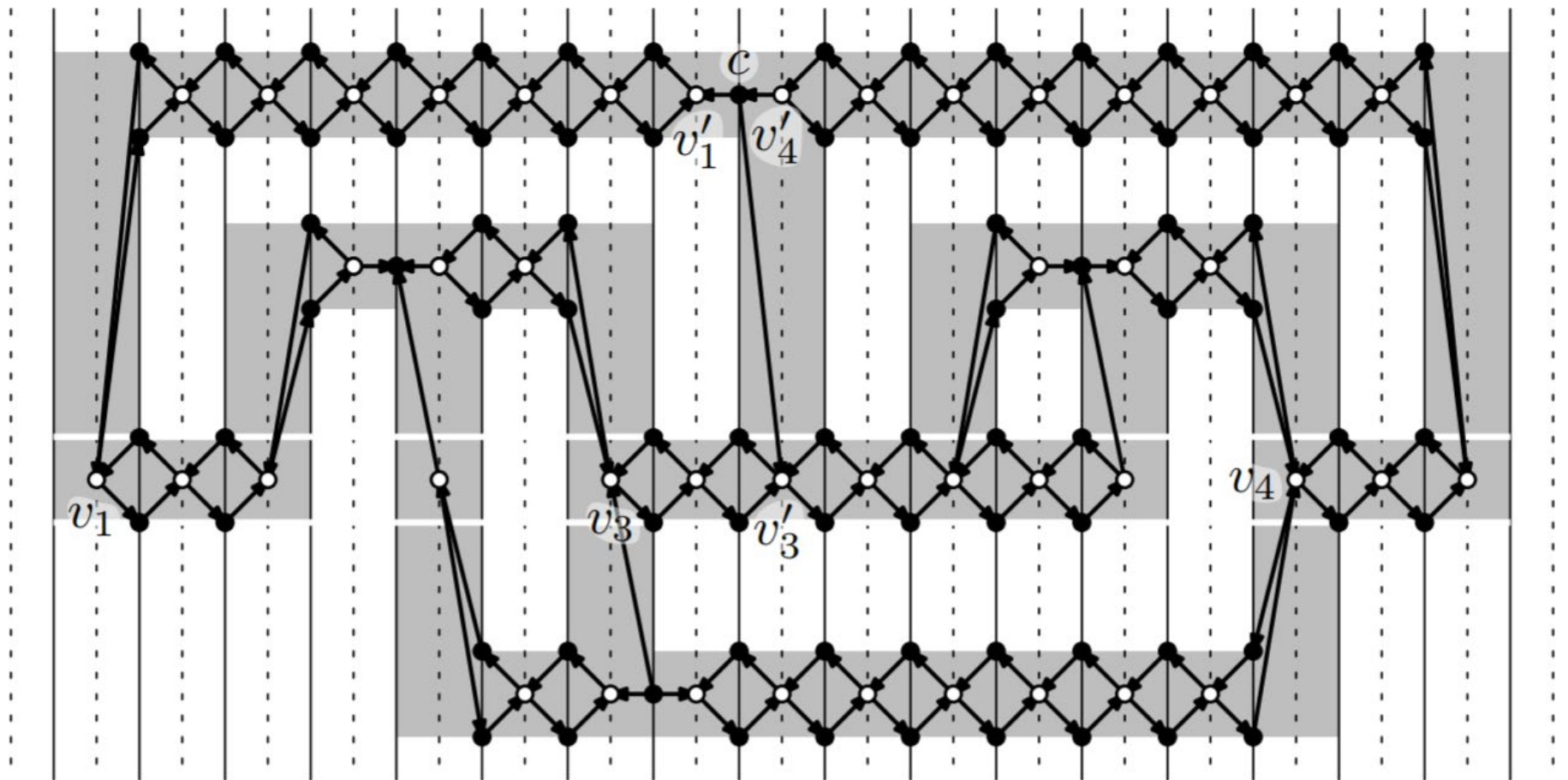
Linked Planar 3SAT is Hard [Pilz 2018]

reduction from Planar [Monotone] Rectilinear 3SAT

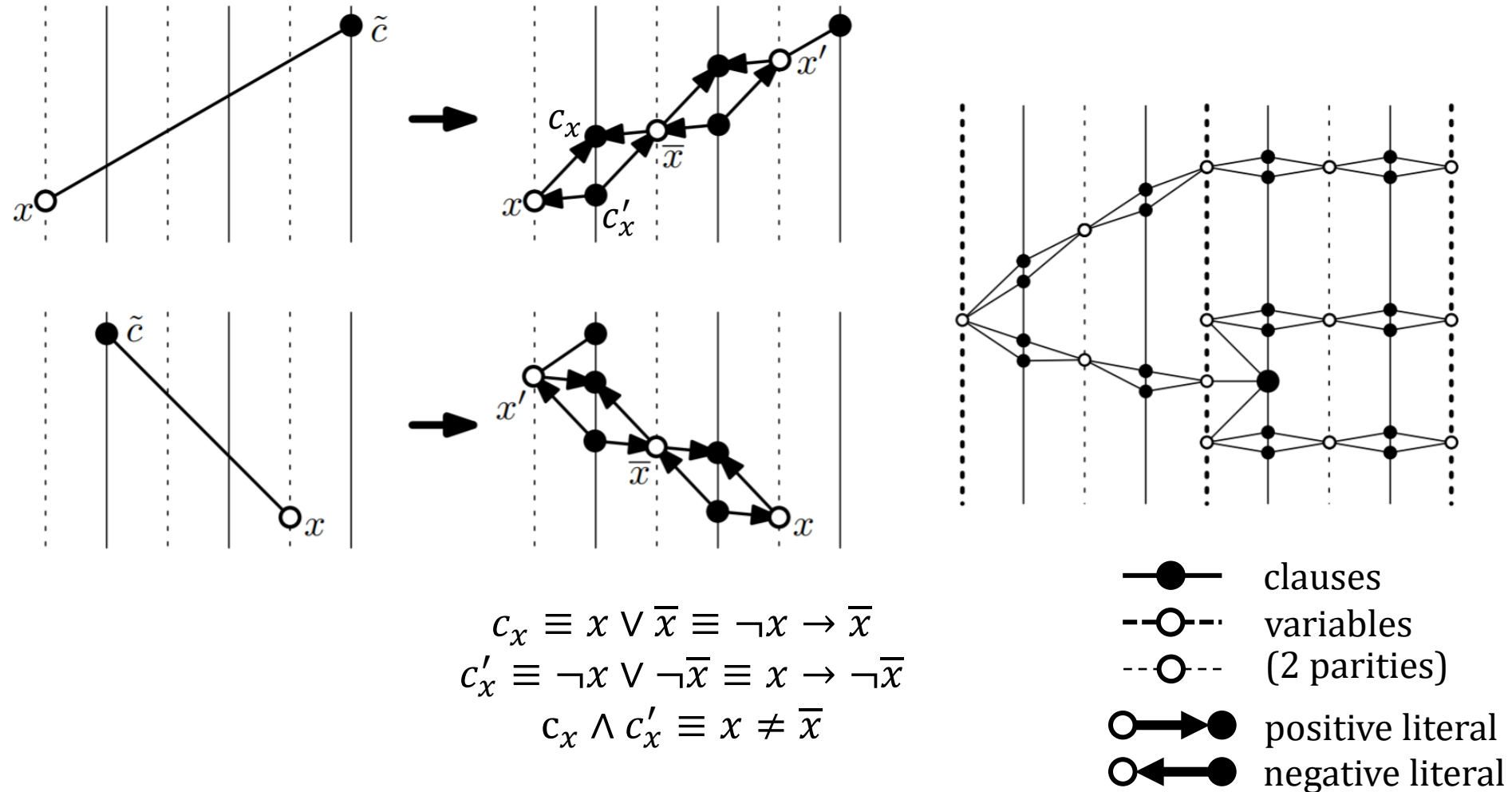


Linked Planar 3SAT is Hard [Pilz 2018]

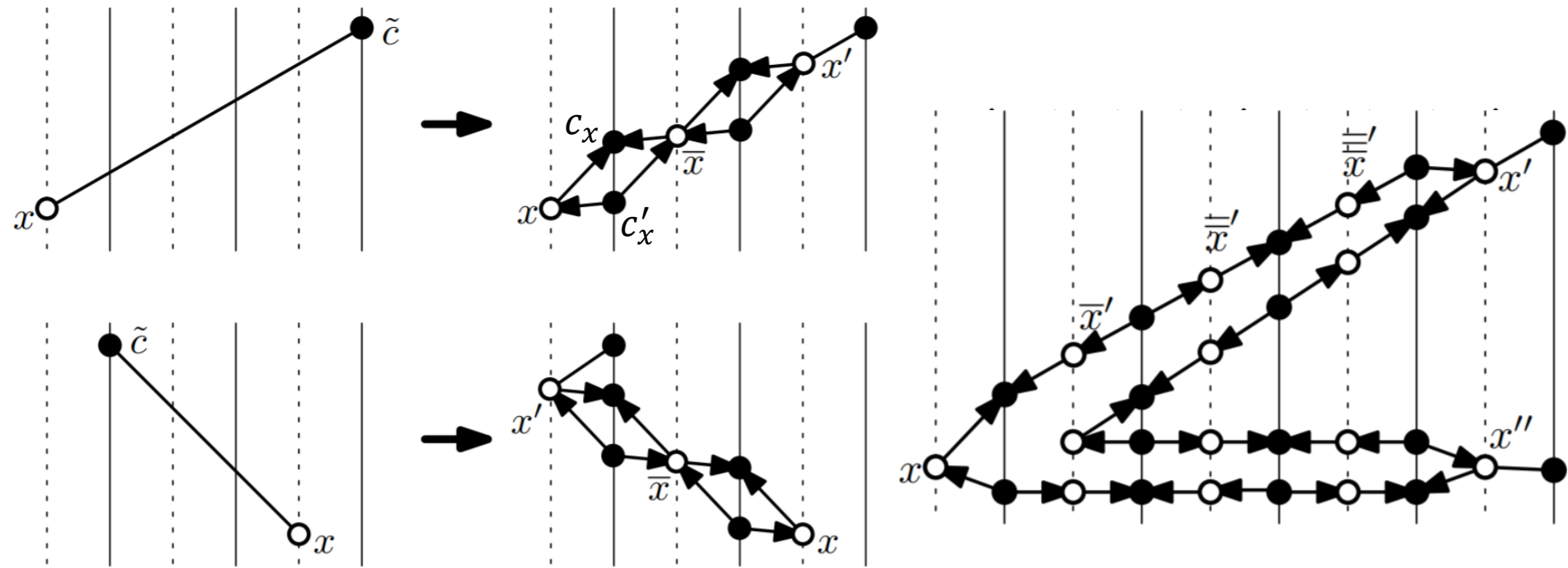
reduction from Planar [Monotone] Rectilinear 3SAT



Linked Planar Monotone 3SAT is Hard [Pilz 2018]



Linked Planar Monotone 3SAT-3 is Hard [Pilz 2018]



$$c_x \equiv x \vee \bar{x} \equiv \neg x \rightarrow \bar{x}$$

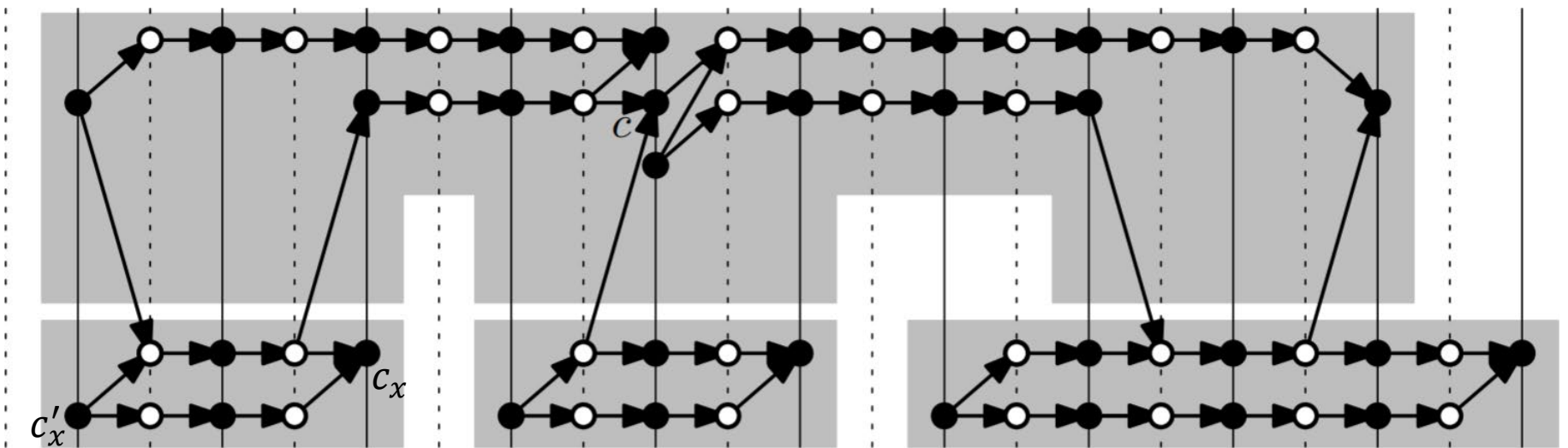
$$c'_x \equiv \neg x \vee \neg \bar{x} \equiv x \rightarrow \neg \bar{x}$$

$$c_x \wedge c'_x \equiv x \neq \bar{x}$$

- clauses
- variables
- (2 parities)
- positive literal
- ←● negative literal

Sided Linked Planar 3SAT-3 is Hard

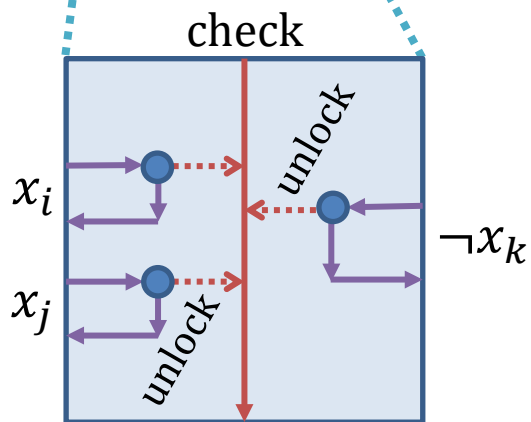
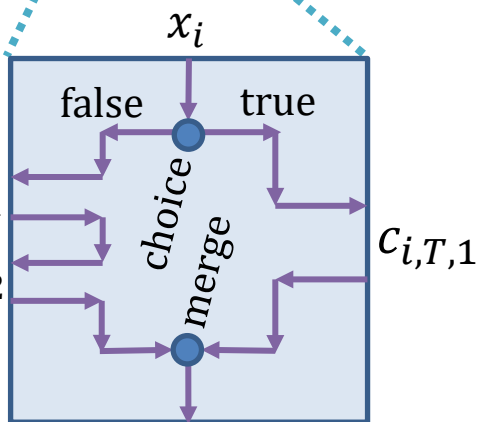
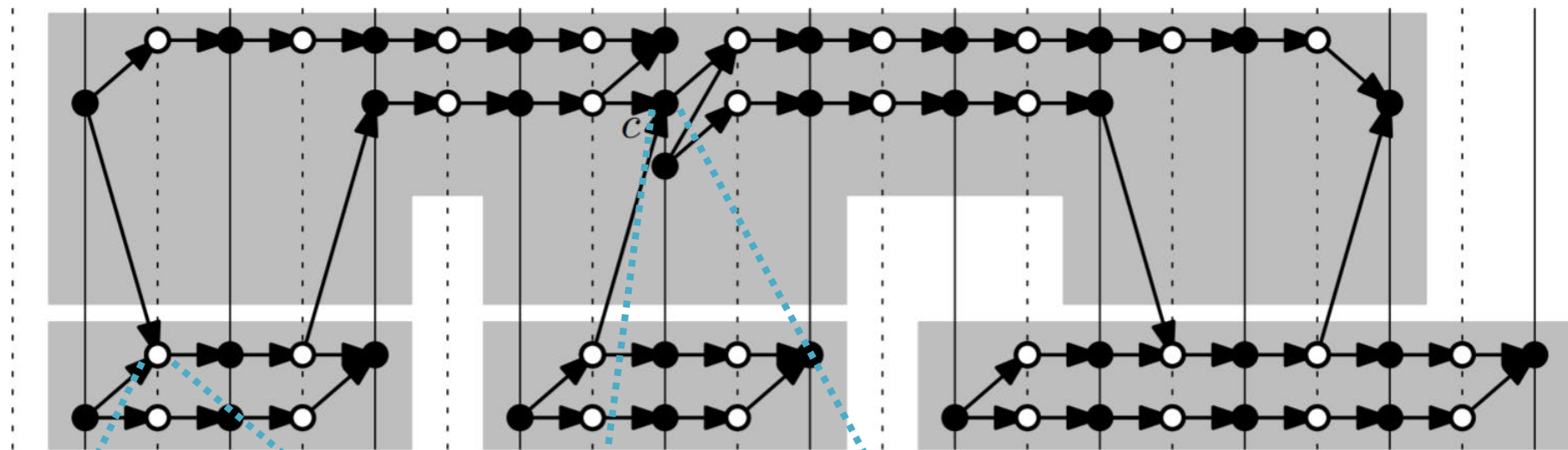
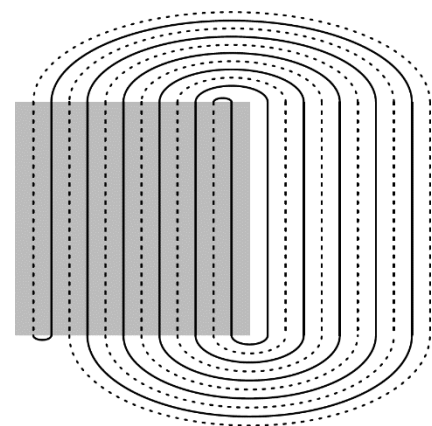
[Pilz 2018]



$$c_x \equiv x \vee \bar{x} \equiv \neg x \rightarrow \bar{x}$$
$$c'_x \equiv \neg x \vee \bar{\bar{x}} \equiv x \rightarrow \bar{\bar{x}}$$
$$c_x \wedge c'_x \equiv x \neq \bar{x}$$

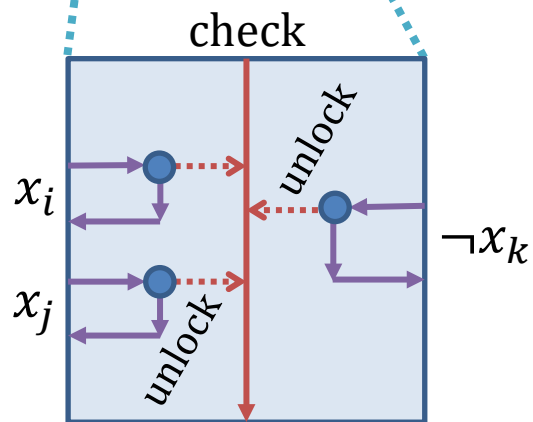
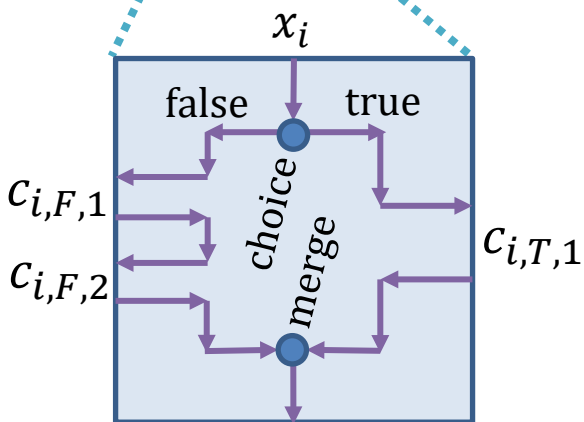
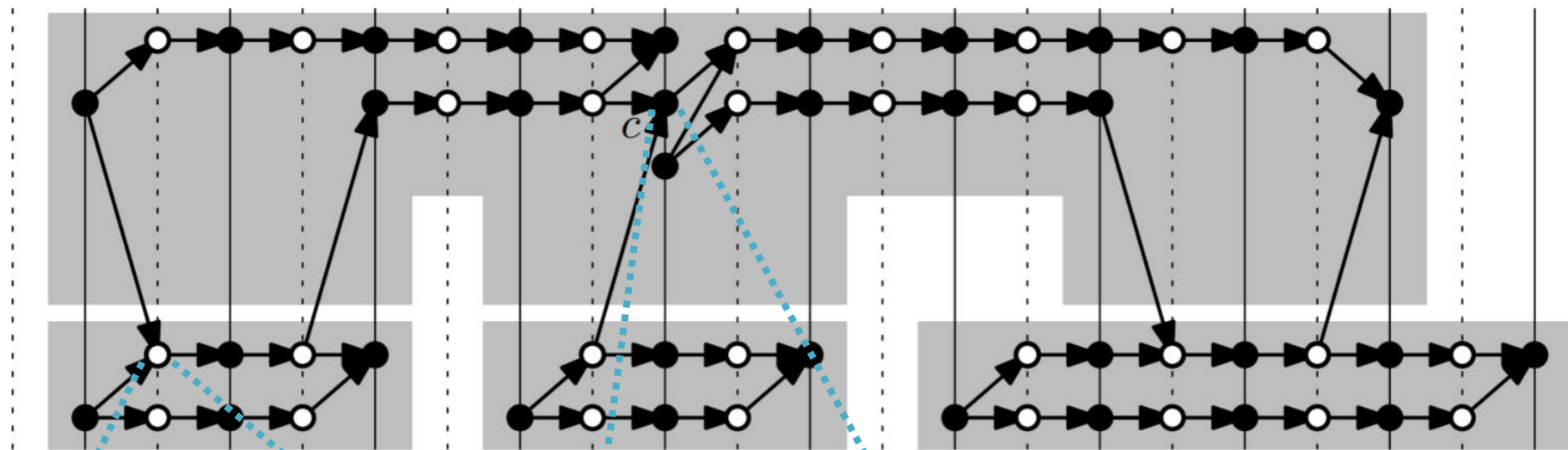
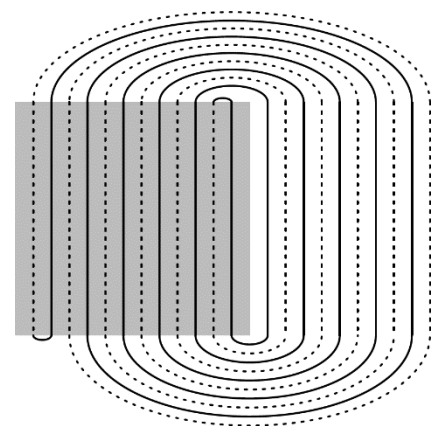
- clauses
- variables
- positive literal
- ←● negative literal





Reducing from Sided Linked Planar 3SAT-3



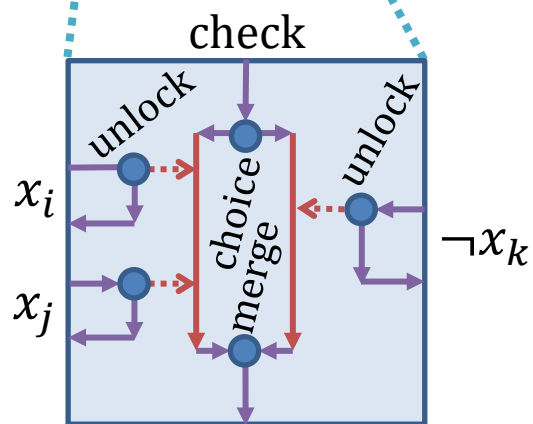
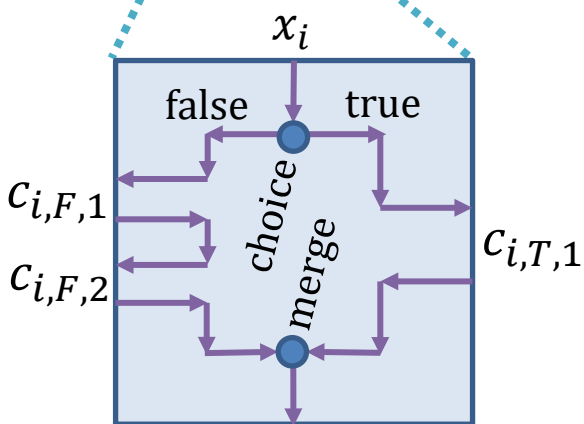
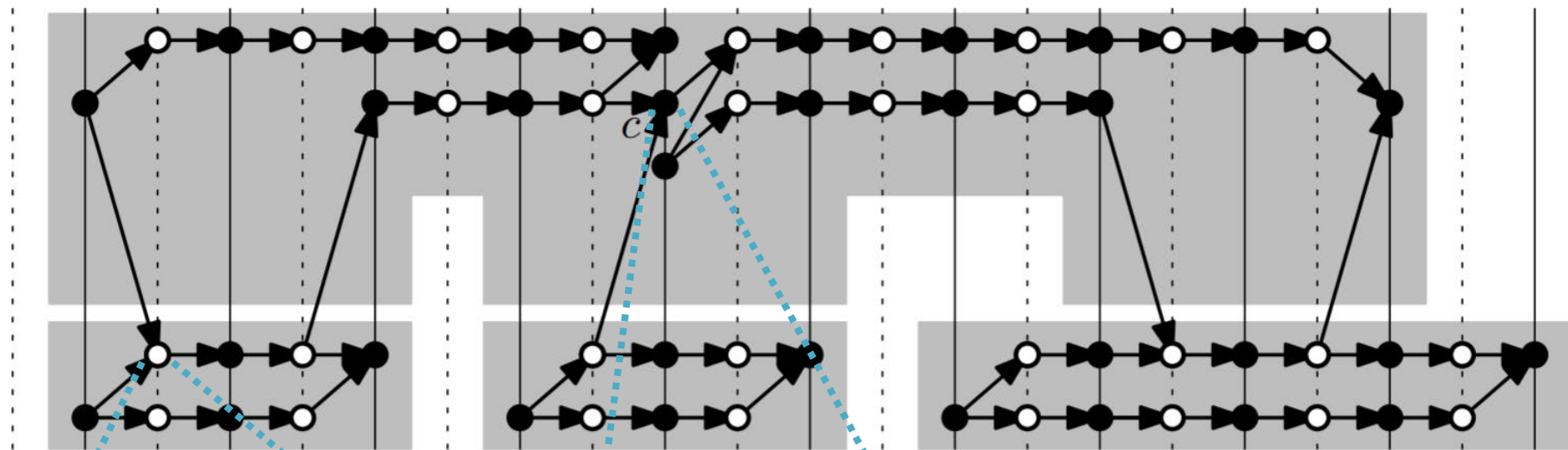
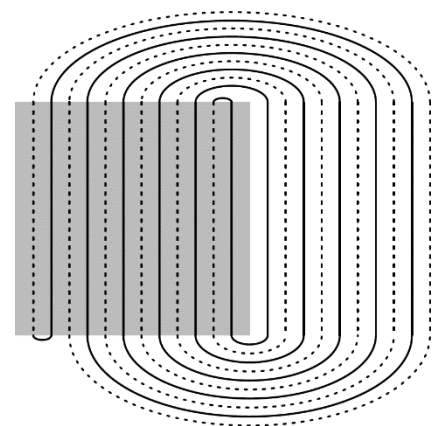
- clause check line
- variable choice line
- positive literal
- negative literal





Reducing from Sided Linked Planar 3SAT-3



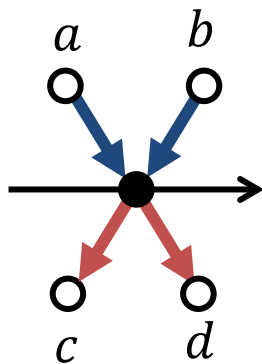
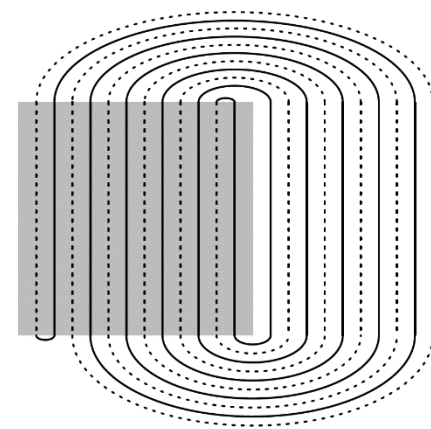
-  clause check line
-  variable choice line
-  positive literal
-  negative literal

Reducing from Sided Linked Planar 3SAT-3



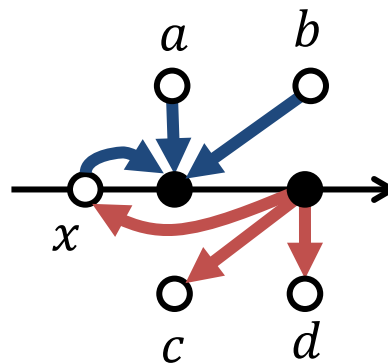
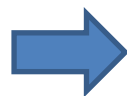
-  clause check line
-  variable choice line
-  positive literal
-  negative literal

Sided Interlinked Planar Monotone 3SAT-3



$$a \vee b \vee \neg c \vee \neg d$$

sided
linked

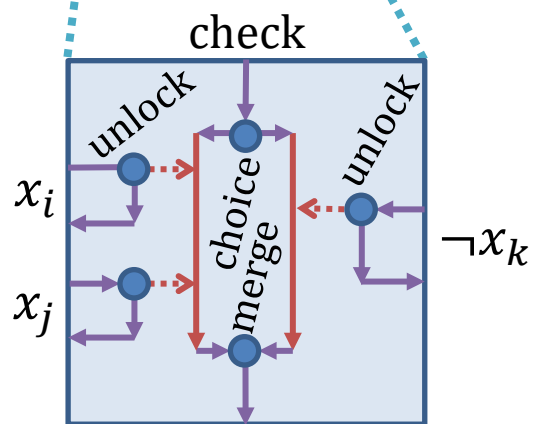
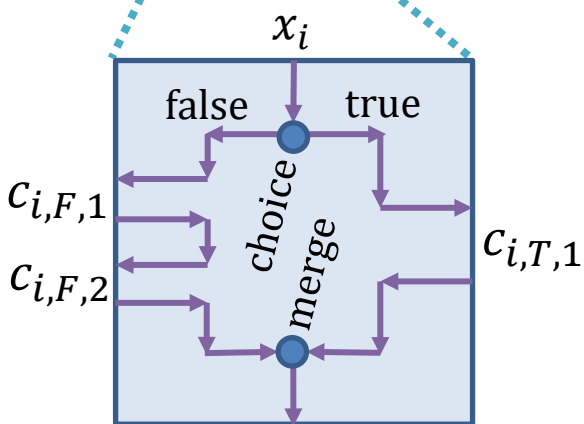
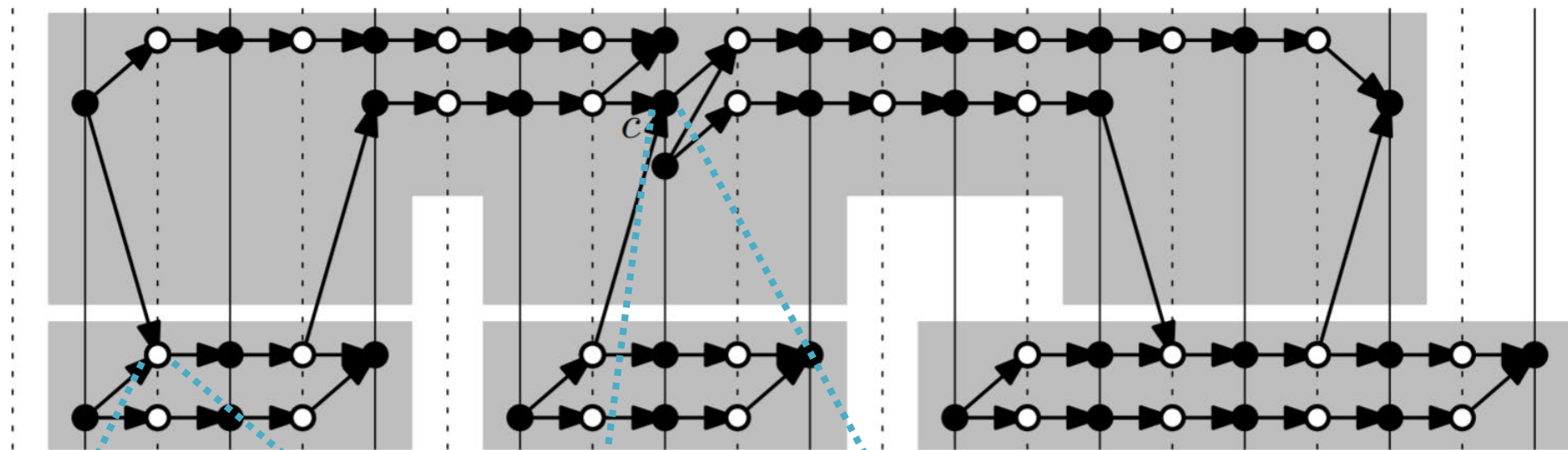
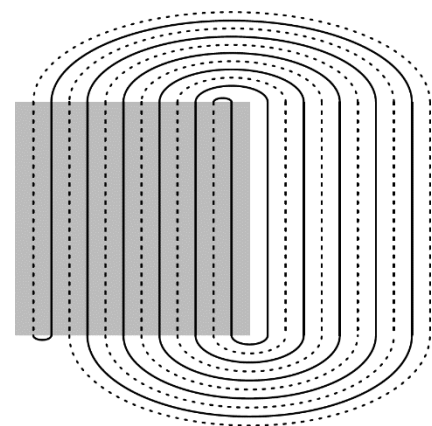


$$(a \vee b \vee x) \wedge (\neg x \vee \neg c \vee \neg d) \equiv (a \vee b \vee x) \wedge (x \rightarrow (\neg c \vee \neg d))$$

sided
interlinked
monotone

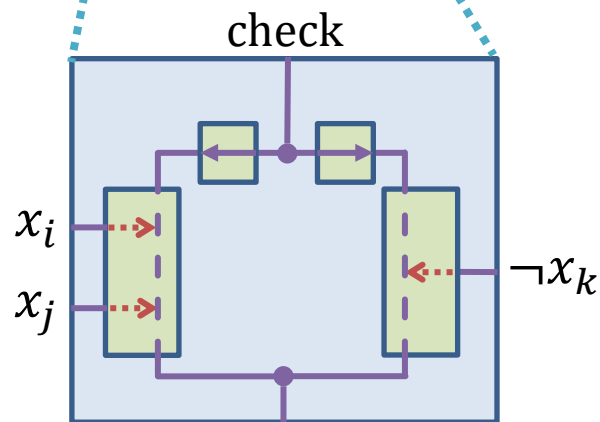
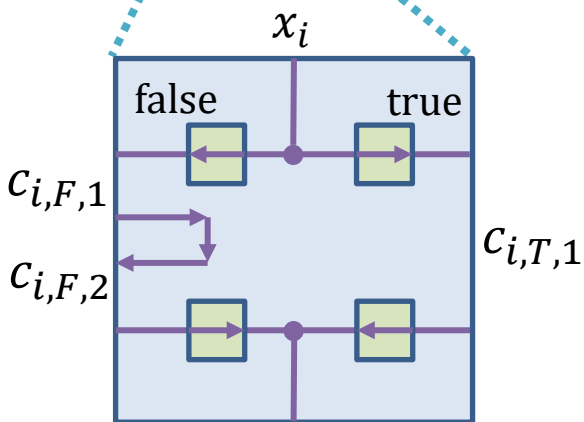
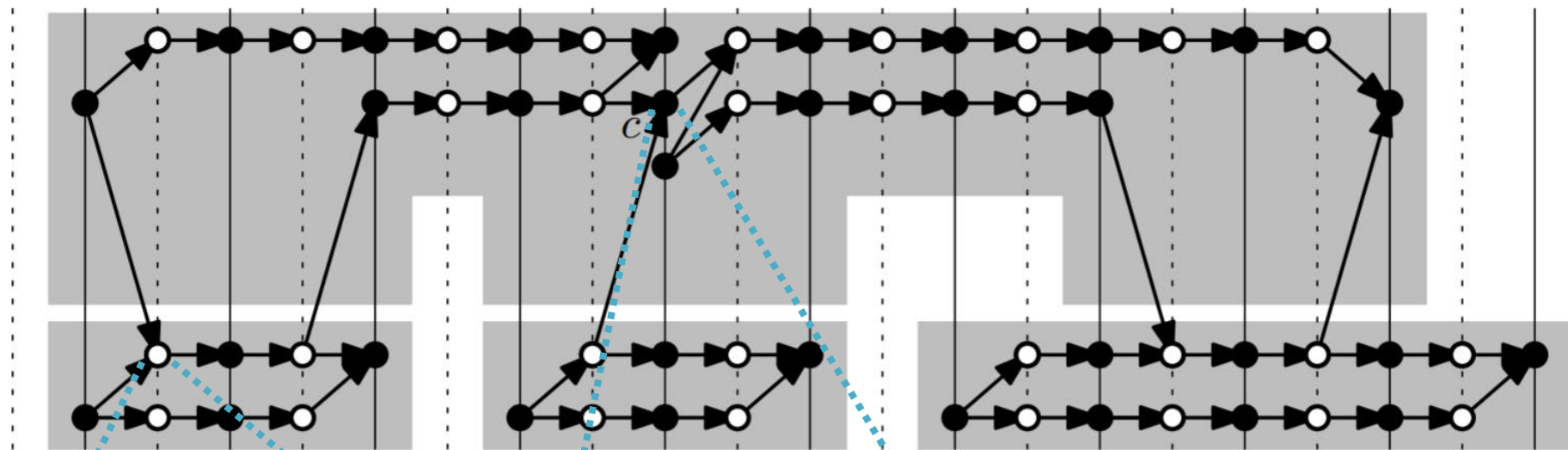
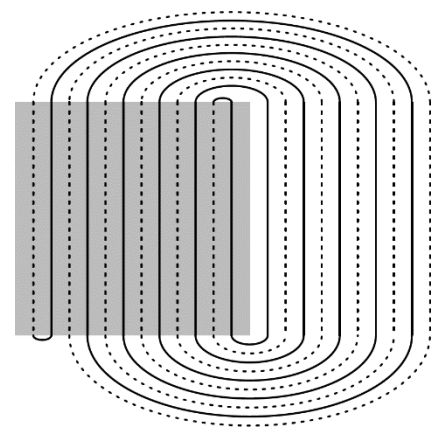
- clauses
- variables
- positive literal
- ←● negative literal





Reducing from Sided Linked Planar 3SAT-3



- clause check line
- variable choice line
- positive literal
- negative literal

Reducing from Sided Linked Planar 3SAT-3



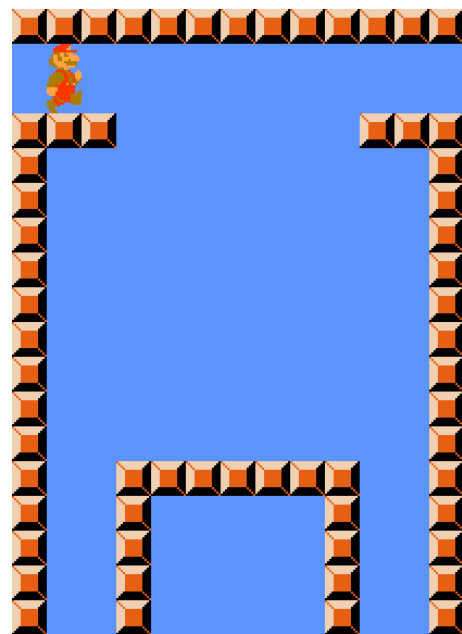
-  clause check line
-  variable choice line
-  positive literal
-  negative literal



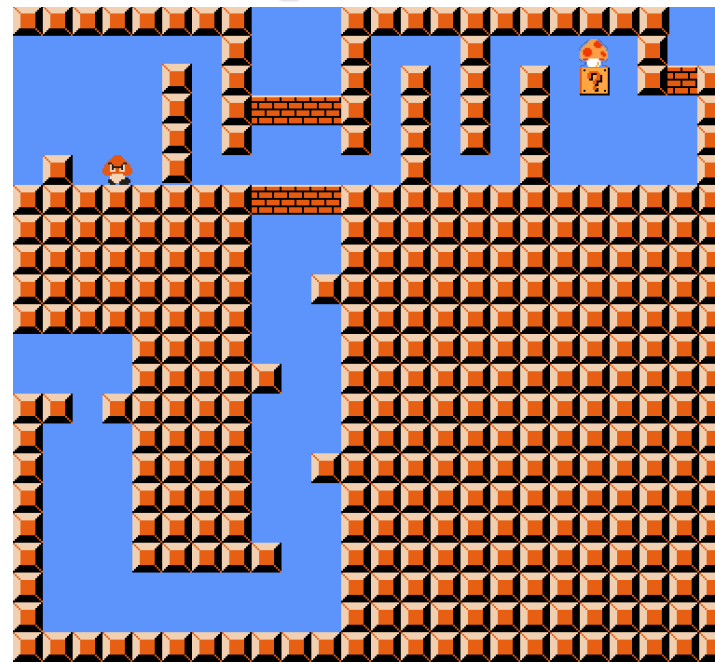
Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

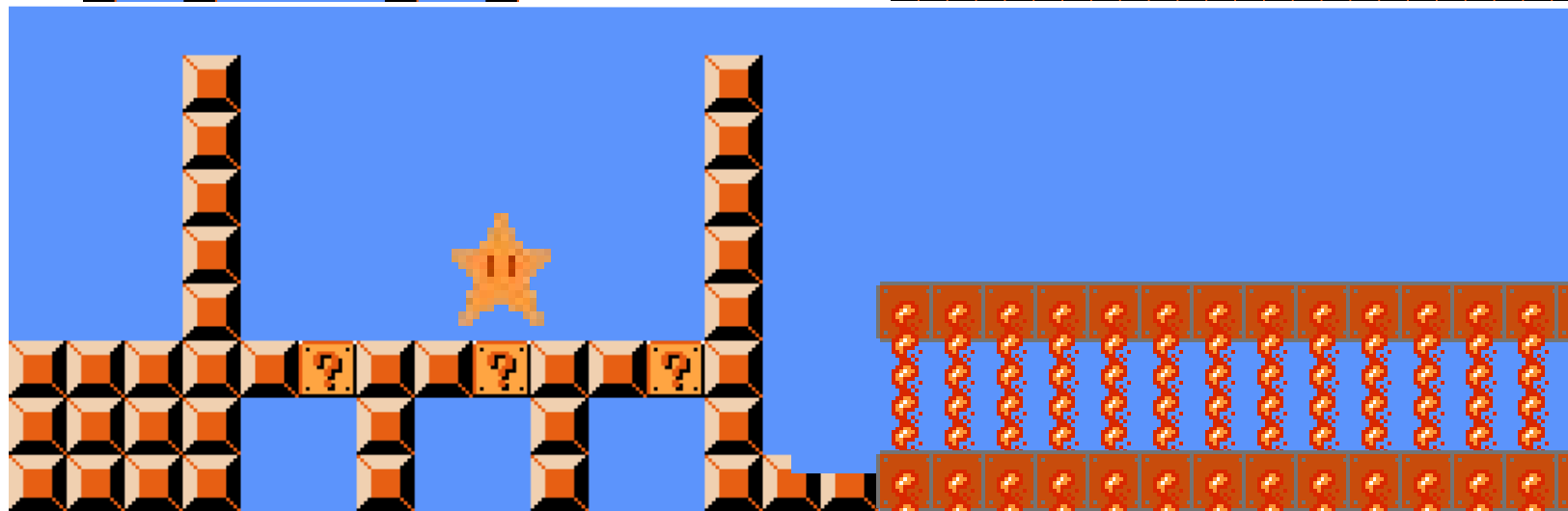
variable



CROSSOVER



clause

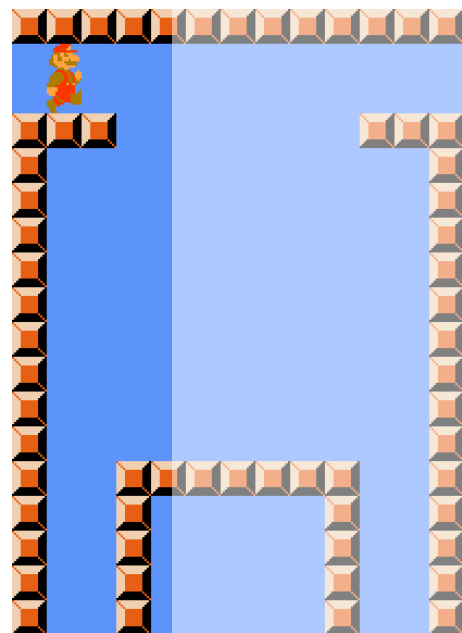




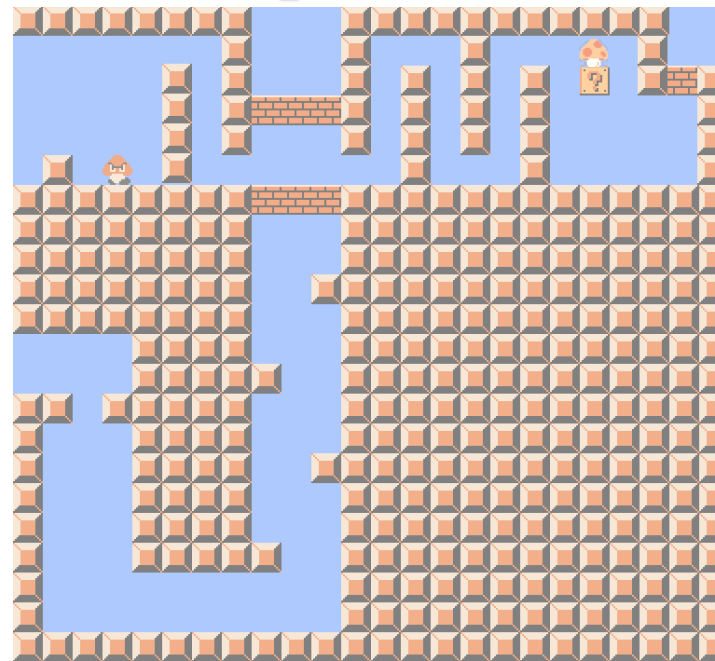
Super Mario Bros. is NP-Hard

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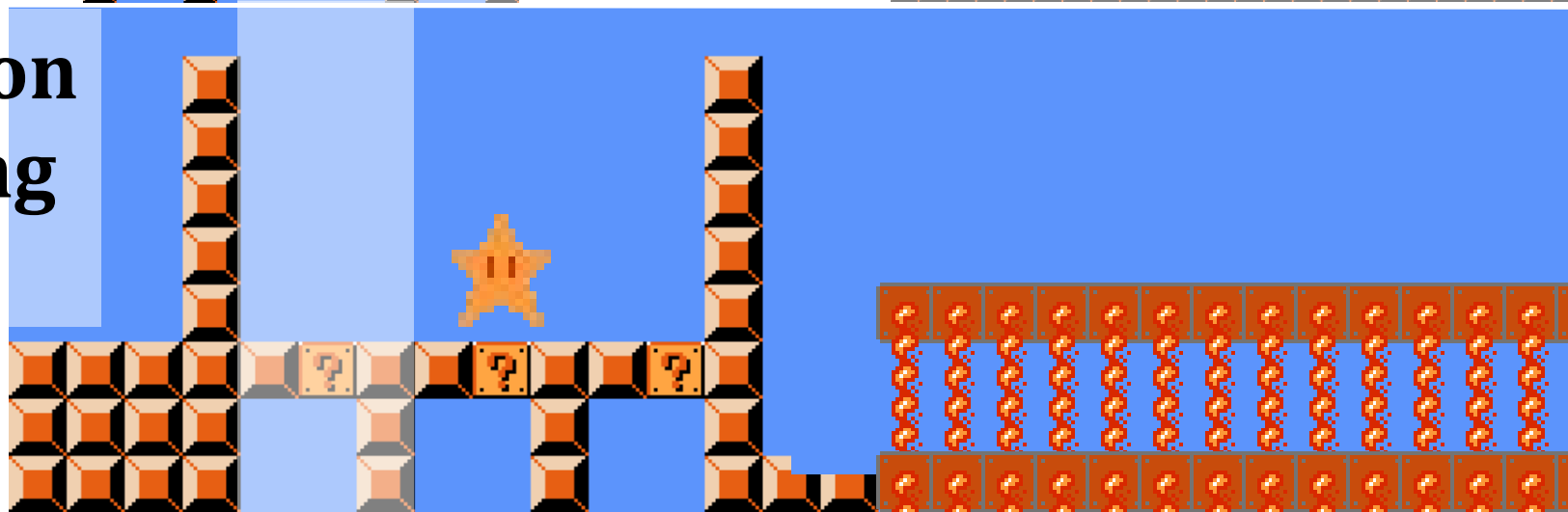
one
way



crossover



2-button
opening
door

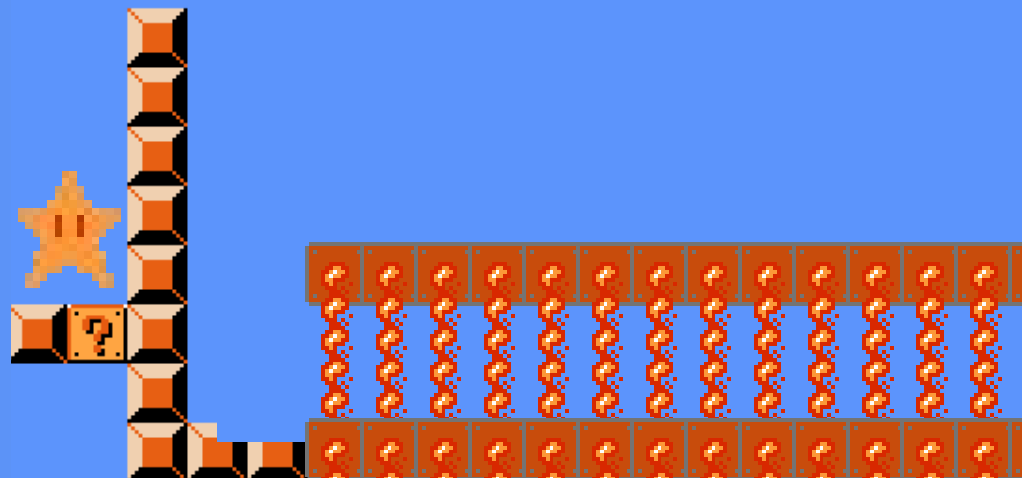




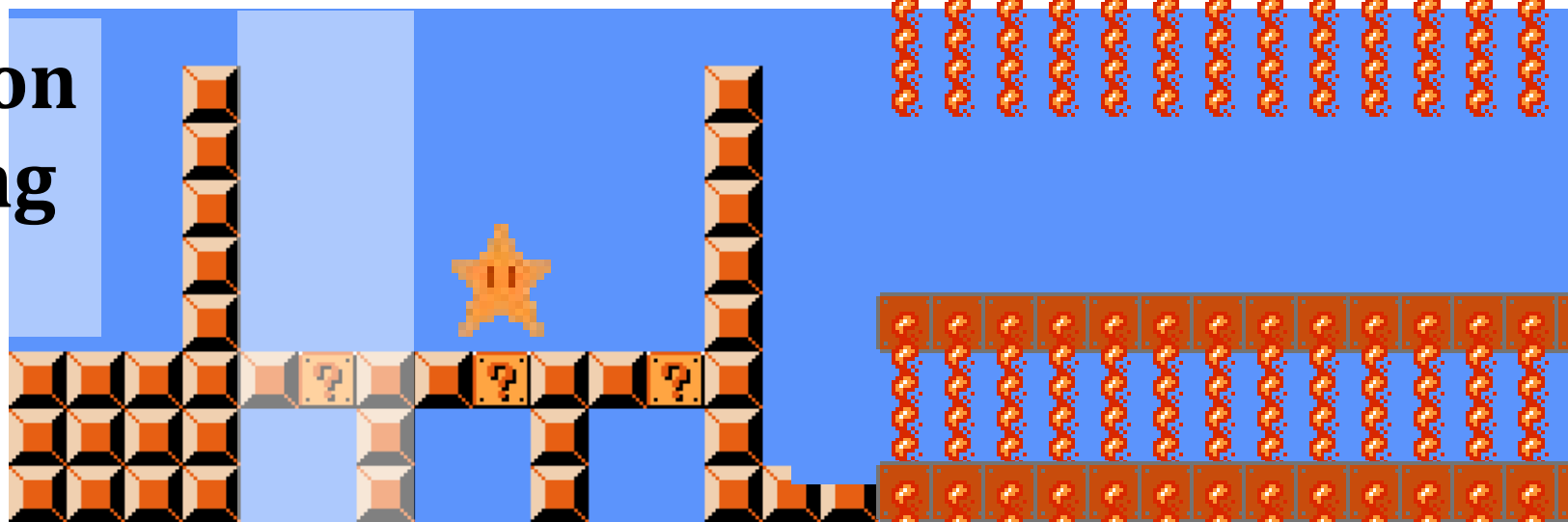
Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

**directed
crumbler**



**2-button
opening
door**





Legend of Zelda Hookshot



Let's Play Zelda: A Link to the Past #13 – Evil Popcorn Chicken & #15 – Flame On
by newfiebangaa http://youtu.be/6i_YGCy5krM & http://youtu.be/ezsLn3_KcGs



Legend of Zelda Hookshot is NP-hard

[Aloupis, Demaine, Guo 2012]

A Link to the Past



variable

clause

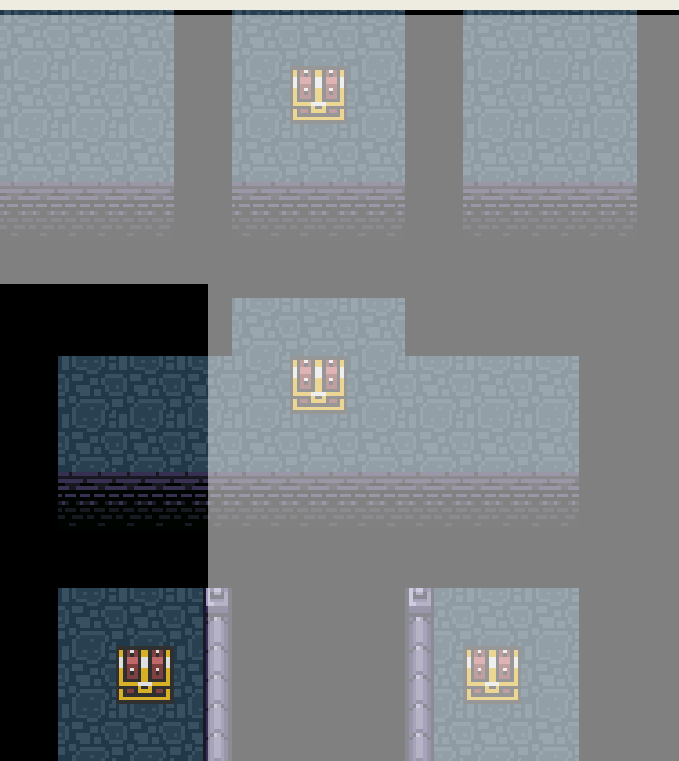
crossover



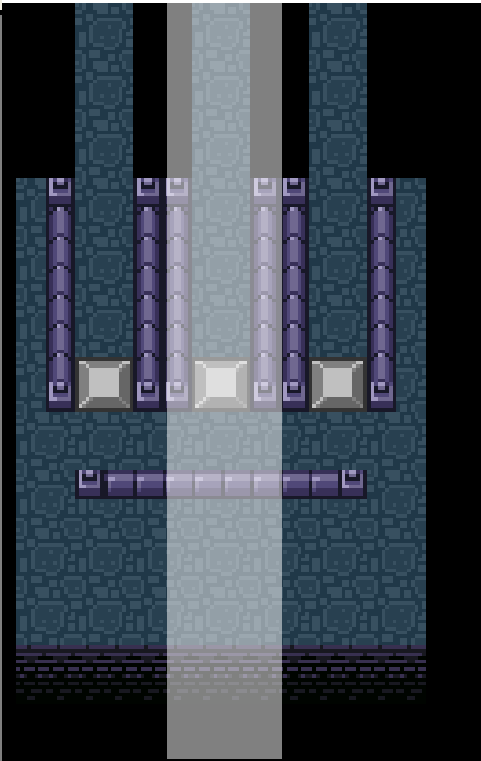
Legend of Zelda Hookshot is NP-hard

[Aloupis, Demaine, Guo 2012]

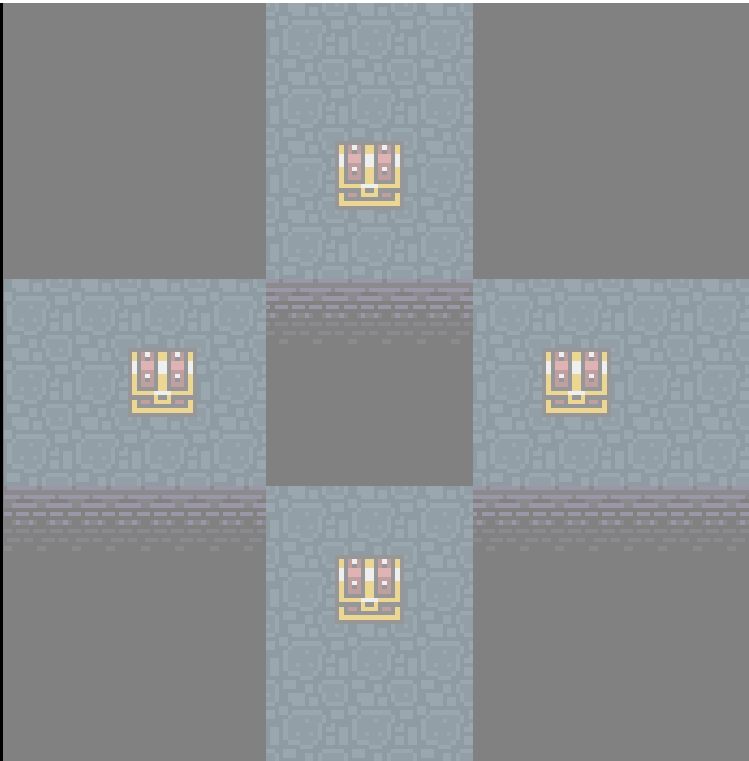
A Link to the Past



one way



2-button opening door



~~crossover~~



Metroid

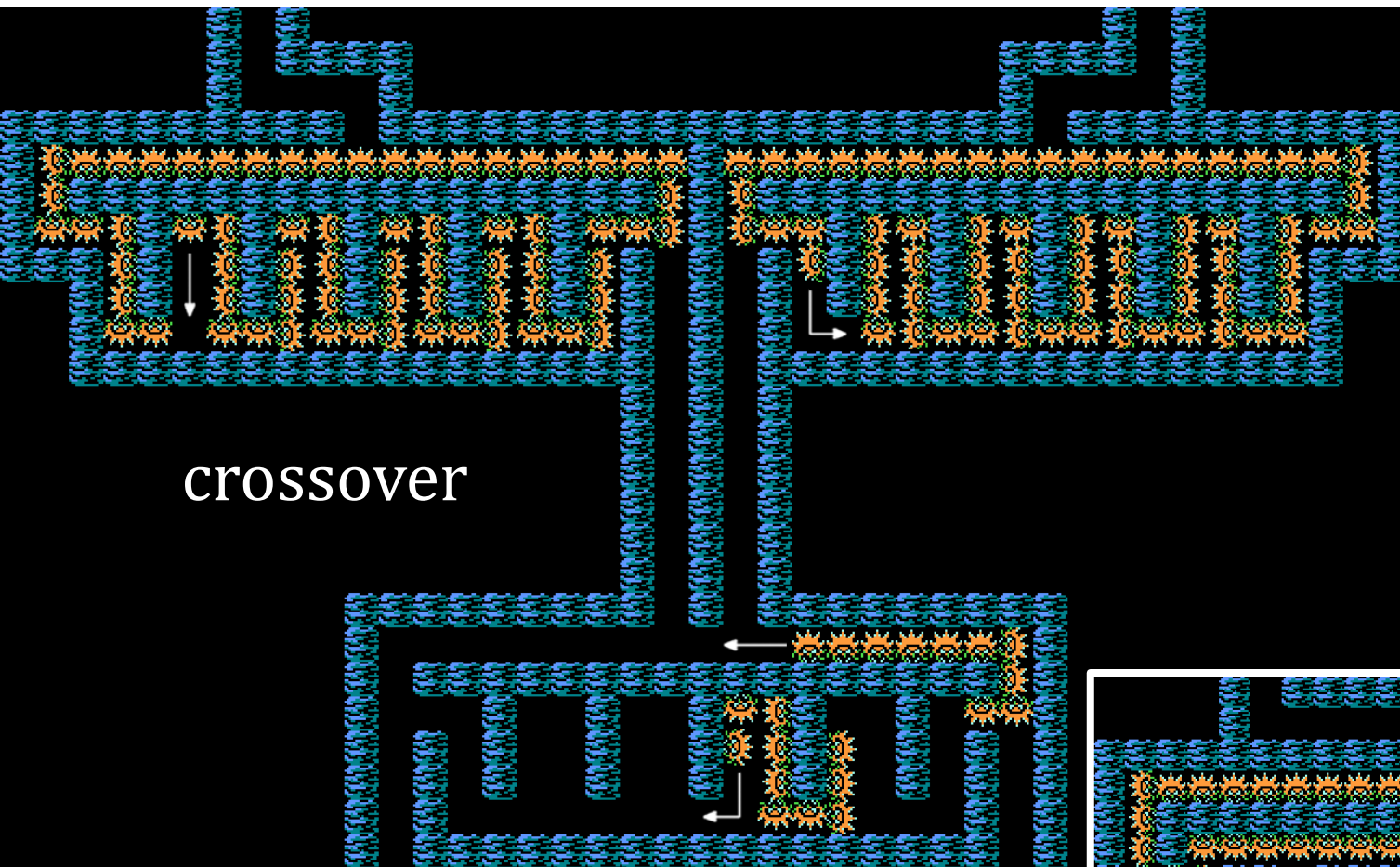


Metroid NES 100% Walkthrough Part 1 Alternate Route
by Timothy Cookson <http://youtu.be/INkHYcWvQag>



Metroid is NP-hard

[Aloupis, Demaine, Guo, Viglietta 2014]



clause



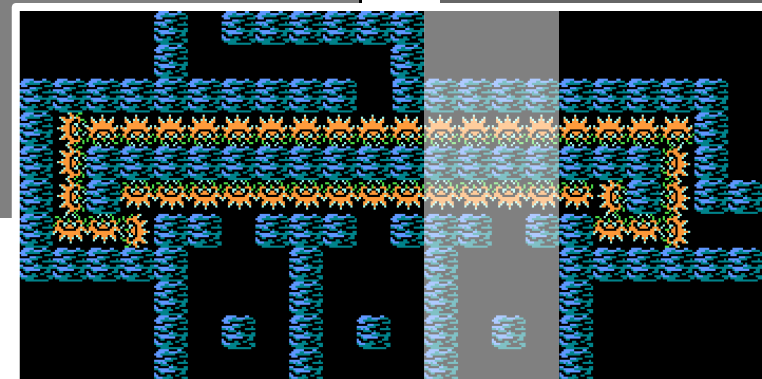


Metroid is NP-hard

[Aloupis, Demaine, Guo, Viglietta 2014]

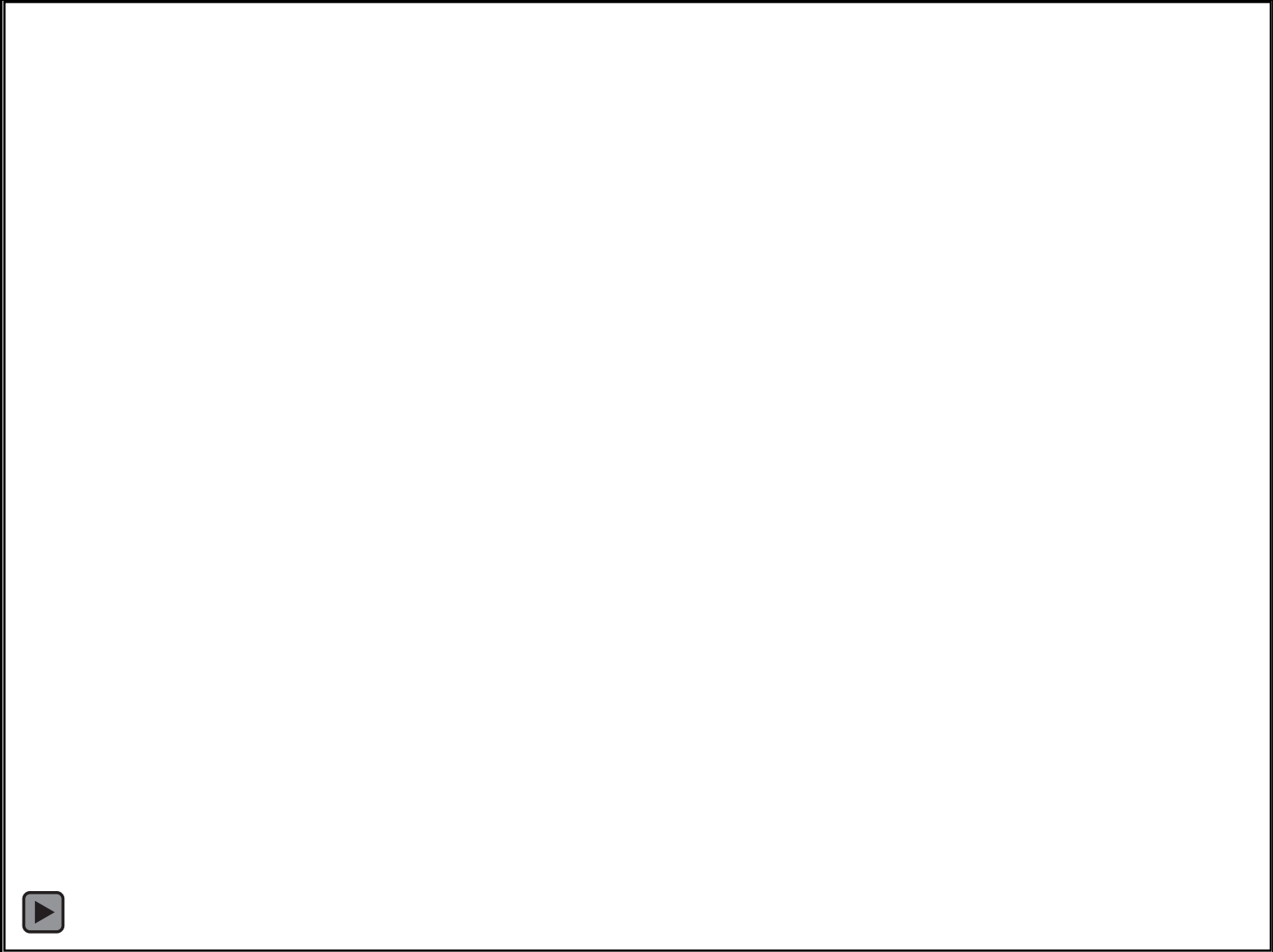


2-button
opening
door





Donkey Kong Country



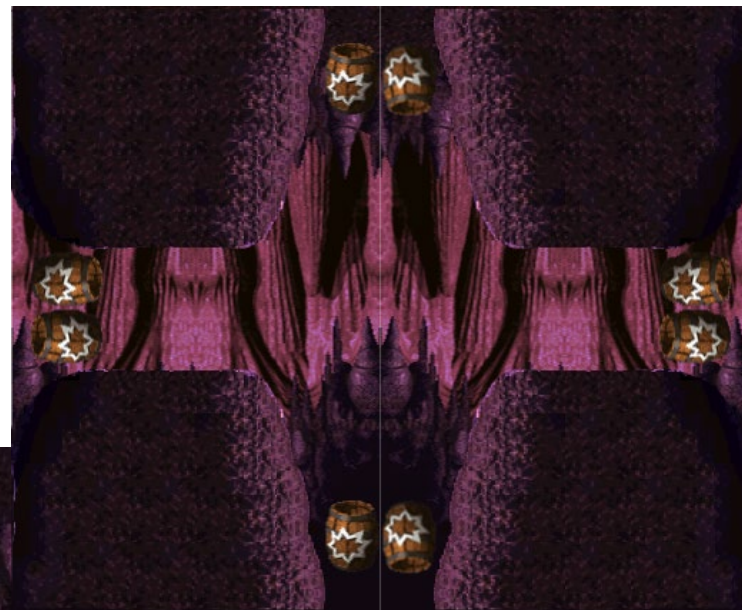
Donkey Kong Country - 101% Walkthrough, Part 5 - Barrel Cannon Canyon
by GarlandTheGreat <http://youtu.be/w6m0muSdXr0>



Donkey Kong Country is NP-hard

[Aloupis, Demaine, Guo 2012]

clause



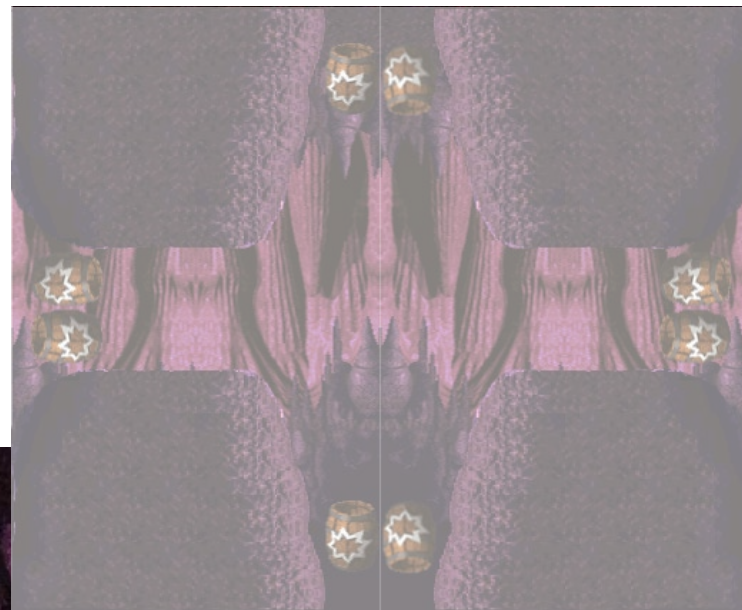
crossover



Donkey Kong Country is NP-hard

[Aloupis, Demaine, Guo 2012]

2-button opening door



~~crossover~~

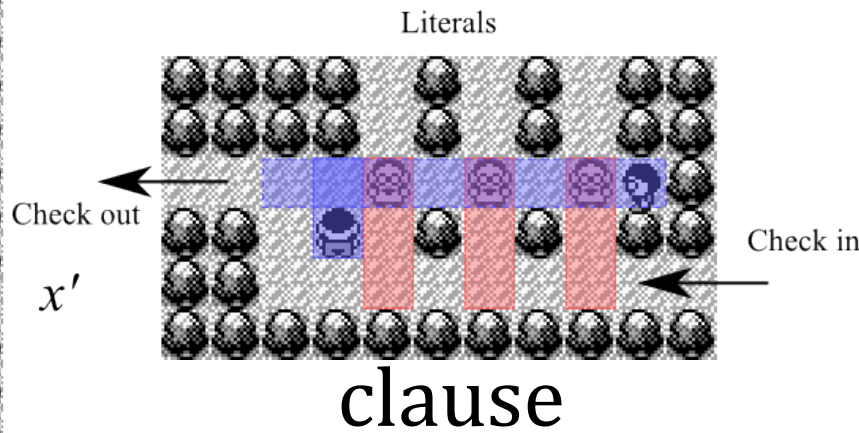
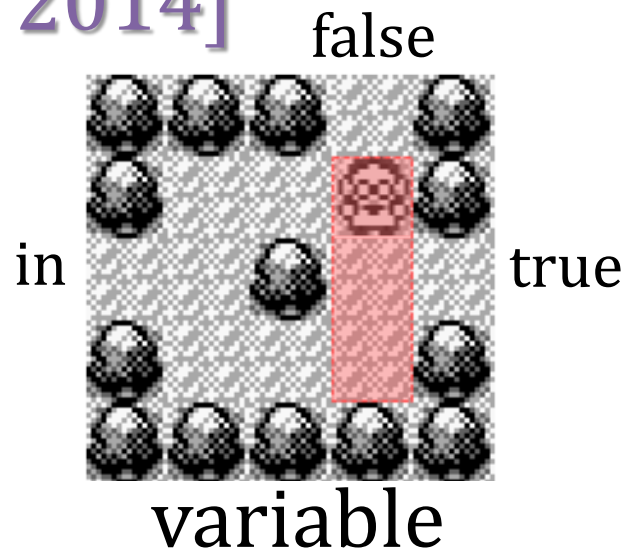
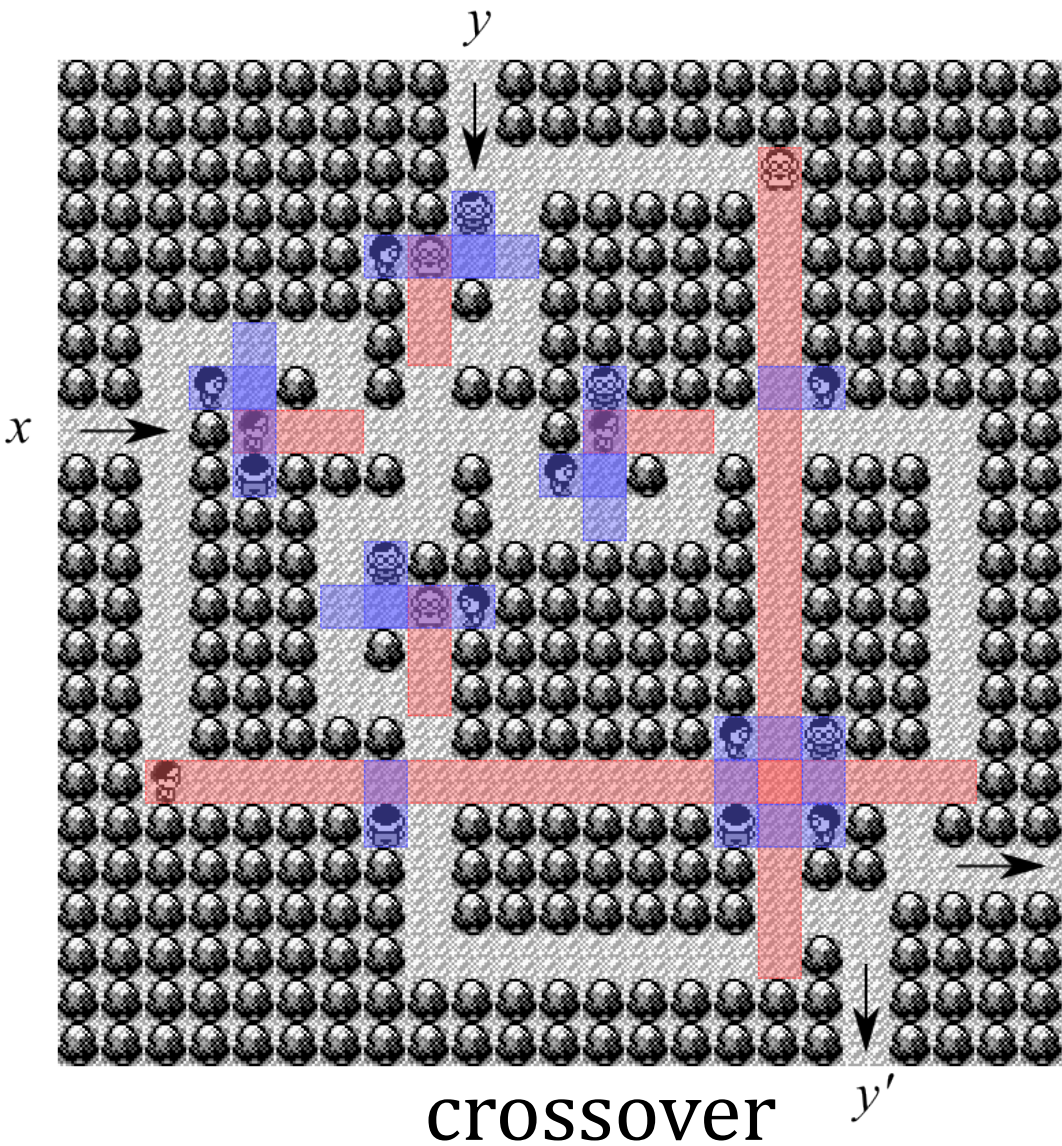
is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

“**Weak Trainers** each hold a **Level 100 Electrode** with maximum **Speed** and equipped with only the **Self Destruct** move. **Strong Trainers** each hold two **Snorlaxes**, with Speed of 30. The player has no items, and only one Pokémon in his team. For Generation I and II games (**Red/Blue/Yellow** and **Gold/Silver/Crystal** versions respectively), the player holds a **Gastly** which has learned Self Destruct using **TM36**, and its **PP** for its other moves have all been expended, so it can only use Self Destruct in battle. When the player encounters a weak Trainer, the enemy Electrode will move first and use Self Destruct, which deals no damage to Gastly since Self Destruct is a **Normal type attack** and Gastly is **Ghost type**, so the weak Trainer immediately loses. When the player encounters a strong Trainer, Gastly moves first and uses Self Destruct, causing the player to lose (even if it defeats the enemy Snorlax, the opponent holds another one). This implementation only works in Generations I and II since TM36 exists only in **Generation I** and the **Time Capsule** feature in **Generation II** allows a Gastly with Self Destruct to be traded from Generation I to Generation II. In **Generations III, IV, and V**, Gastly can be replaced by **Duskull**, which is allowed to learn the move **Memento**, which serves the same purpose as Self Destruct, via **breeding**.”

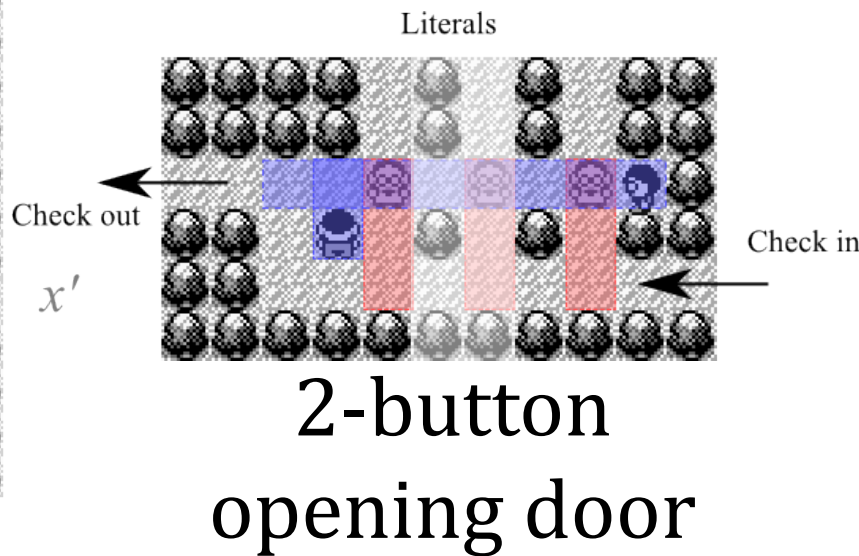
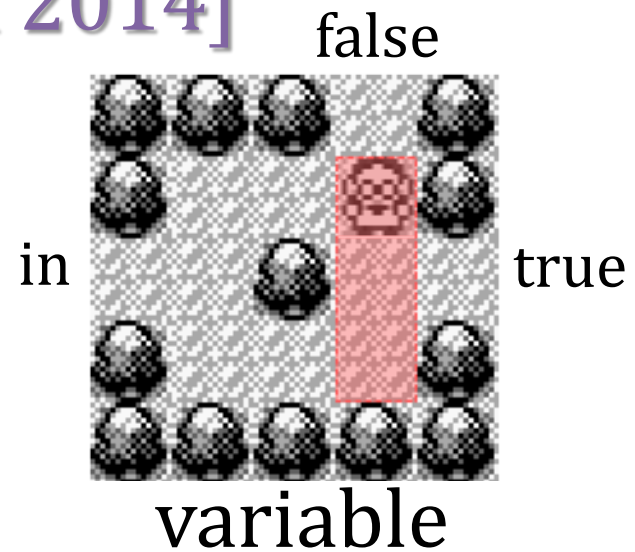
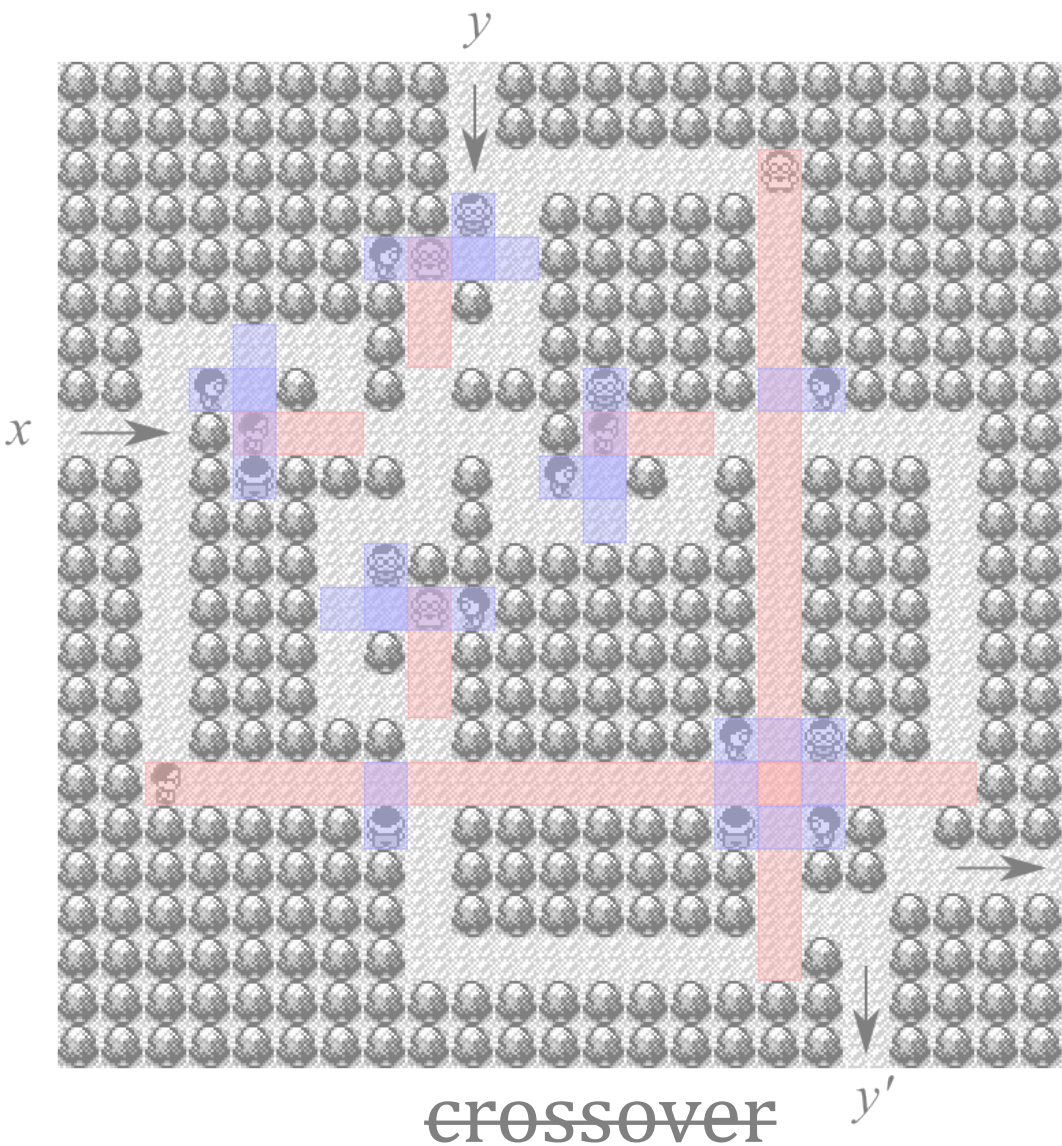
POKÉMON is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]



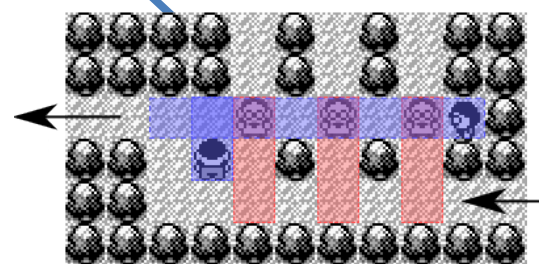
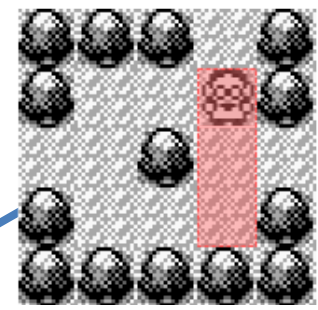
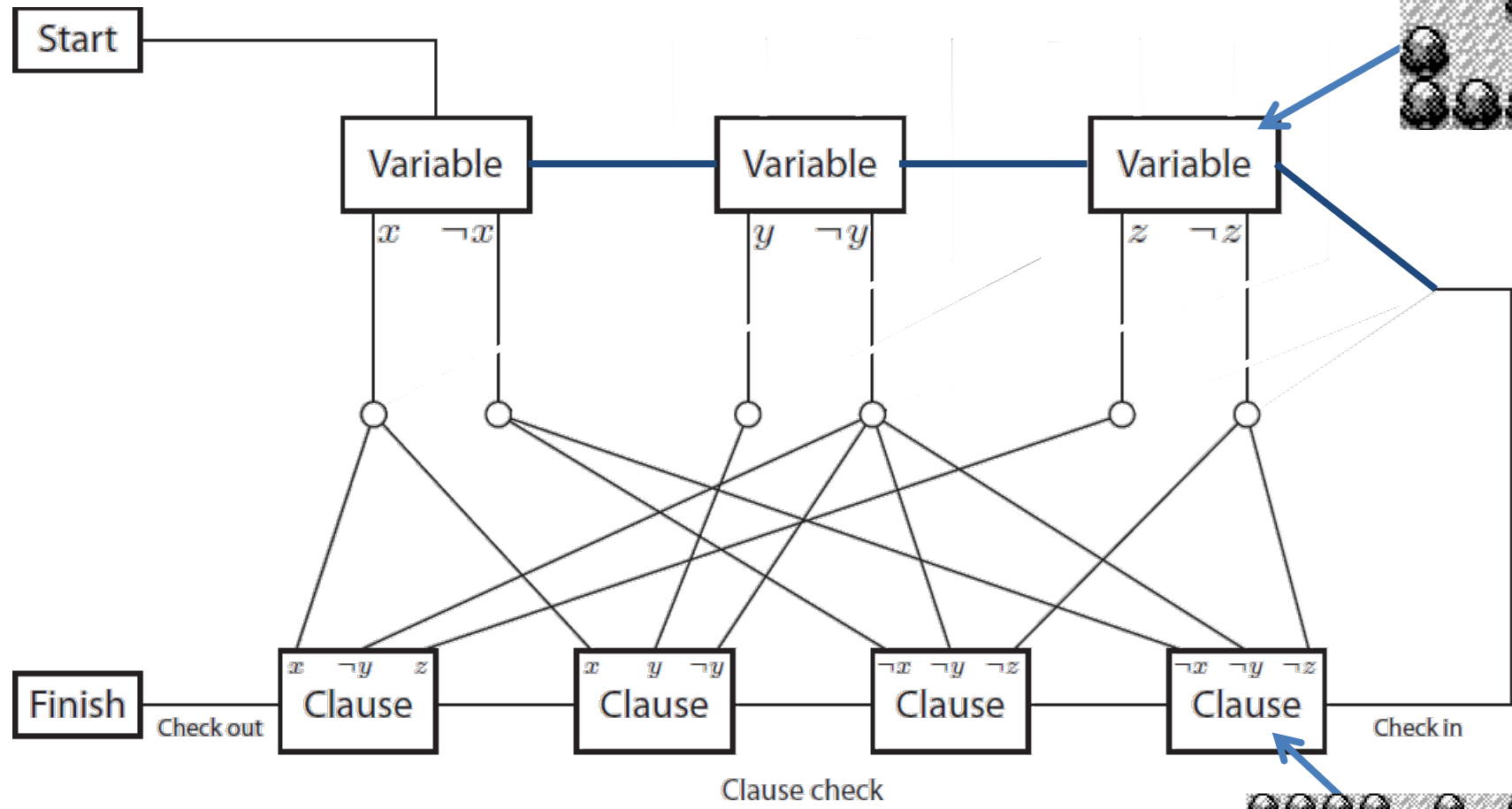
POKÉMON is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]



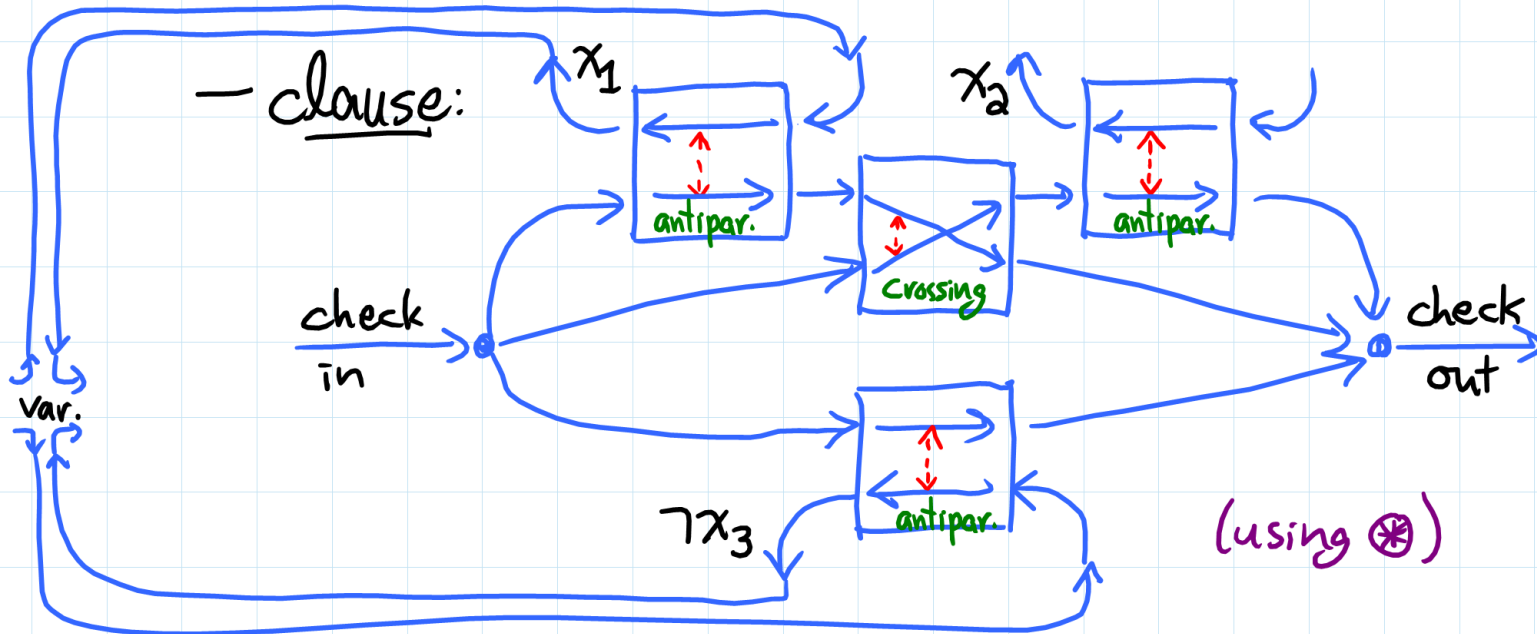
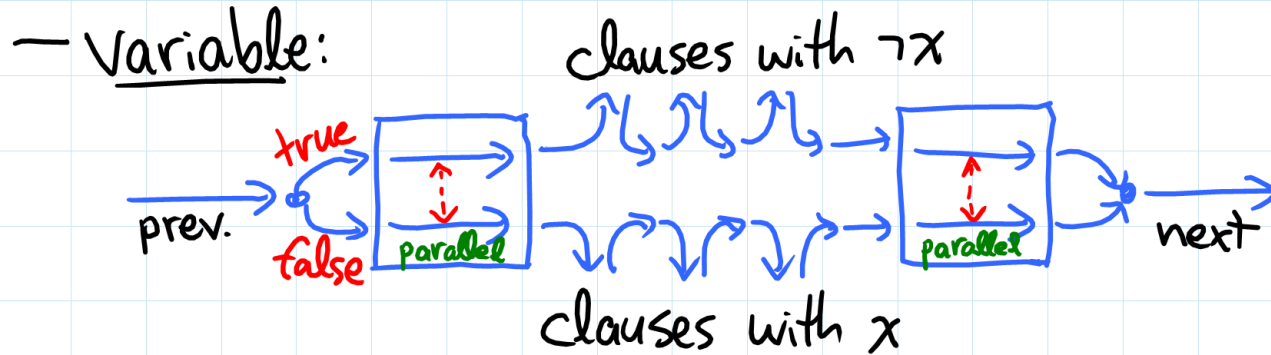
POKÉMON is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]



$$(x \text{ OR } \neg y \text{ OR } z) \ \& \ (x \text{ OR } y \text{ OR } \neg y) \ \& \\ (\neg x \text{ OR } \neg y \text{ OR } \neg z) \ \& \ (\neg x \text{ OR } \neg y \text{ OR } \neg z)$$

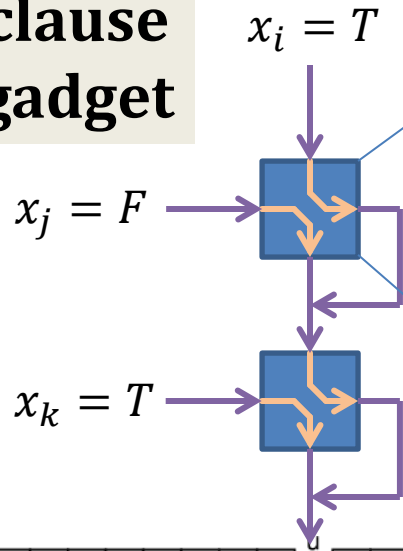
Sided Linked Planar 3SAT → Distant Closing Gadgets



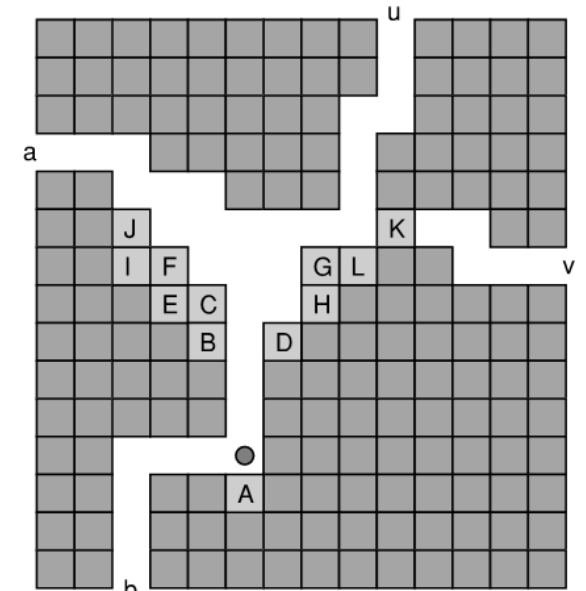
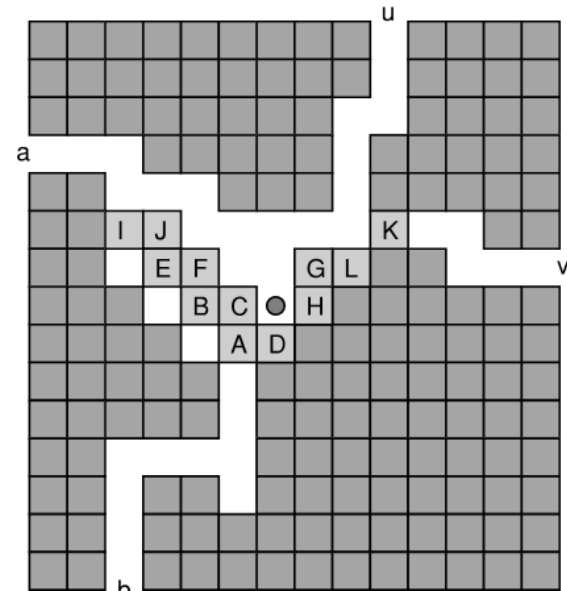
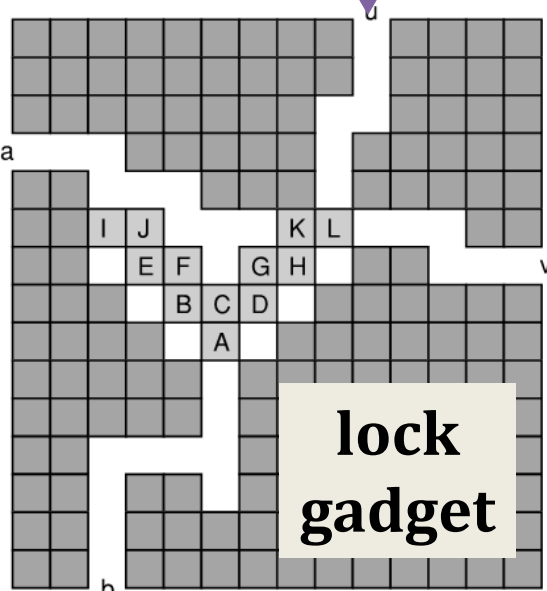
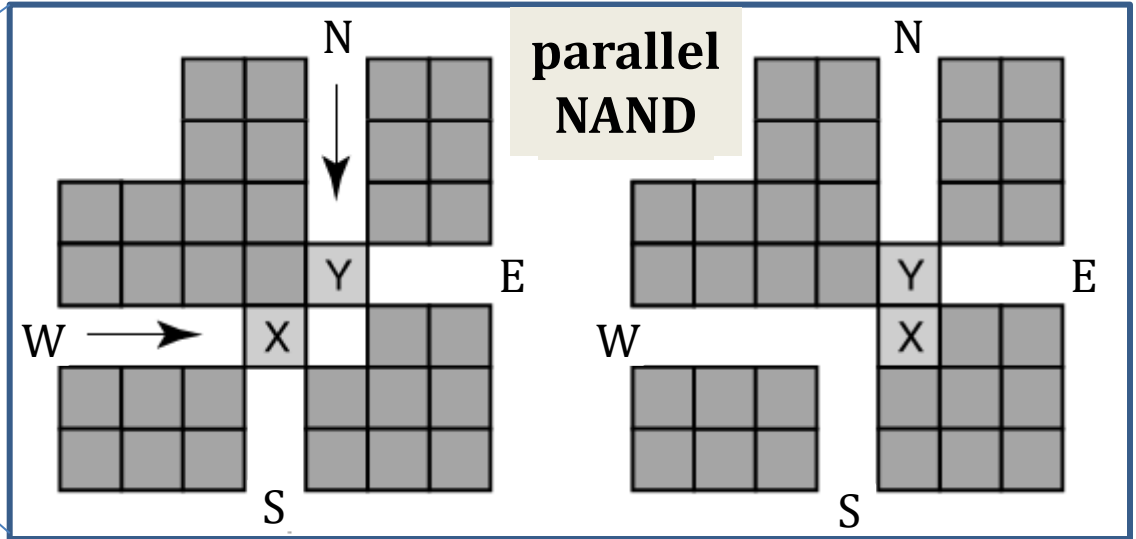
(Push)Push-1 is NP-hard in 2D

[Demaine, Demaine, O'Rourke 2000]

clause gadget



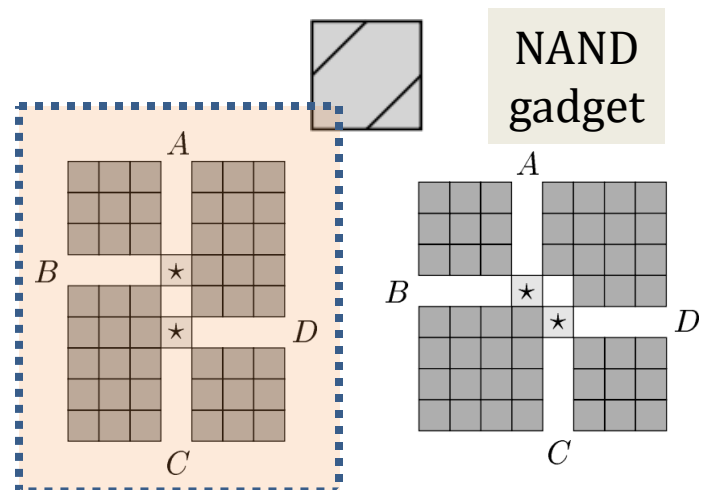
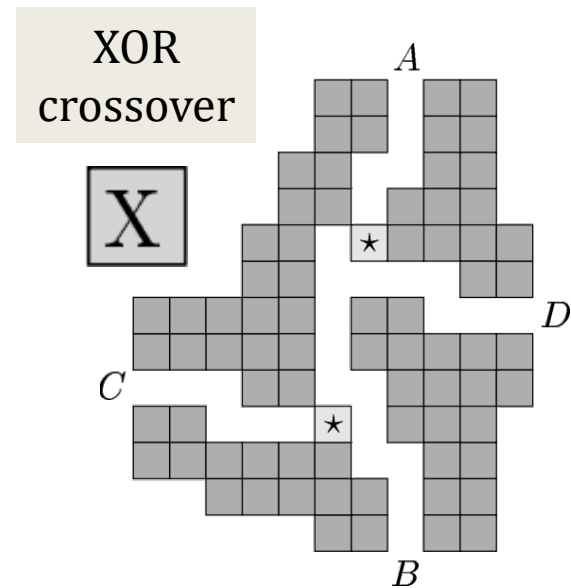
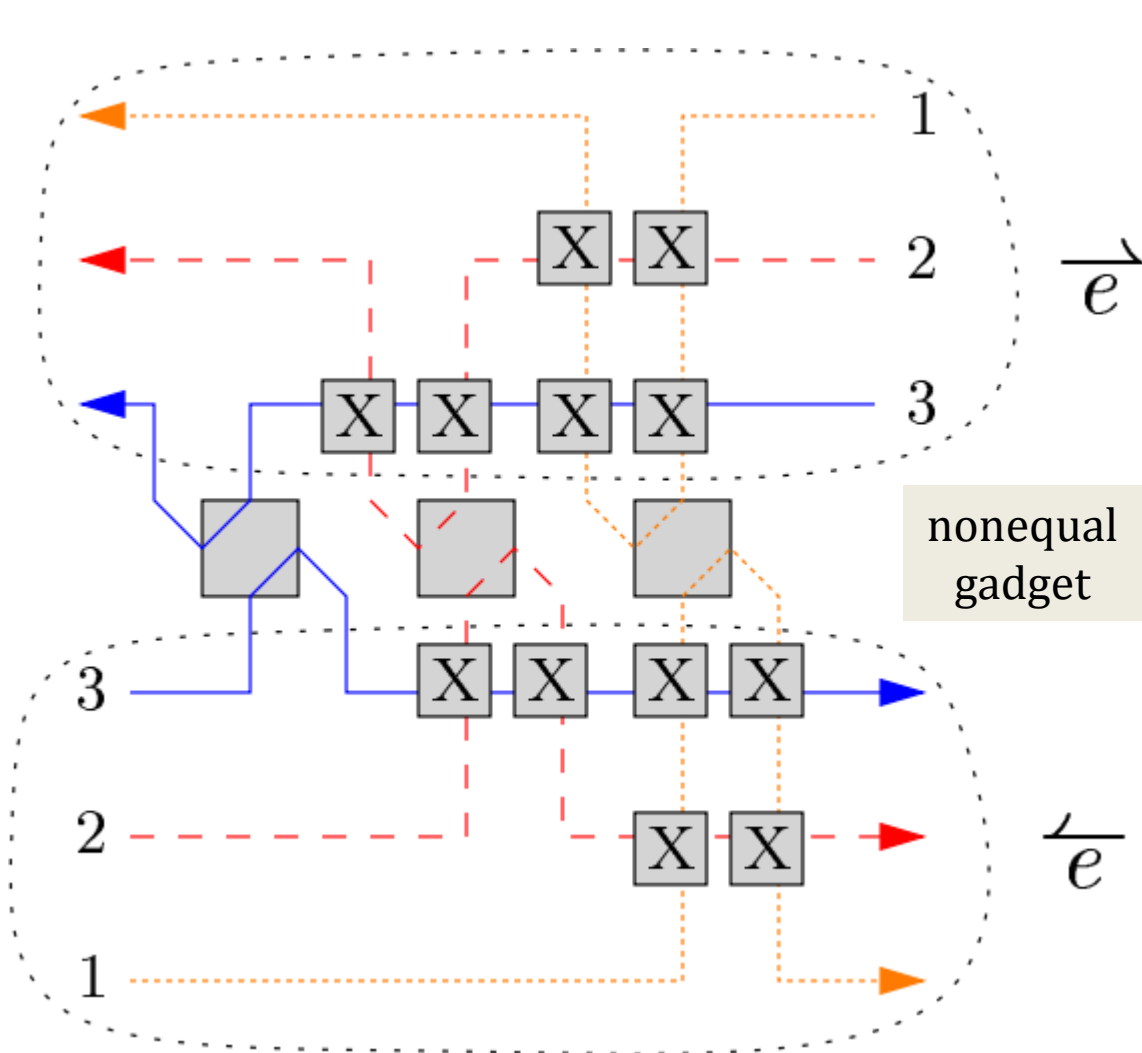
parallel NAND



lock gadget

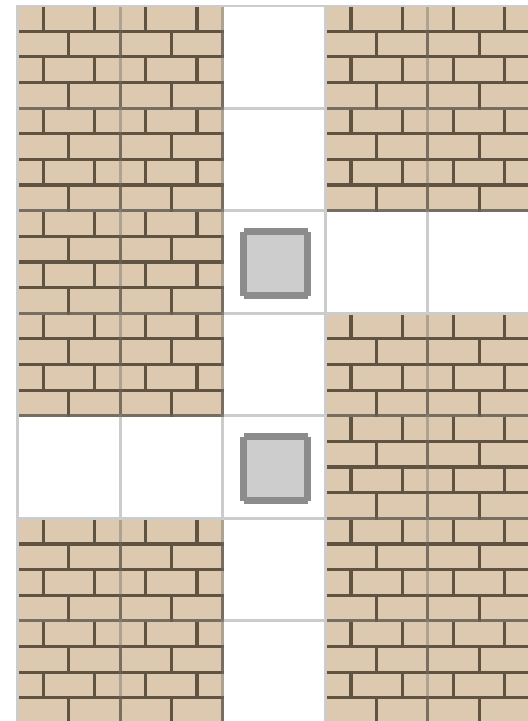
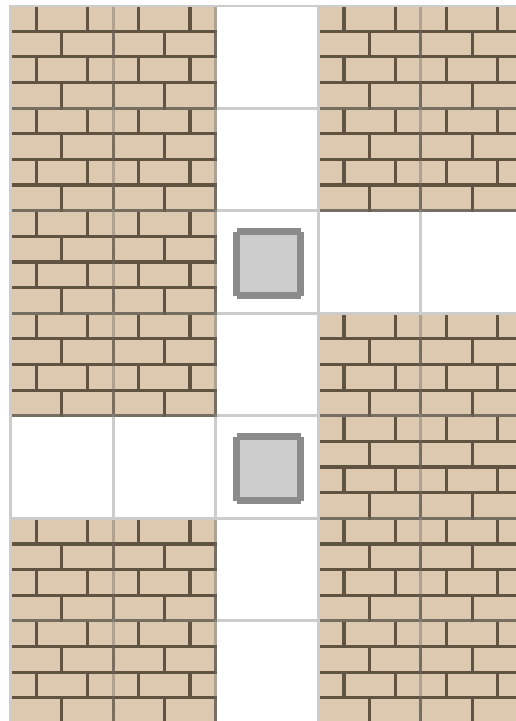
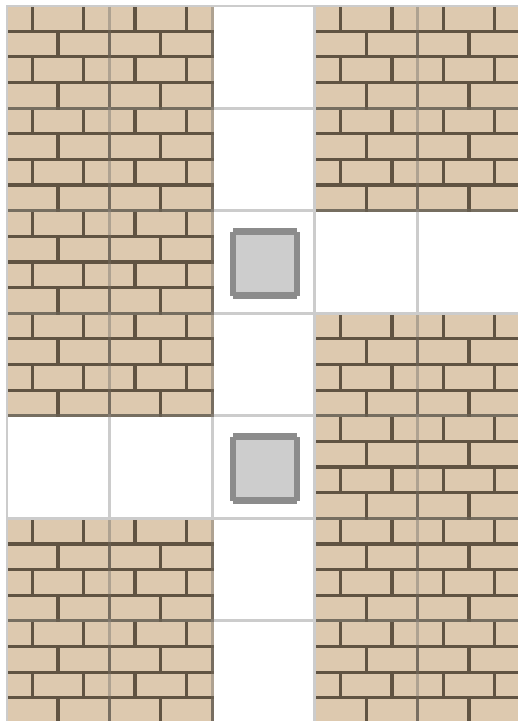
Push-1X is NP-complete

[Demaine, Demaine, Hoffmann, O'Rourke 2003]





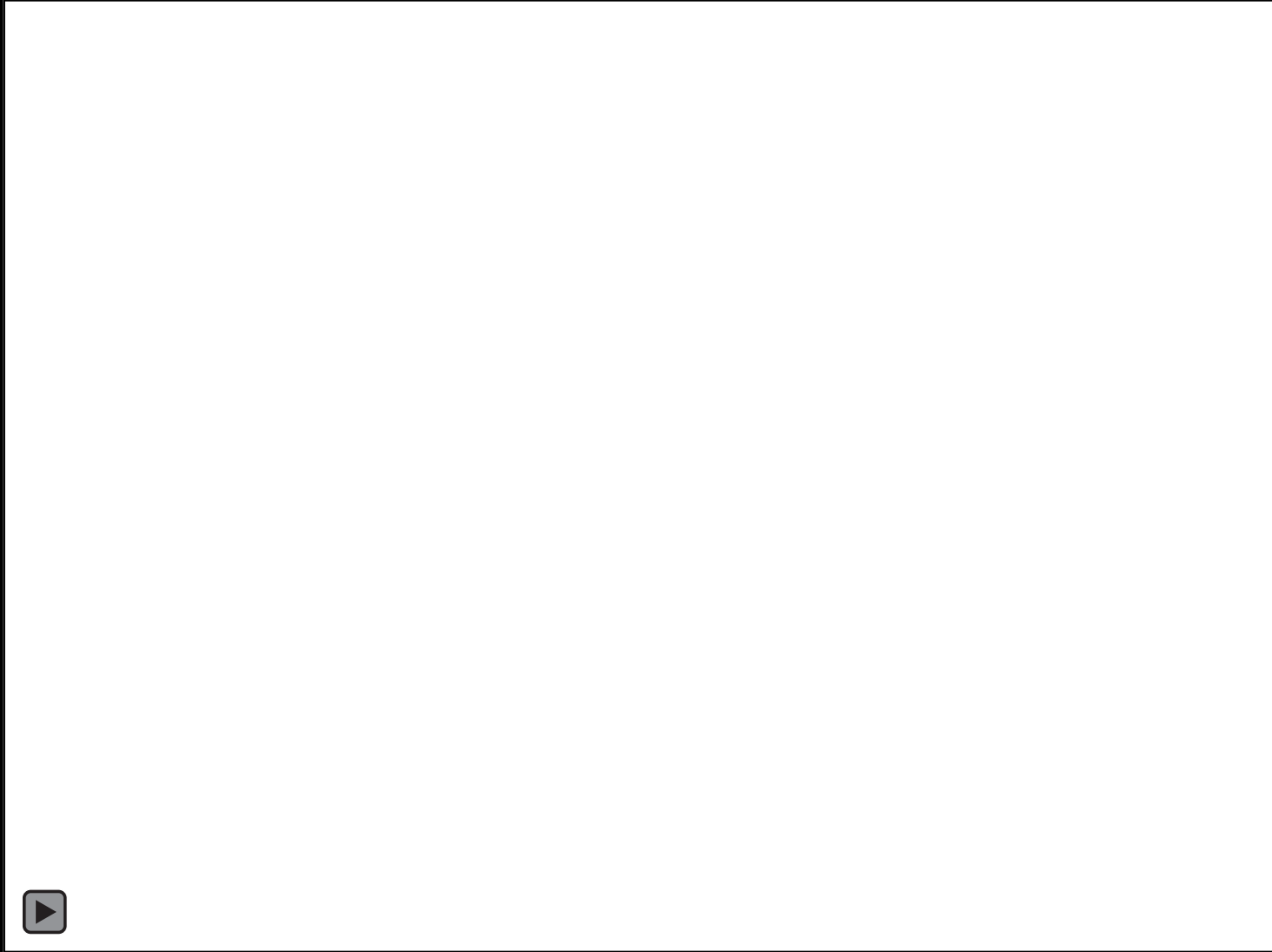
(Push)Push-1(X) is NP-hard in 2D



**antiparallel
NAND**



Legend of Zelda Block Pushing

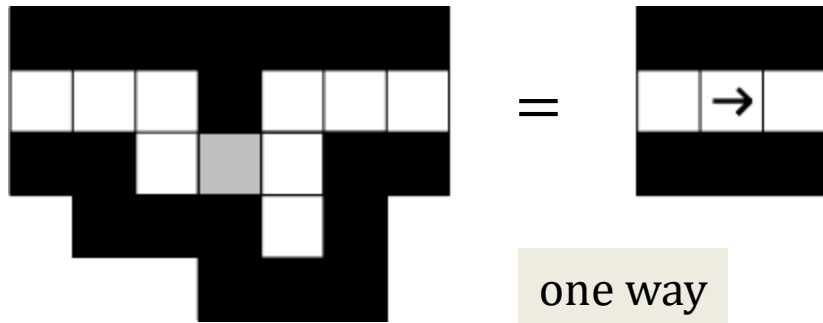


Let's Play Zelda: A Link to the Past #12 – Hookshot
by newfiebangaa <http://youtu.be/ZznLKBYcvc0>

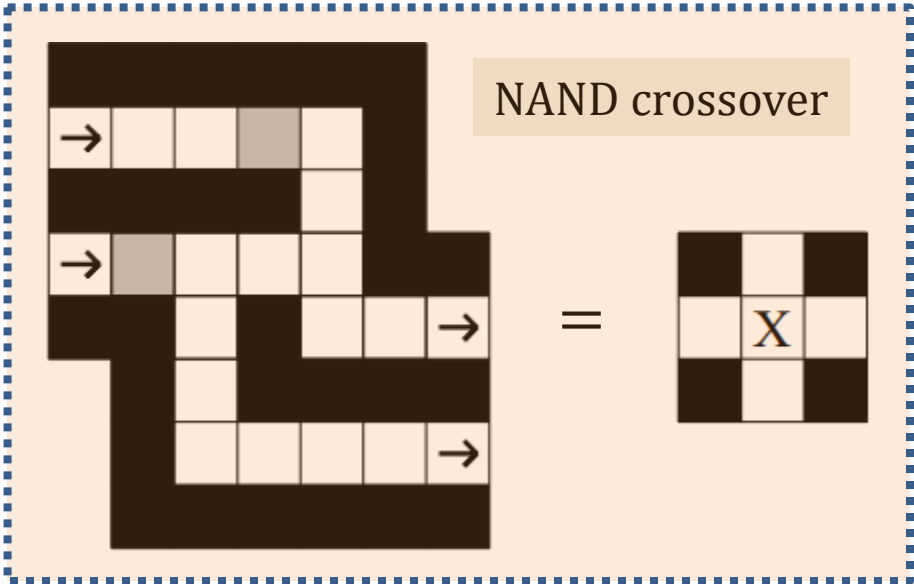
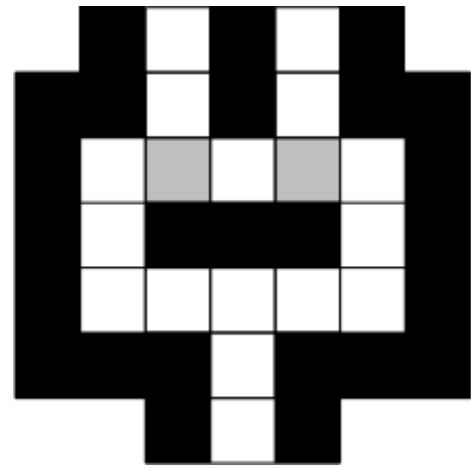


Push-1FG is NP-hard

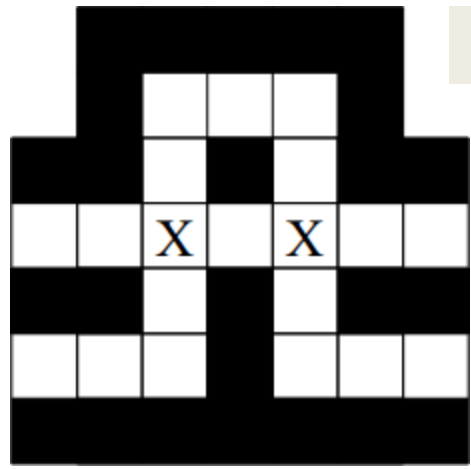
[Friedman 2002]



fork



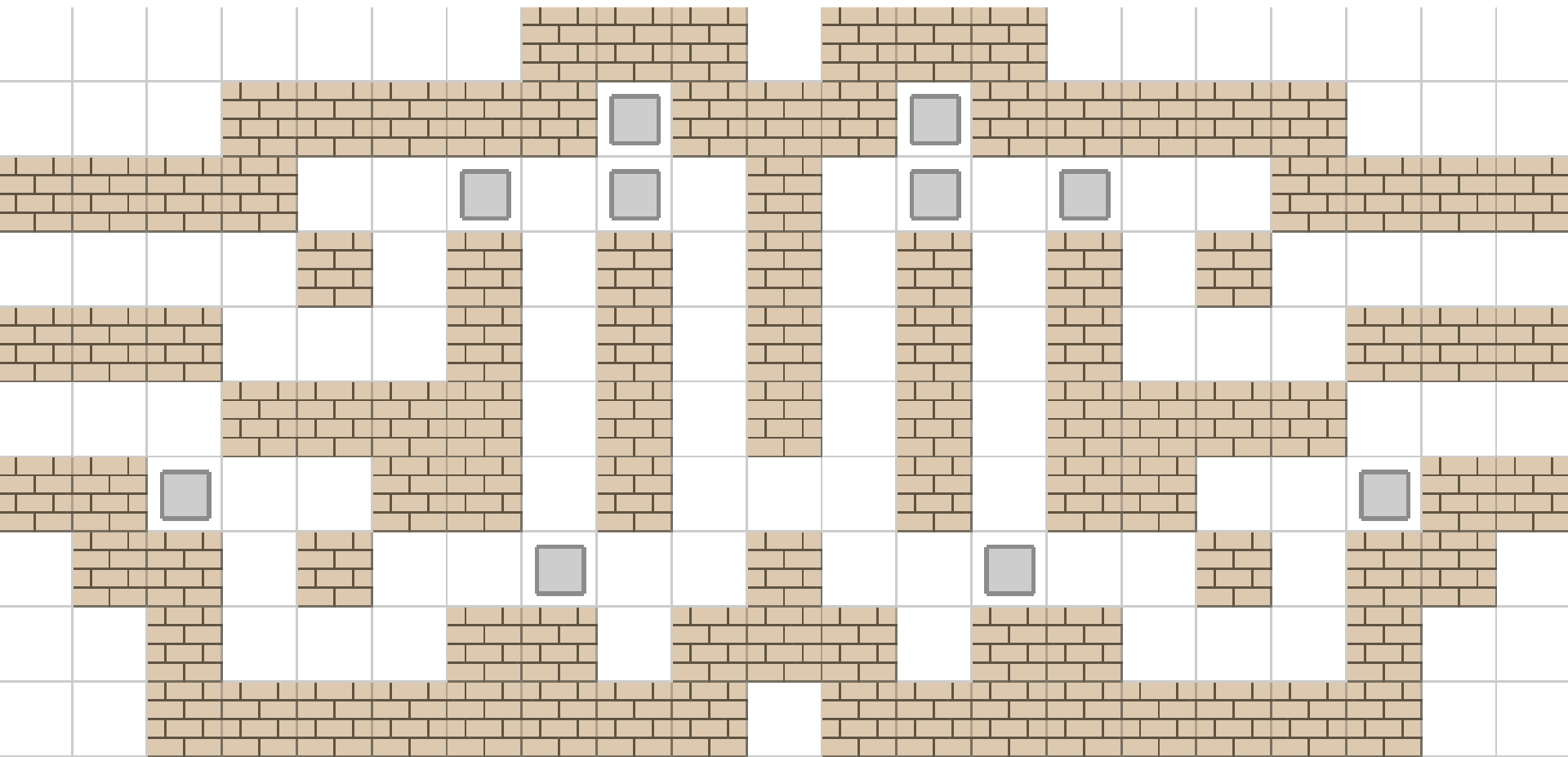
NAND





Pull?-1FG is NP-hard

[Ani, Asif, Demaine,
Diomidov, Hendrickson,
Lynch, Scheffler, Suhl 2020]



Crossing NAND