

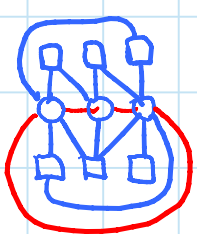
Linked planar 3SAT (a.k.a. "Mario SAT") [Pilz 2018]
 a stronger form of planar 3SAT motivated by...

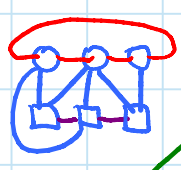
Super Mario Bros. is NP-hard [Aloupis, Demaine, Guo 2012]
 [Aloupis, Demaine, Guo, Viglietta - FUN 2014]

- reduction from 3SAT
- variable gadget via long falls
- clause via
 - koopa shells breaking blocks [SMB3]
 - koopa shells revealing POW block [SMW]
 - invincibility stars through fire bars [SMB]
- crossover is complicated
 - mushroom can go left
 - big Mario can run/crouch/jump under height 1
 - small/big Mario management
 - leakage from horizontal to vertical if both traversed: worry can revisit & set past vars.
 - fix: ensure horizontal happens before vertical

Can we avoid Mario-specific crossovers via some form of planar 3SAT?

Recall: [L5]

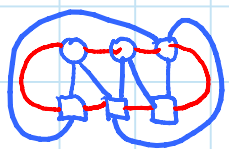
- 3SAT remains NP-hard when
 Variable-clause bipartite graph + cycle through all variables is planar, even with pos./neg. edges on opposite sides of the variable cycle \Rightarrow monotone

- 3SAT is polynomial when  Variable-clause bipartite graph + cycle through all variables + path through all clauses is planar
 \leftarrow e.g. x axis
 \rightarrow forced on one side of var. cycle

\checkmark what if path? hard!

Linked Planar 3SAT: [Pilz 2018] "Mario SAT"

3SAT-3 remains NP-hard when
variable-clause bipartite graph +

 cycle that visits all variables & then visits all clauses is planar

- also for monotone 3SAT-3

- also sided: pos./neg. edges on opposite sides of the cycle \nRightarrow monotone

- also assuming each clause has ≤ 2 on each side \otimes

- also for E3SAT

- also for positive 1-in-3SAT

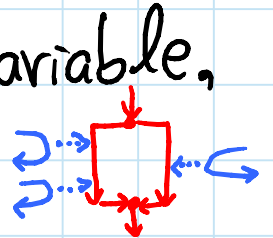
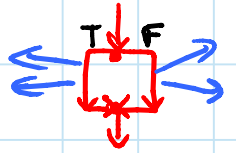
- Careful: planar \Rightarrow E4SAT & monotone E3SAT

& sided linked monotone 3SAT are $\in P!$

Back to Mario:

[6.892: Demaine, Diomidov, Lynch 2019]

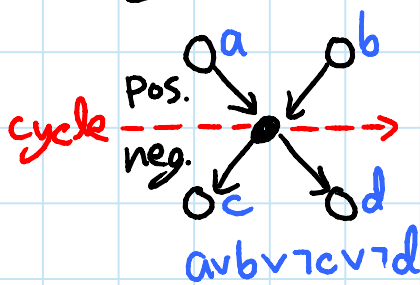
- reduce from Sided Linked Planar 3SAT
- sided \Rightarrow positive & negative uses of each variable are consecutive
- need clause that can unlock from both sides of the clause line $\ddot{}$
- simpler: re-use split/merge from variable, and use 2 unlockable paths - one for connections on each side



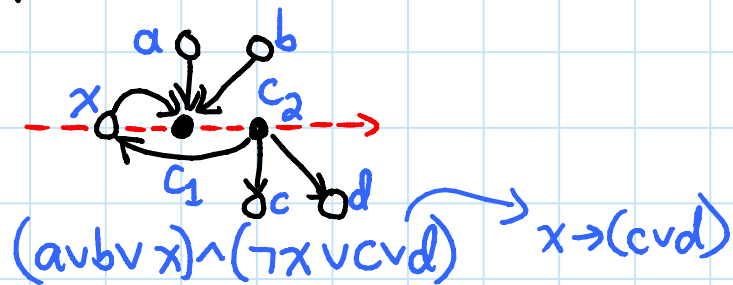
Sided interlinked planar monotone 3SAT: formula view

monotone 3SAT-3 remains NP-hard when variable-clause bipartite graph + cycle that visits all variables & all clauses, with each clause after its 3 variables is planar, even when sided:
pos./neg. edges on opposite sides of cycle

with specified start location & direction


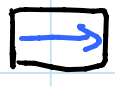
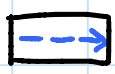

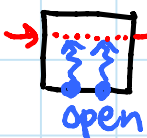


sided linked



sided interlinked monotone

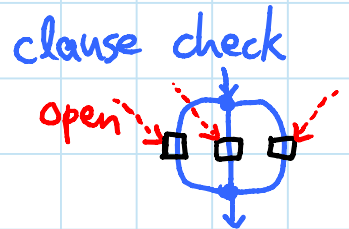
$x \rightarrow (c \vee d)$

- Opening doors: planar motion planning ^{→ see L11} is NP-hard given
- branching hallways 
 - one-way mechanism: one of
 - diode  (e.g. game has gravity + "long" falls) ^{jumps}
 - crumbler: closes after traversal
 - directed  or undirected 
 - no-return: allow \leftarrow before \rightarrow (not argued here) but prevent \leftarrow (immediately) after \rightarrow
 - opening door with 2 buttons  ^{traverse (possible only after ≥ 1 open)}
 - directed or undirected
 - (NO CROSSOVER NEEDED)



OR: opening door with 1 button + no-return + CROSSOVER

OPEN: needed?



[Lynch - Ph.D. 2020]

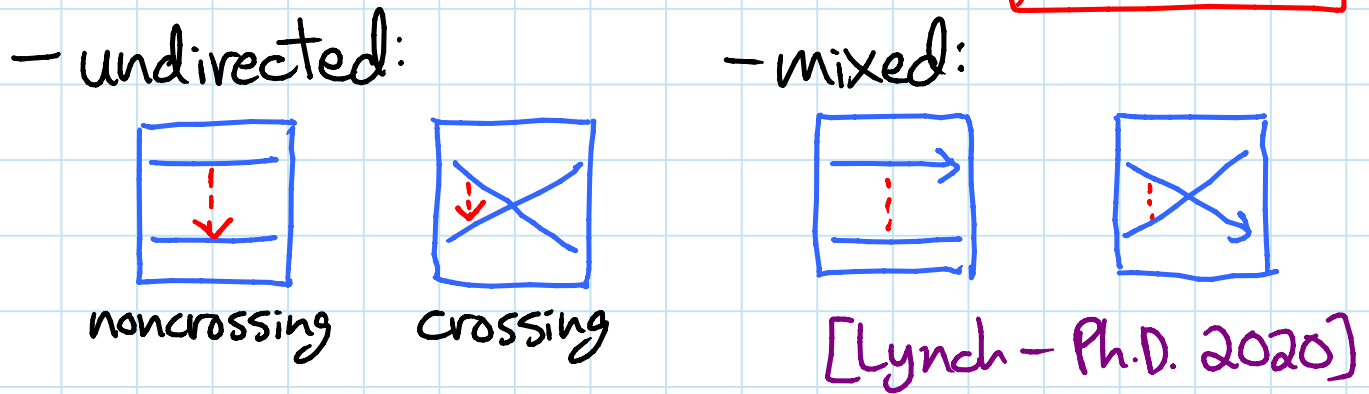
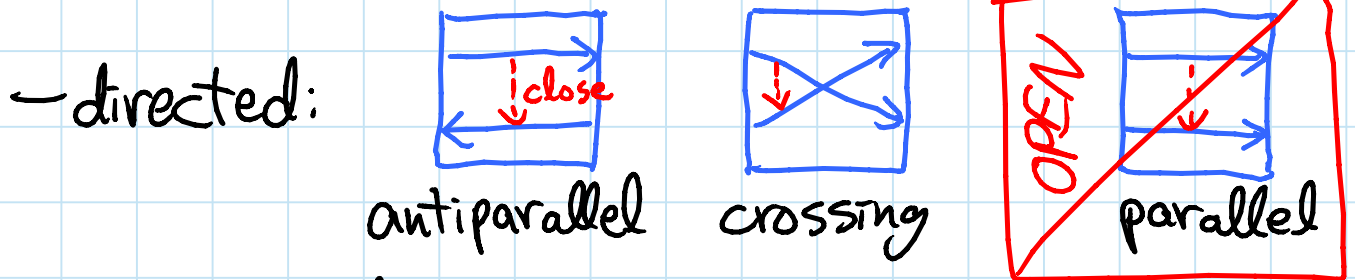
\approx Metathm. 3 of [Forišek - FUN 2010]

Applications:

- Super Mario Bros.
 - door self-closes so also serves as no-return ("directed crumbler" = one way, one use)
- Legend of Zelda hookshots + push-once blocks
- Metroid
- Pokémon (using variable gadget, not no-return)

Closing doors: planar motion planning is NP-hard with

- branching hallways
- distant closing: 2 paths, at least one of which fully closes the other (and possibly itself) with one of the following geometries:



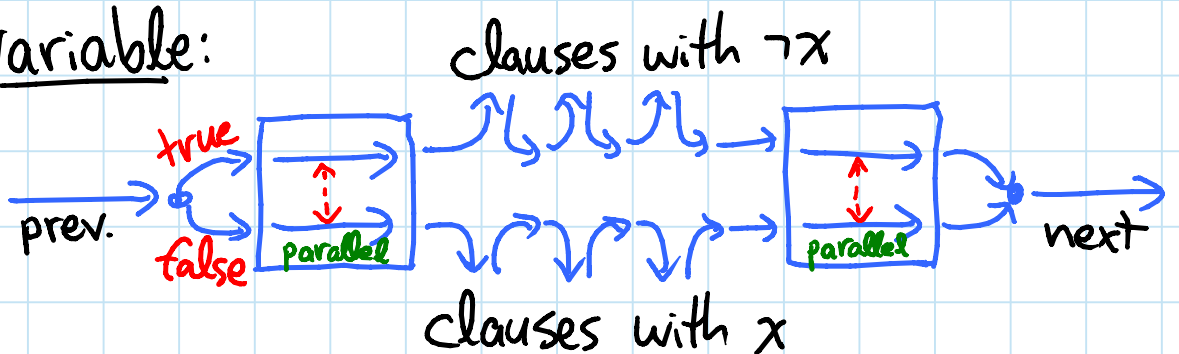
missing in lecture

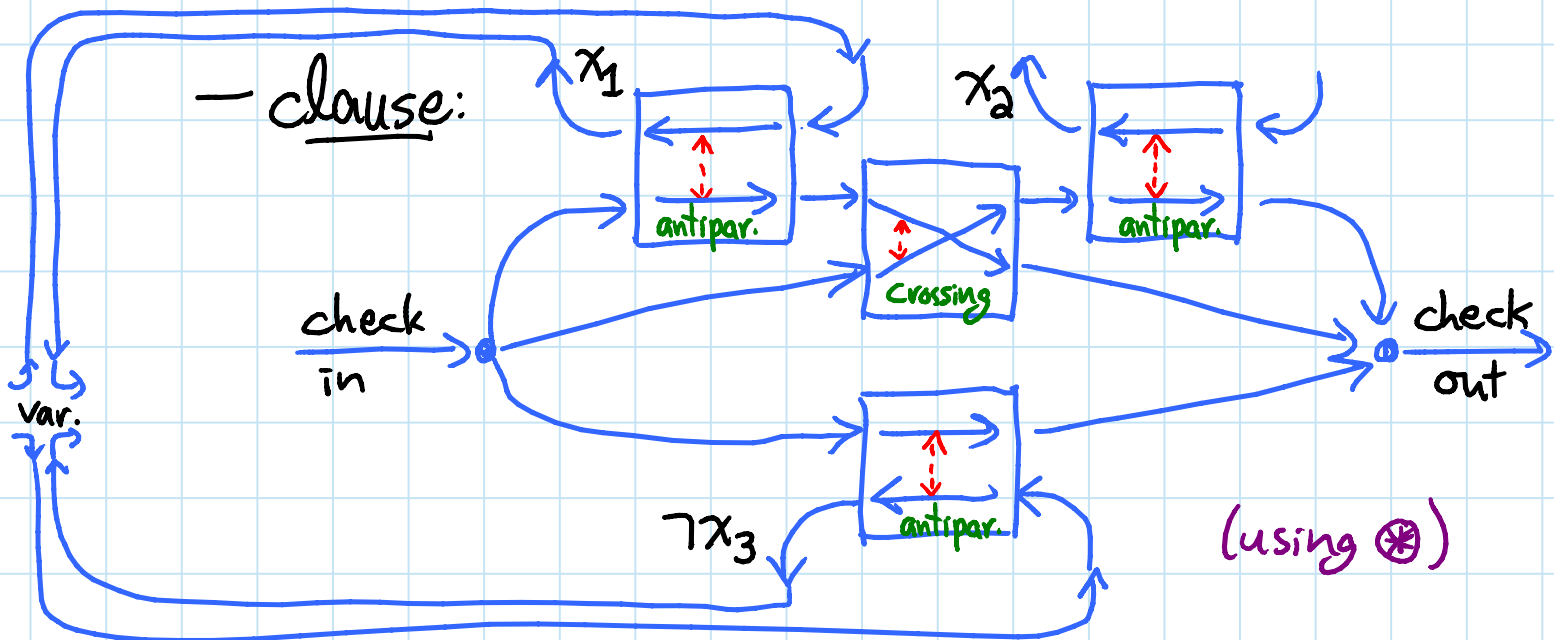
Reduction from Sided Linked Planar 3SAT:

- assume parallel and crossing and antiparallel directed or undirected or mixed NANDs or matched crumblers

each path closes the other ↙ ↘ each path closes both

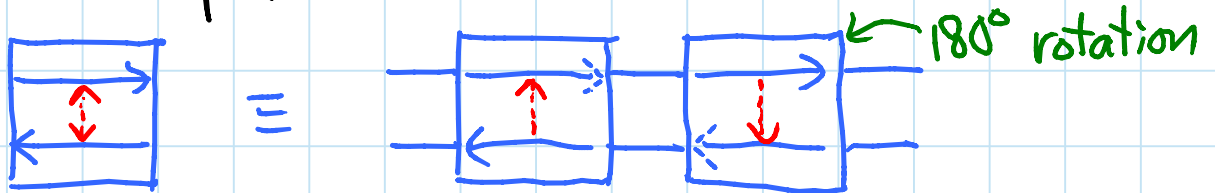
- Variable:





Distant closing → NAND or matched crumblers

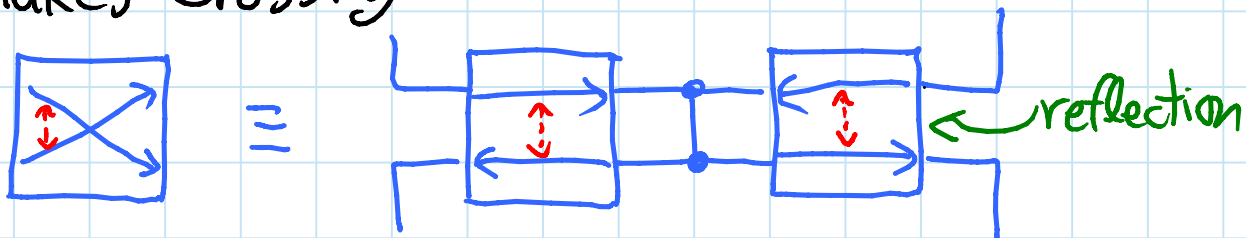
- antiparallel/undir./mixed distant closing $\times 2$ make antipar./undir. NAND or matched crumblers:



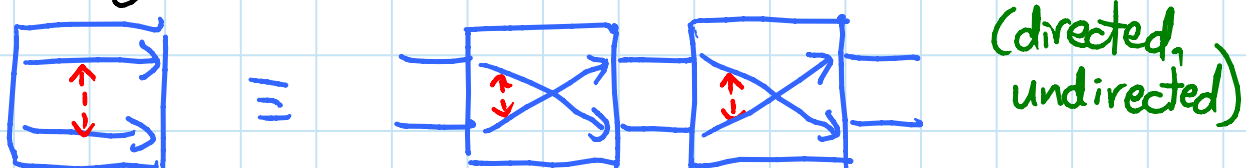
- similarly, 2/3 crossing distant closings make parallel/crossing NAND or matched crumblers

Crossing vs. not

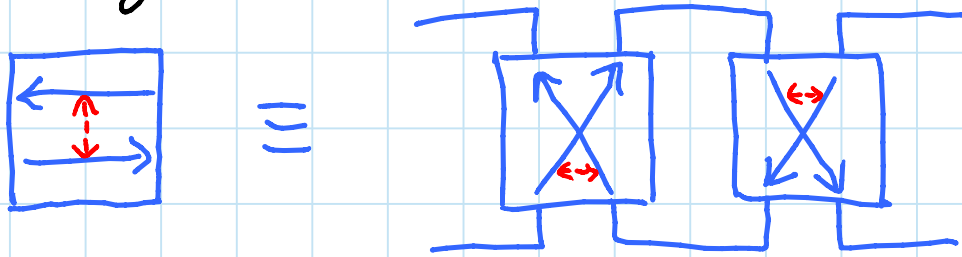
- antiparallel NAND/matched crumblers makes crossing:



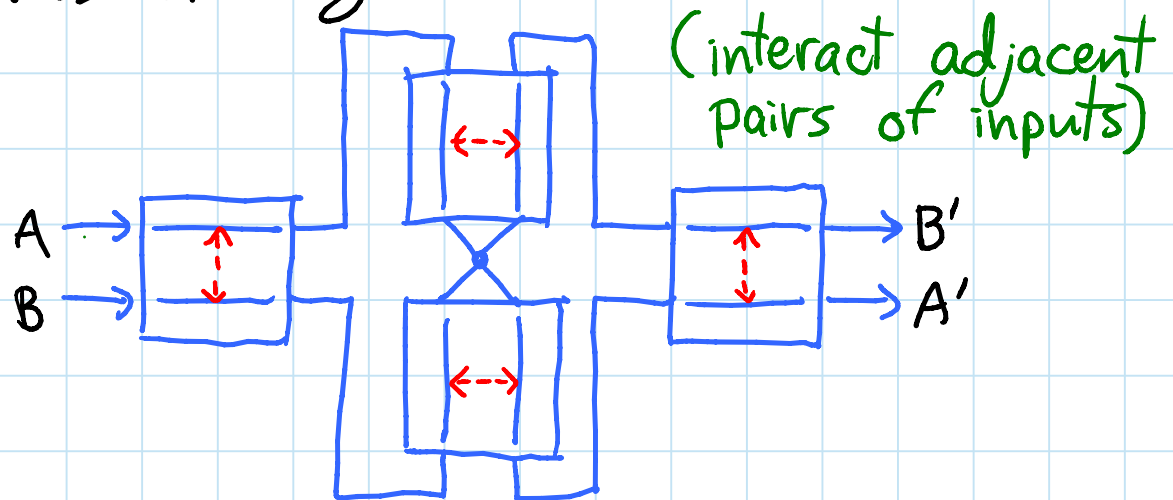
- crossing NAND/matched crumblers makes parallel:



- crossing NAND/mat. crumb. makes antiparallel:



- undirected noncrossing NAND/matched crumblers makes crossing:



(part of a general theory of "motion planning with gadgets" - see L11)

Applications:

- (Push)Push-1(X) via antiparallel NAND
- Legend of Zelda push-once blocks
- Push-1G (gravity) via crossing NAND
- Pull?-1FG (from 6.892 Spring 2019!)

optional
pulls