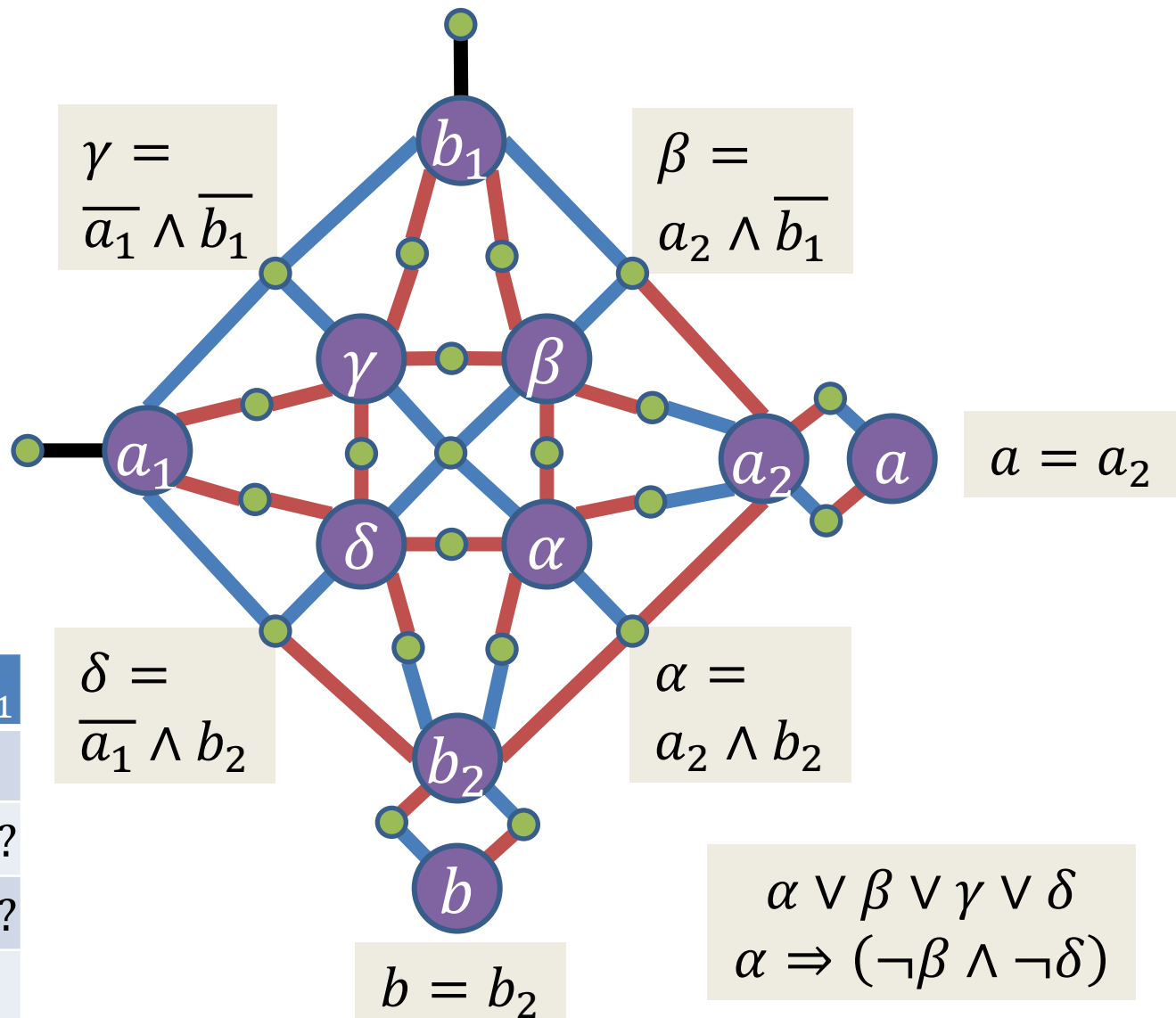
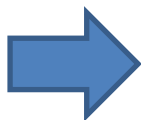
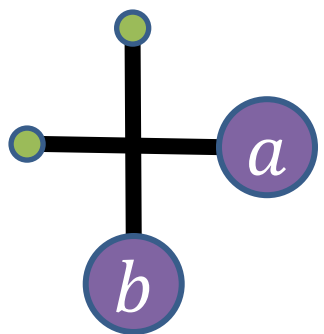


Planar 3SAT is NP-hard

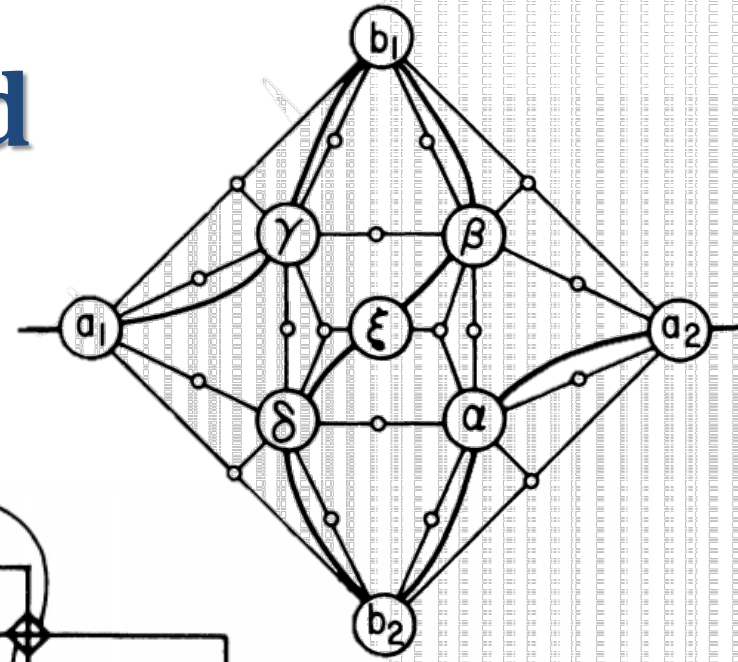
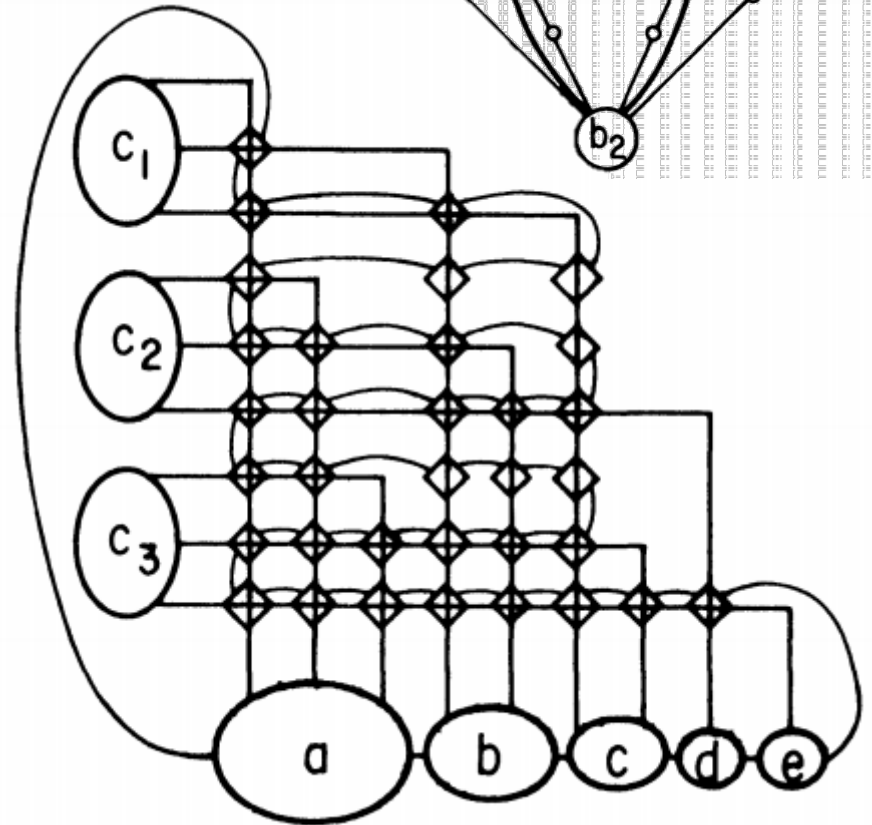
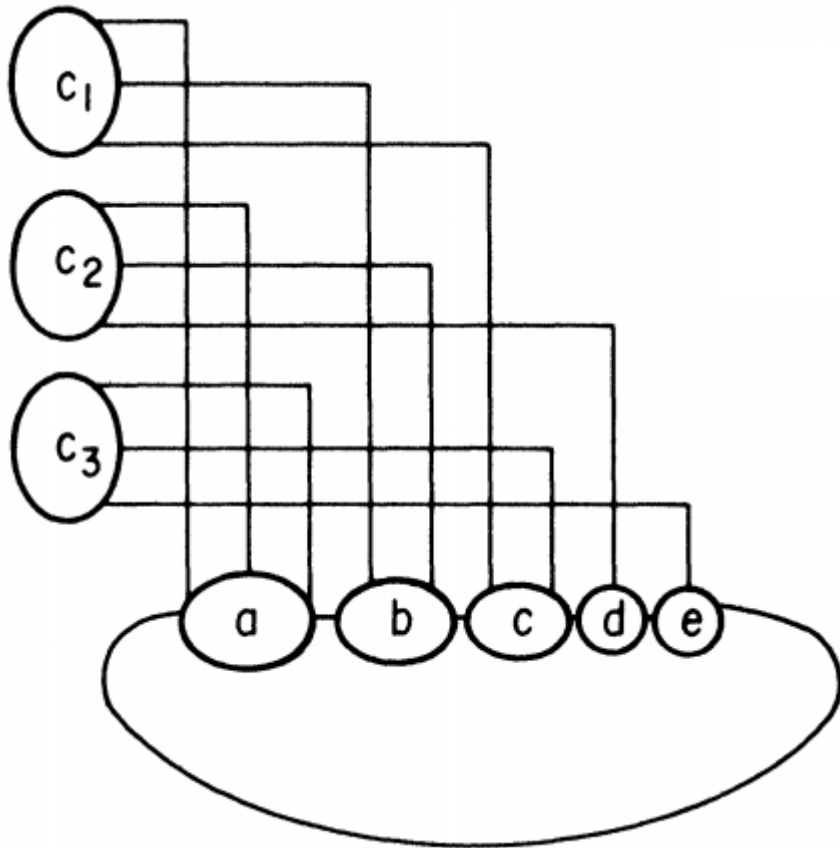
[Lichtenstein 1982]



a_2	b_2	α	β	δ	γ	a_1	b_1
0	0	0	0	0	1	0	0
0	1	0	0		1?	0?	0?
1	0	0		0	1?	0?	0?
1	1	1	0	0		1	1

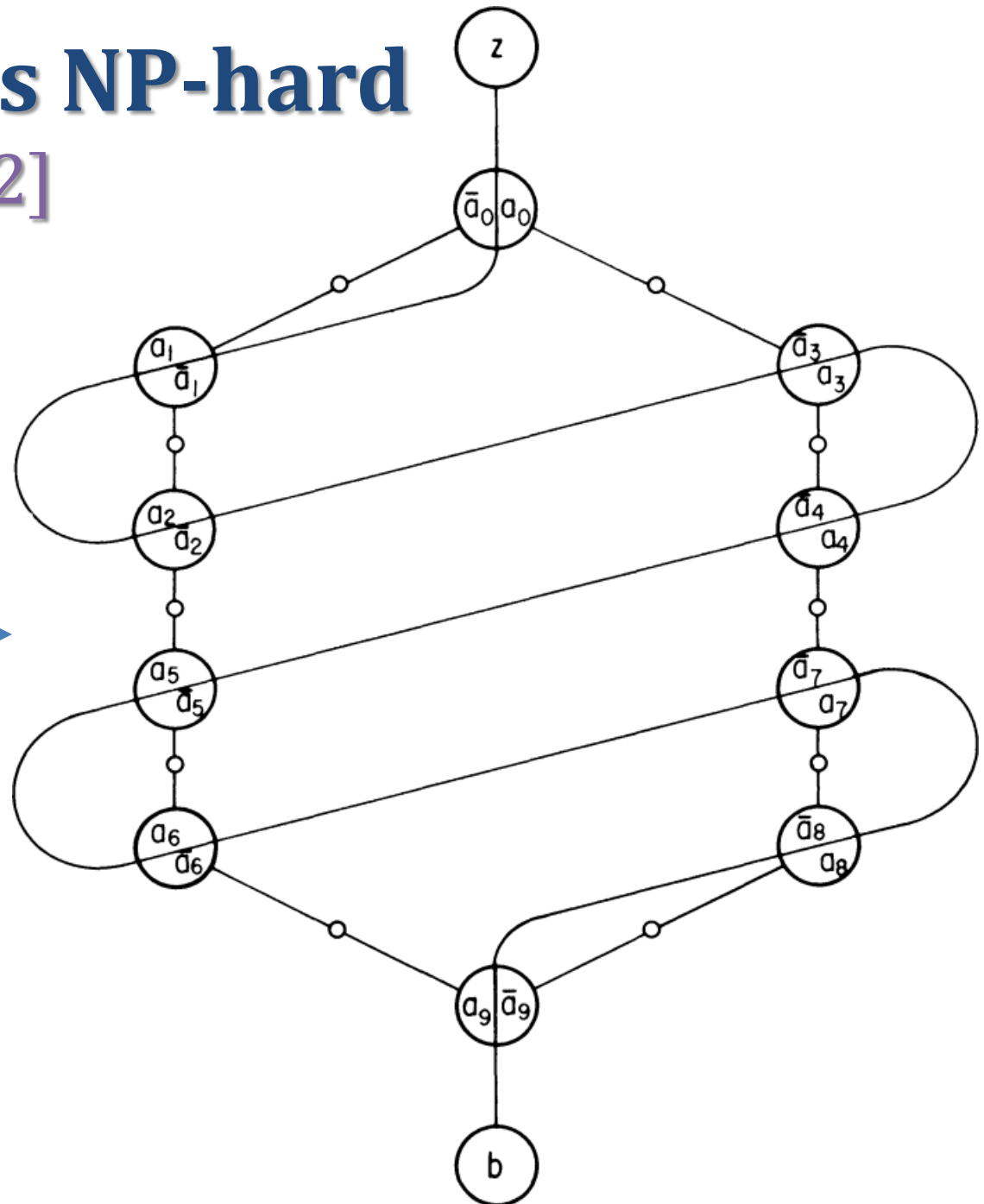
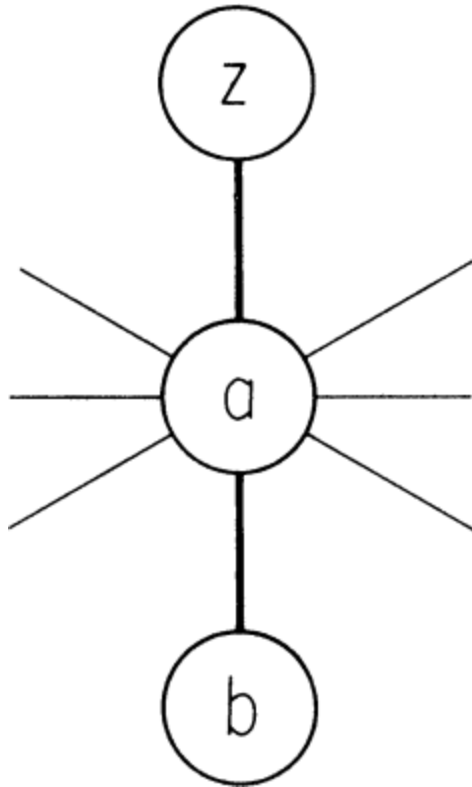
Planar 3SAT is NP-hard

[Lichtenstein 1982]

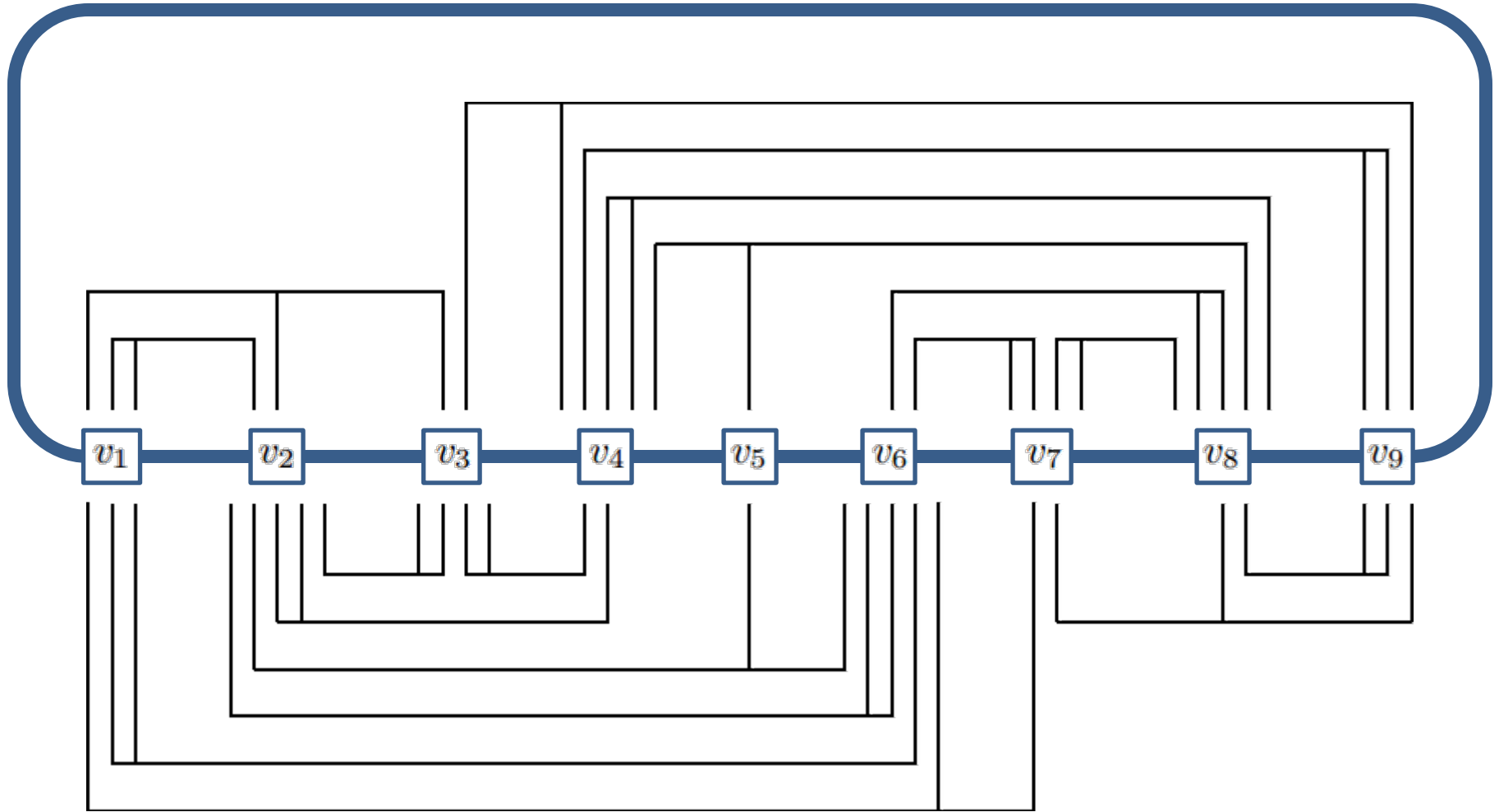


Planar 3SAT is NP-hard

[Lichtenstein 1982]



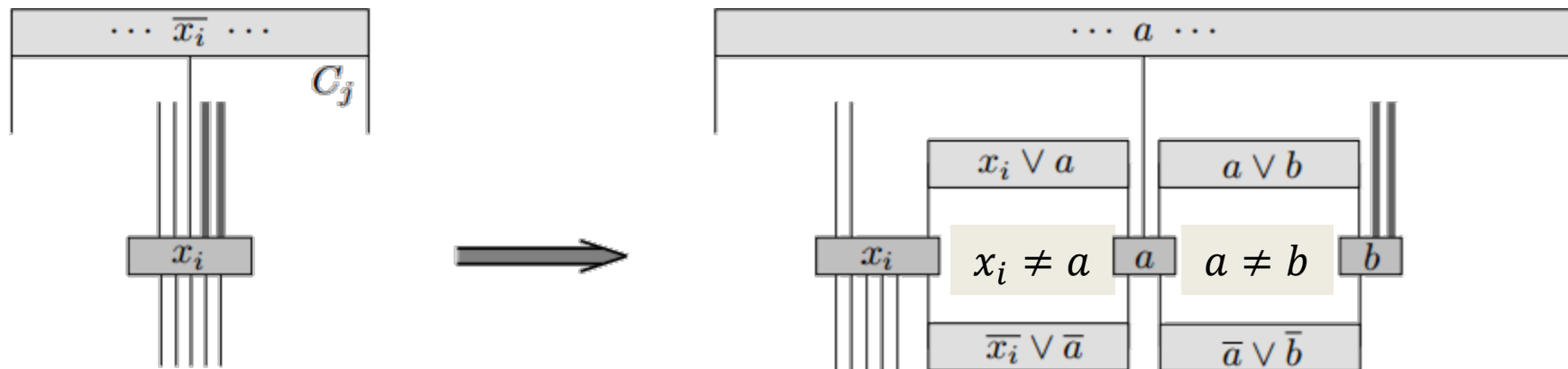
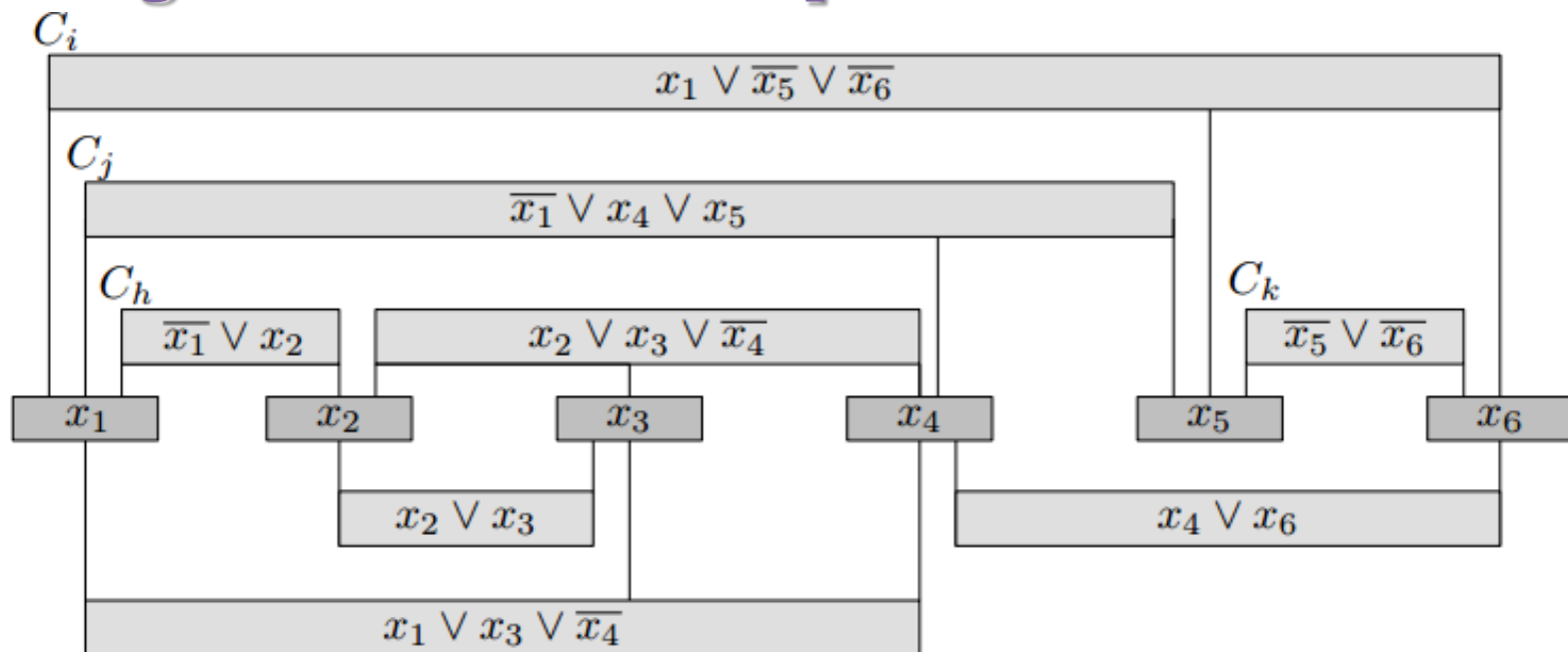
Planar Rectilinear 3SAT



[Knuth & Raghunathan 1992]

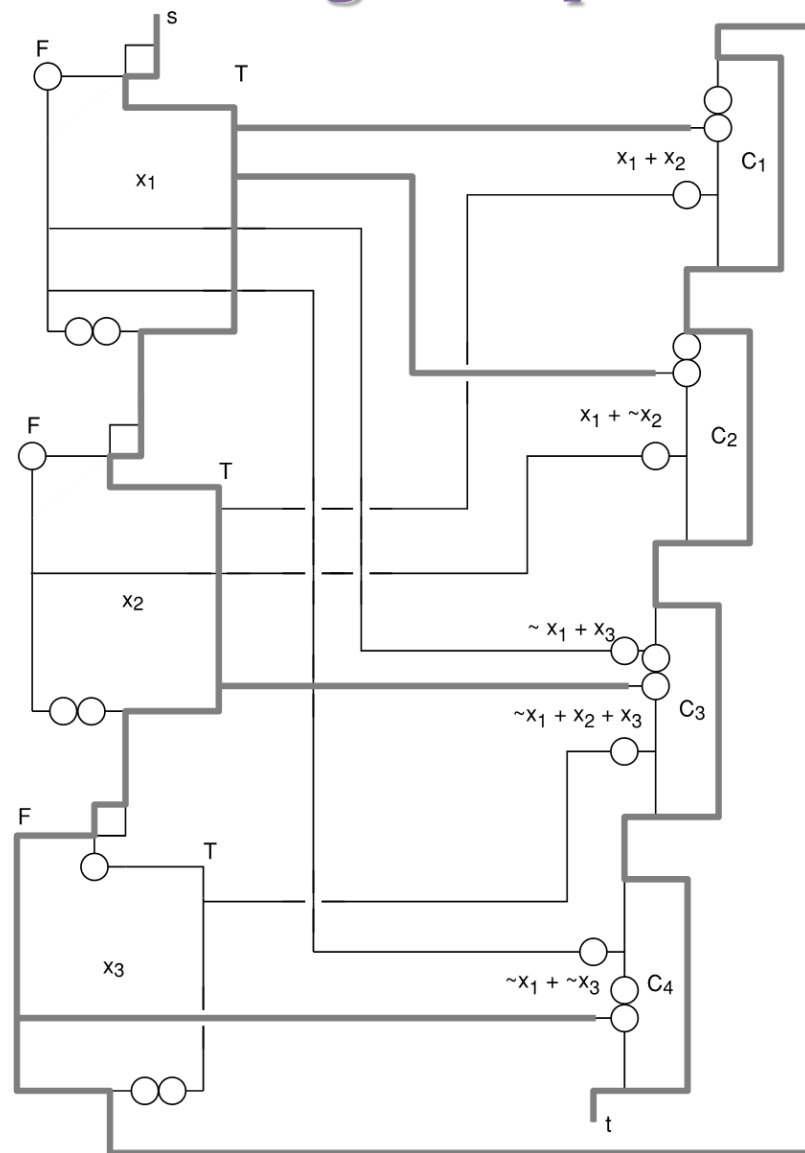
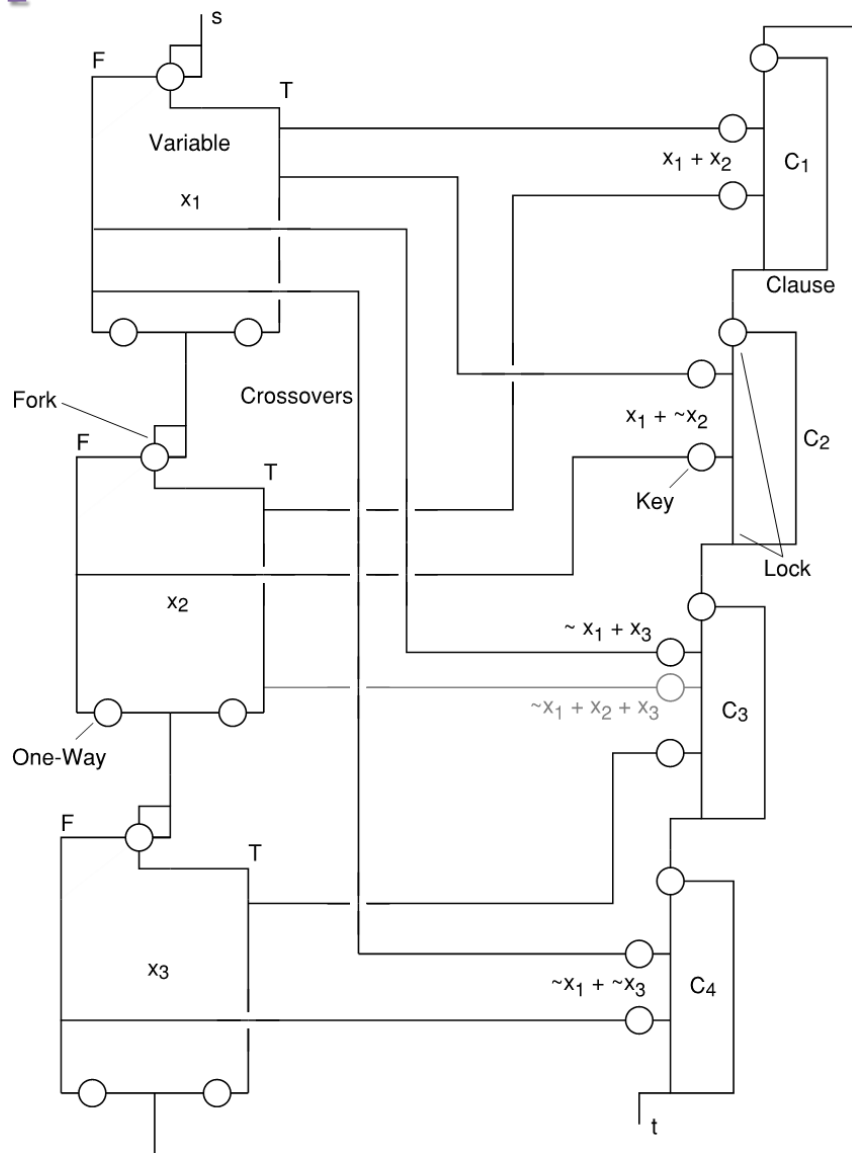
Planar Monotone Rectilinear 3SAT

[de Berg & Khosravi 2010]



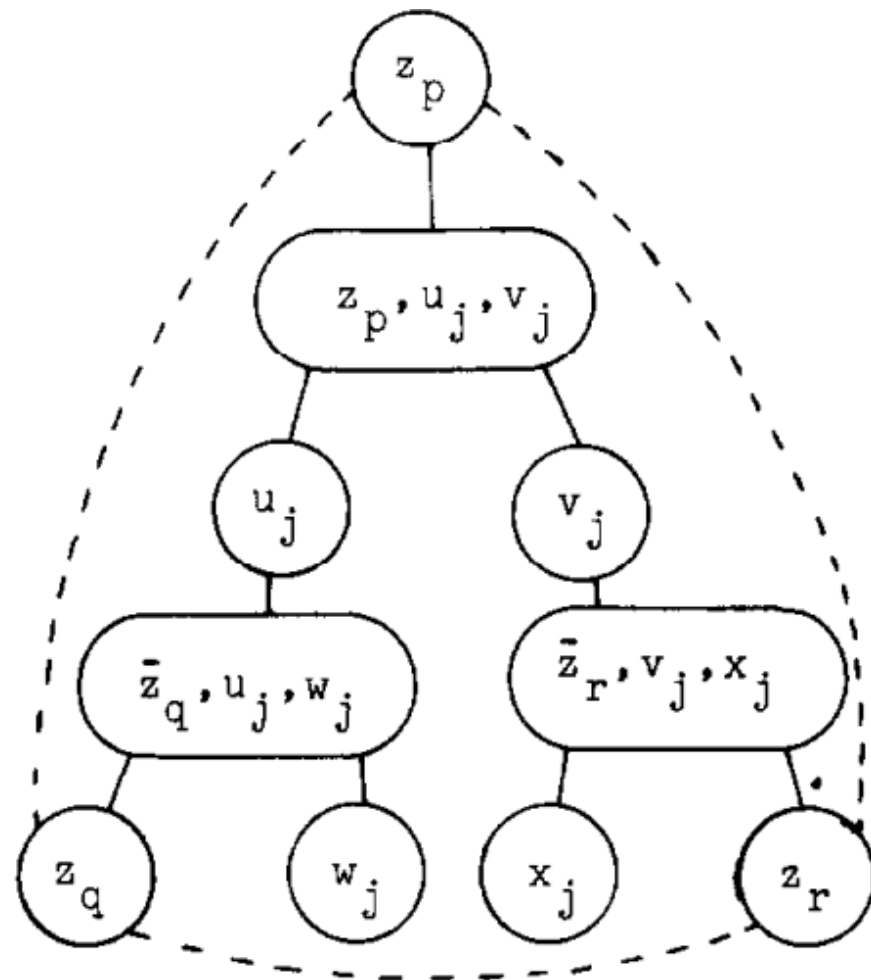
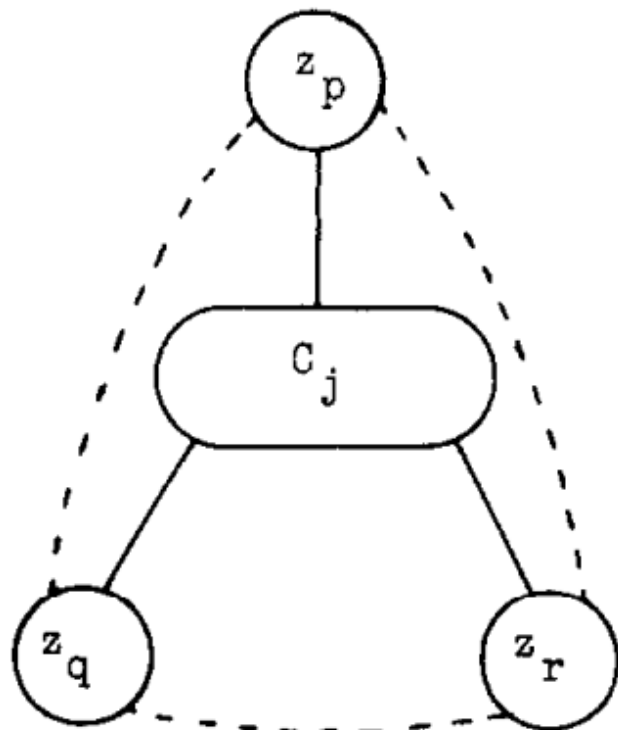
PushPush-1 is NP-hard in 3D

[O'Rourke & Smith Problem Solving Group 1999]



Planar 1-in-3SAT

[Dyer & Freeze 1986]

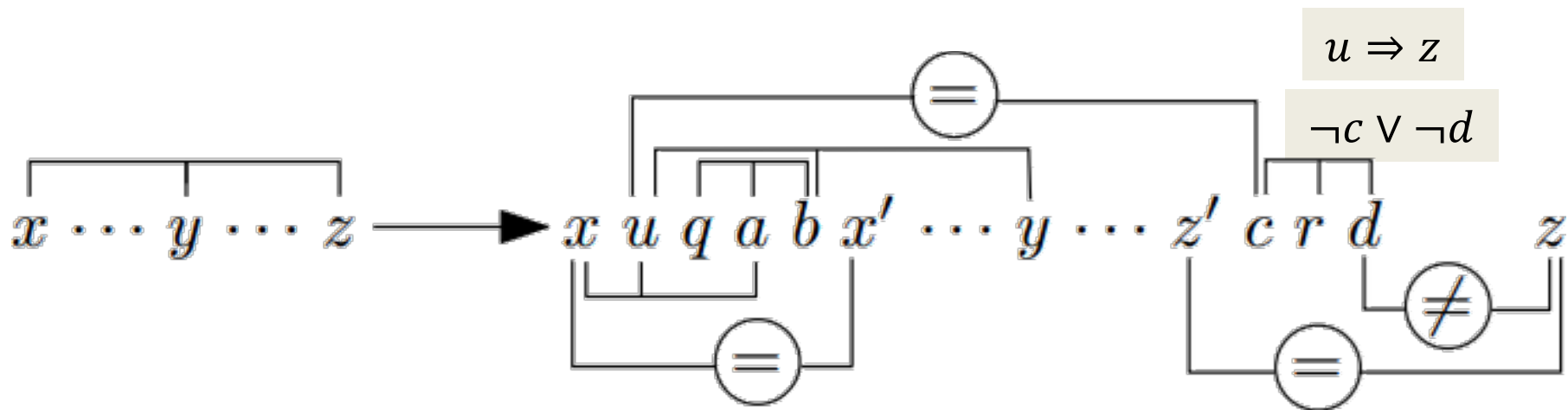
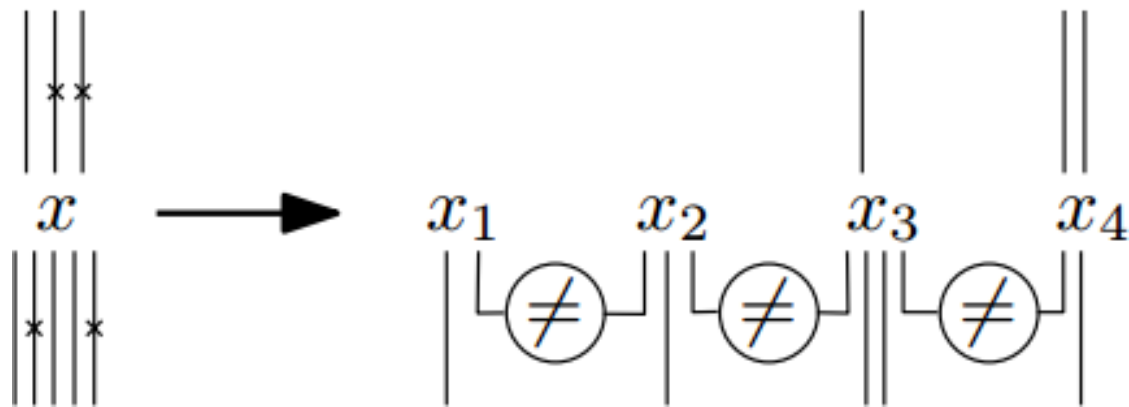


$$C_j = \{z_p, z_q, z_r\}$$

$$\{z_p, u_j, v_j\}, \{\bar{z}_q, u_j, w_j\}, \{\bar{z}_r, v_j, x_j\}$$

Planar Positive Rectilinear 1-in-3SAT

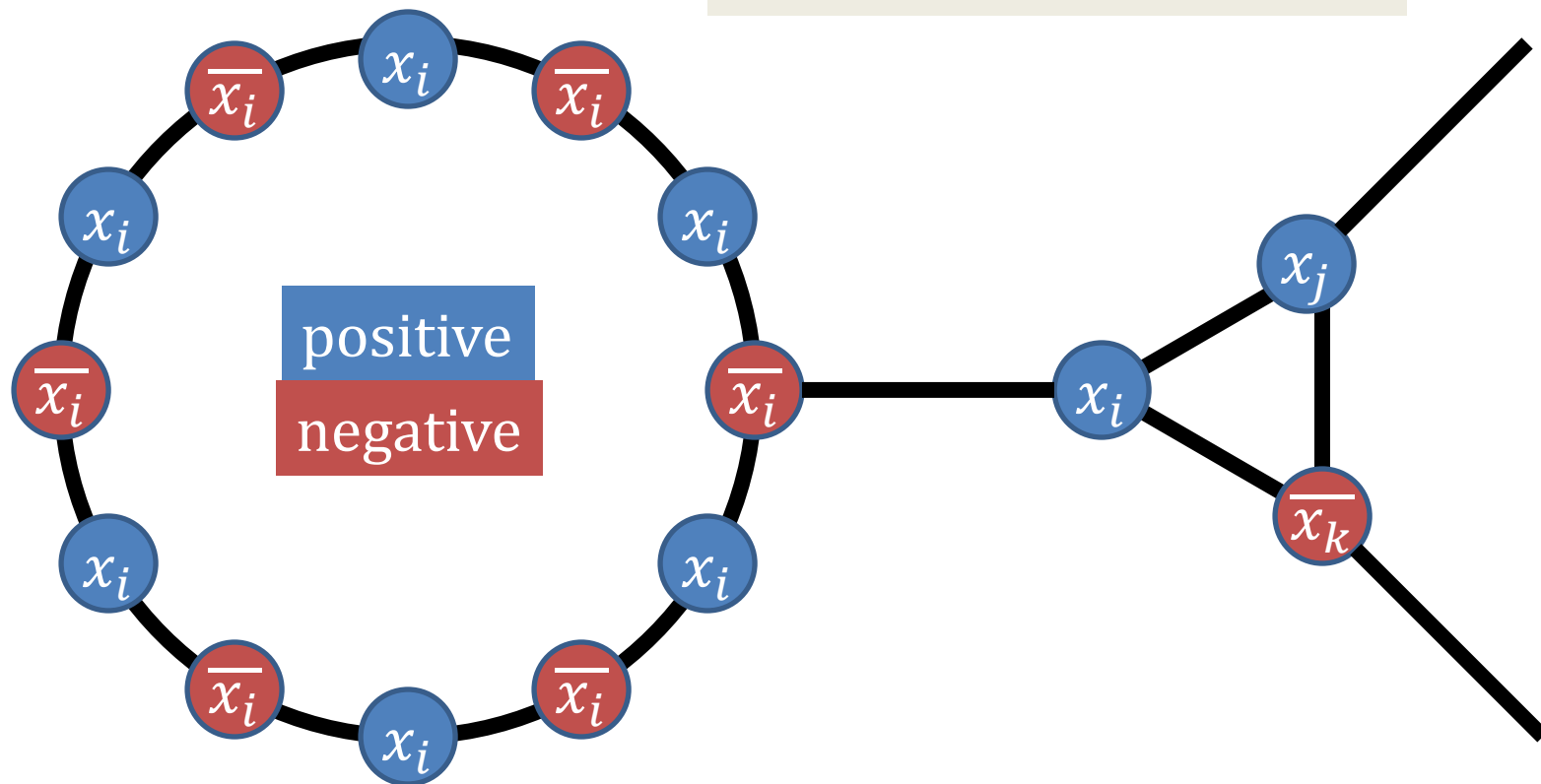
[Mulzer & Rote 2008]



Planar NAE 3SAT is Polynomial

[Moret 1988]

reduction to Max Cut



cut size = $2n_i$

variable

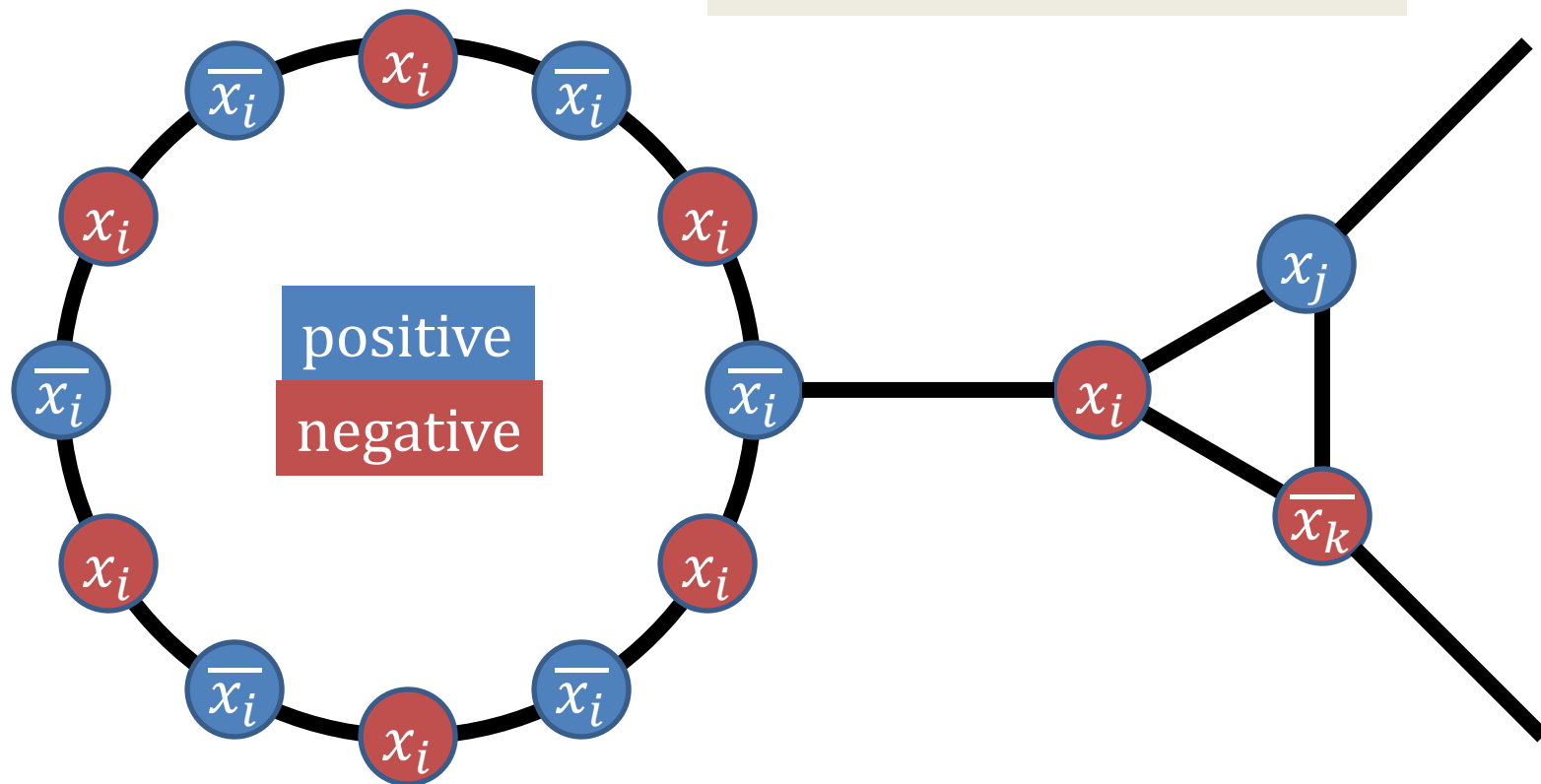
cut size = 5

NAE clause

Planar NAE 3SAT is Polynomial

[Moret 1988]

reduction to Max Cut



cut size = $2n_i$

variable

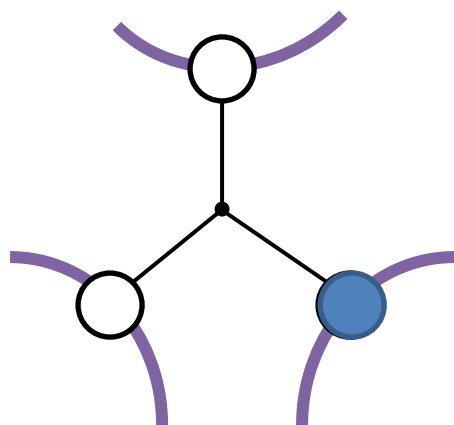
cut size = 5

NAE clause

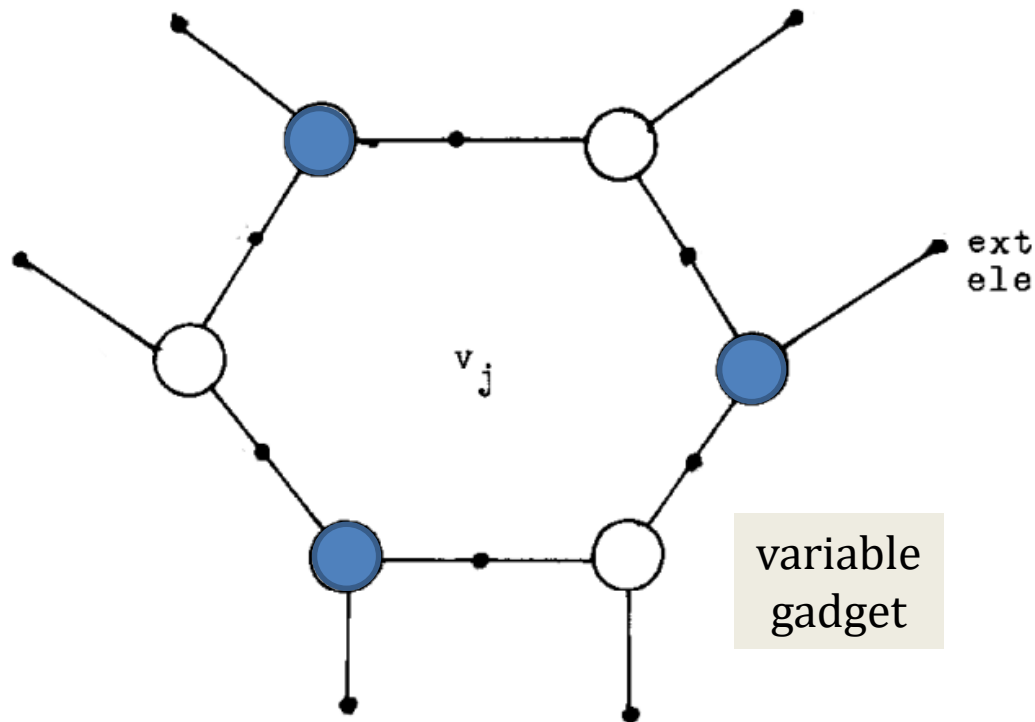


Planar X3C

[Dyer & Freeze 1986]

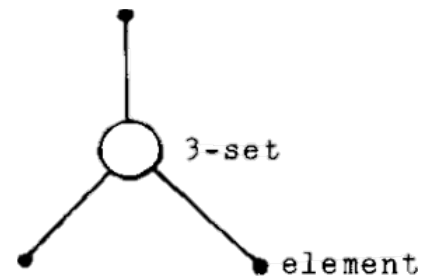


simple clause gadget



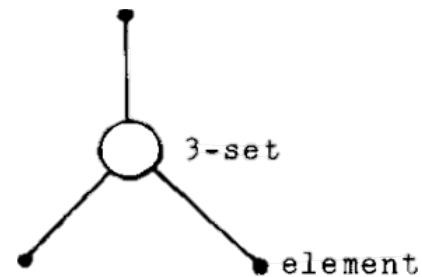
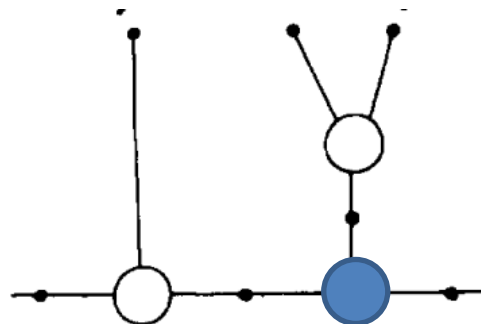
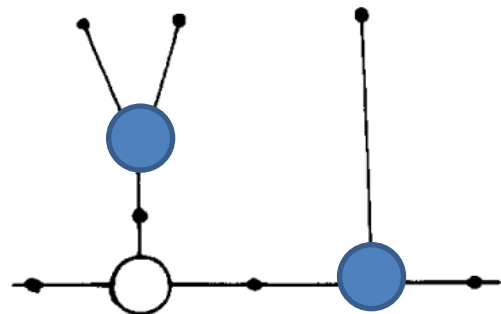
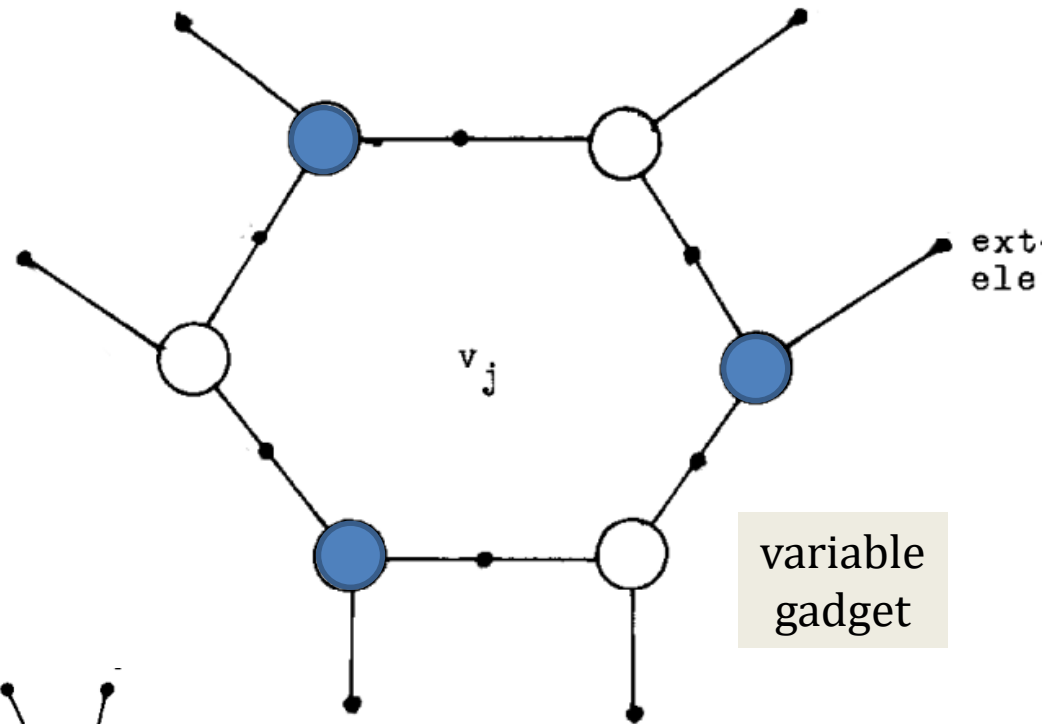
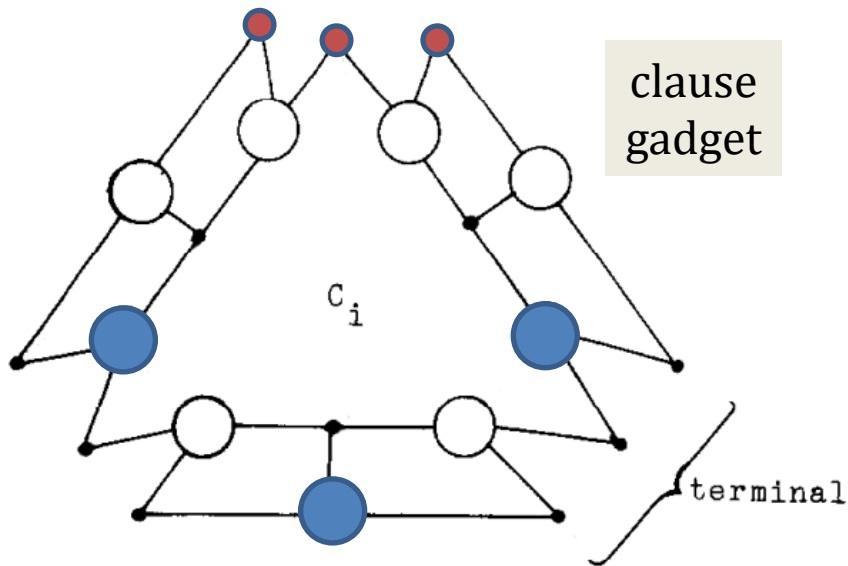
variable gadget

$$\text{size of cover} = \sum_i \frac{1}{2} n_i$$



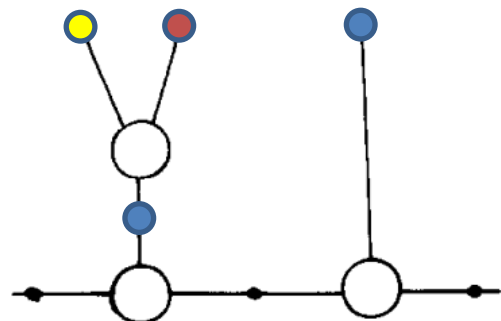
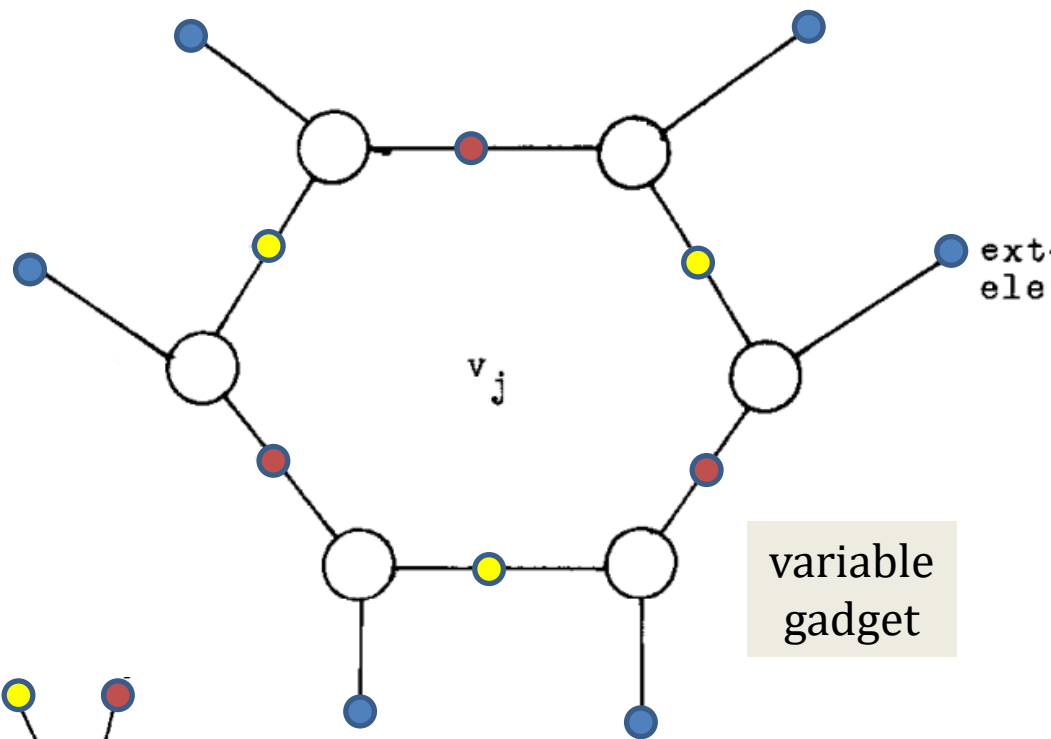
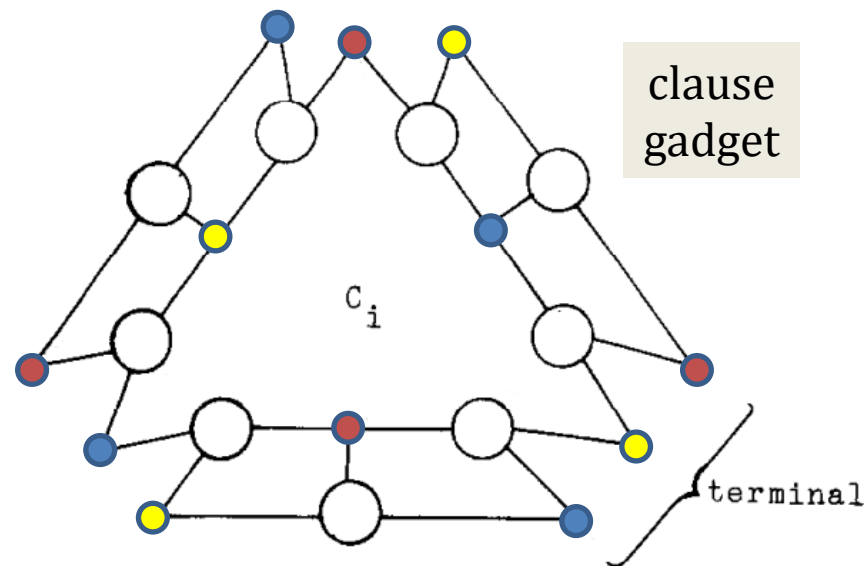
Planar X3C

[Dyer & Freeze 1986]

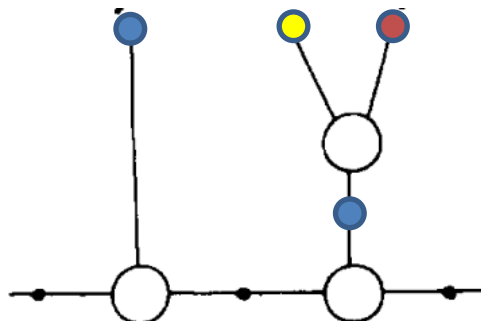


Planar 3DM

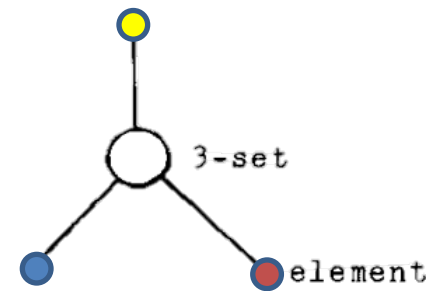
[Dyer & Freeze 1986]



positive connector

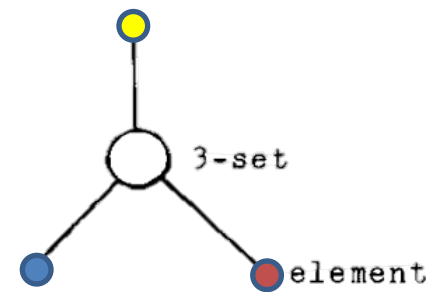
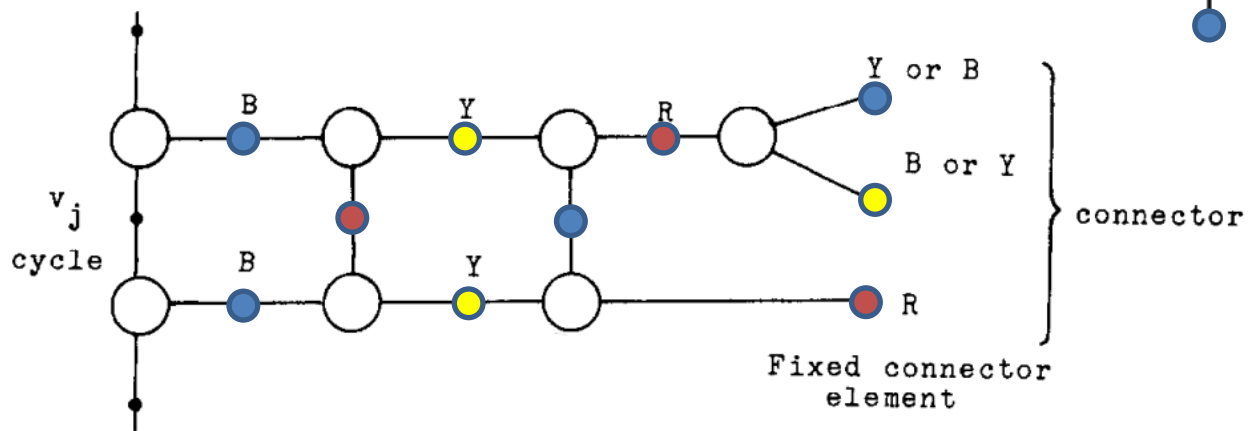
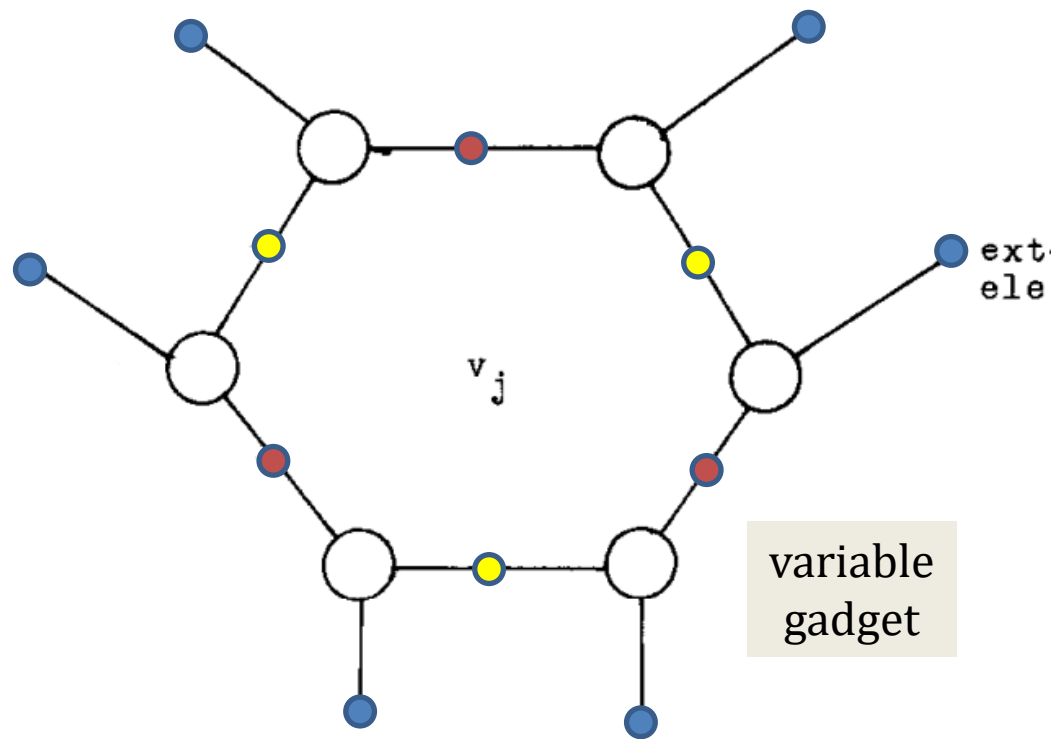
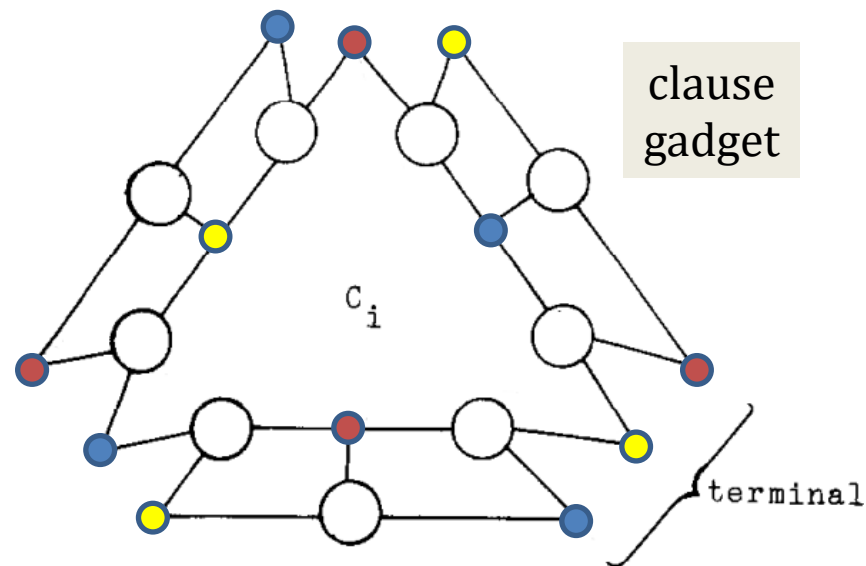


negative connector



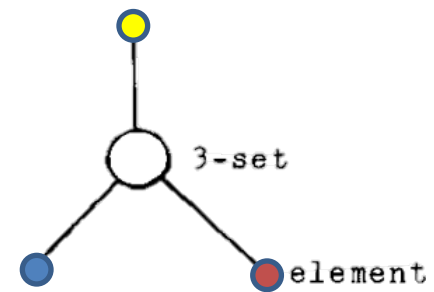
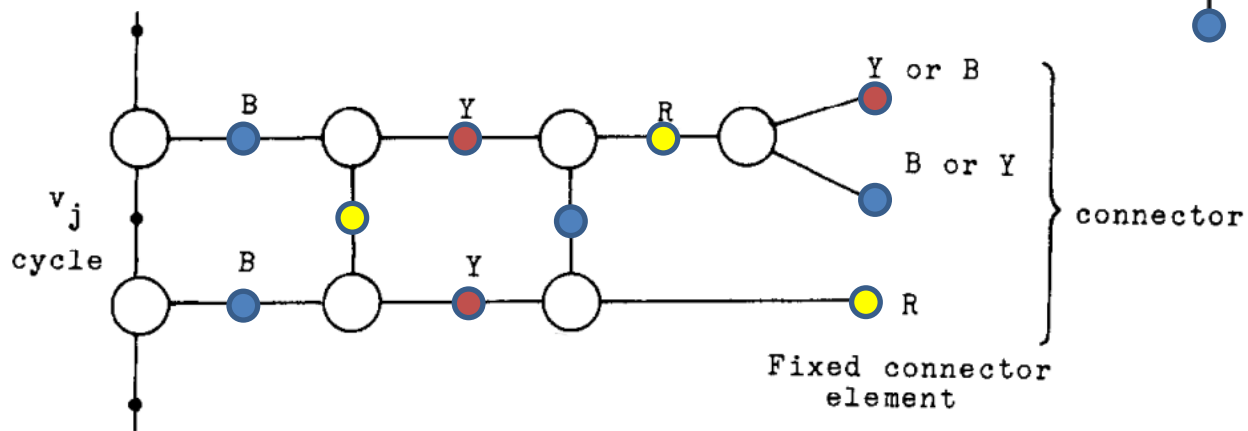
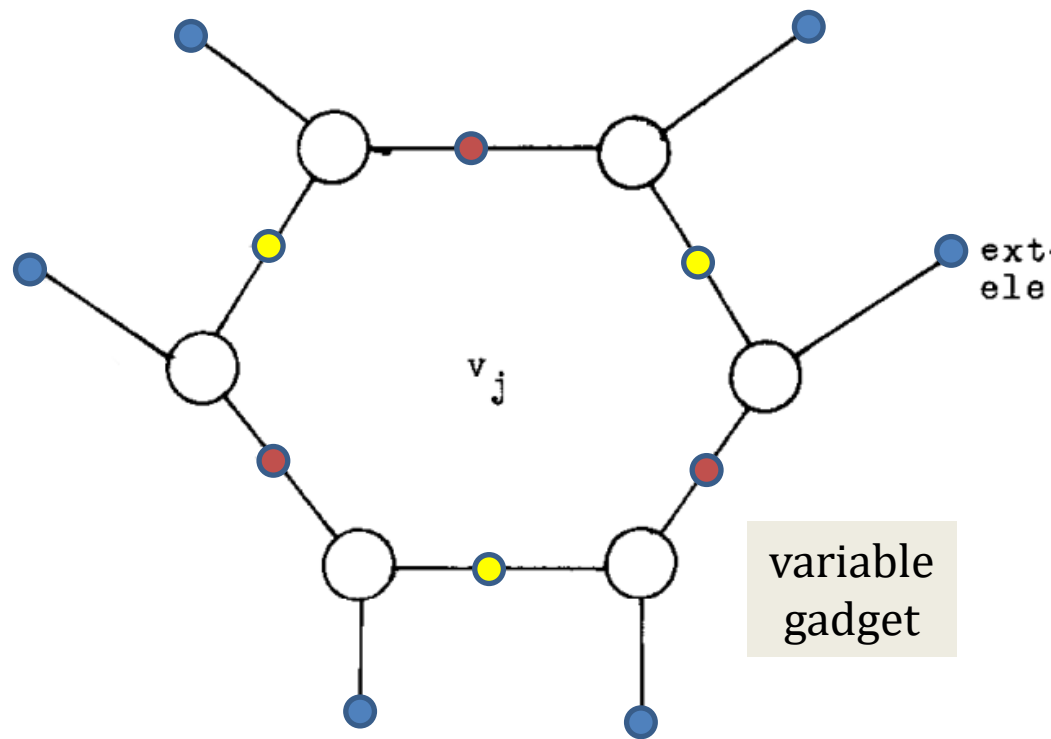
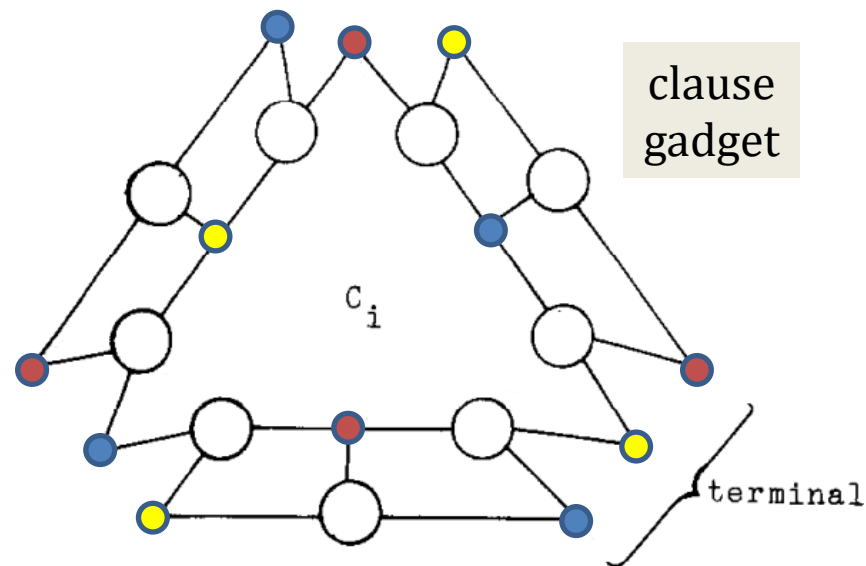
Planar 3DM

[Dyer & Freeze 1986]



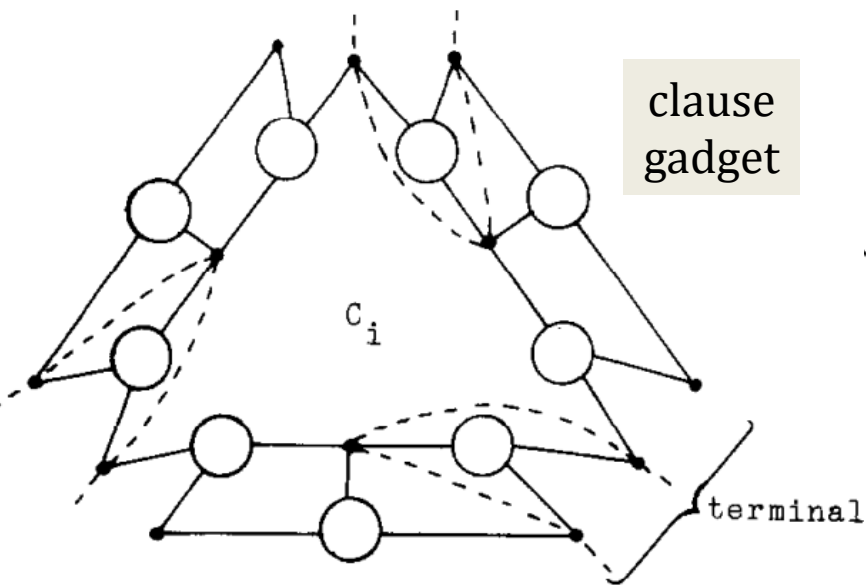
Planar 3DM

[Dyer & Freeze 1986]

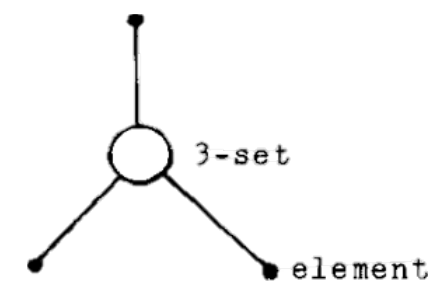
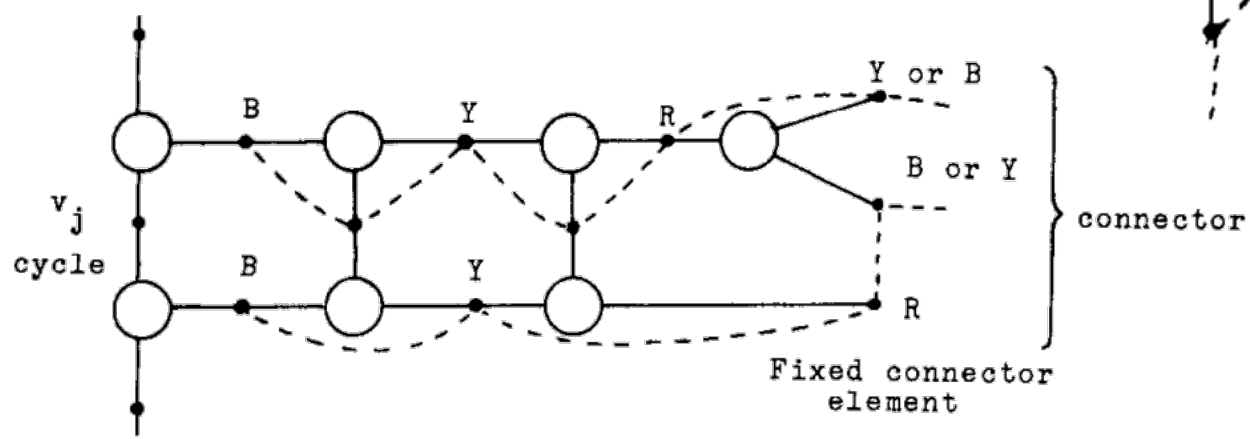
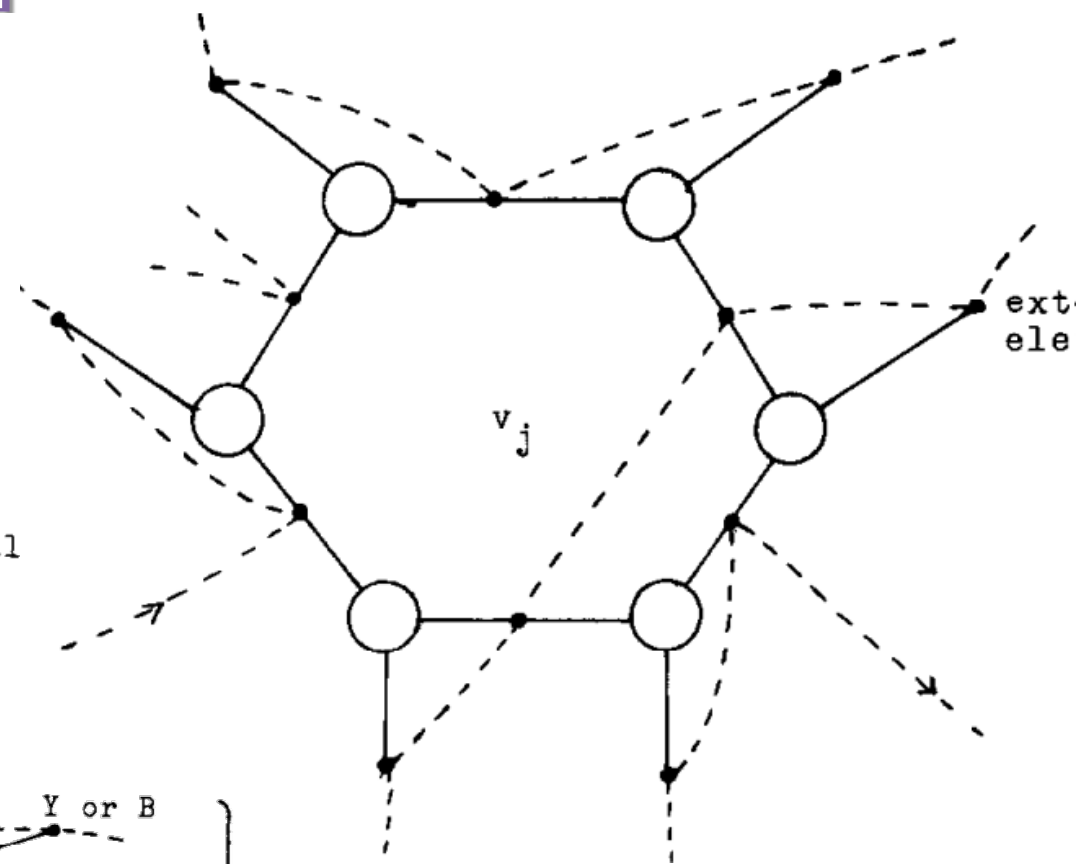


Planar 3DM with Element Cycle

[Dyer & Freeze 1986]

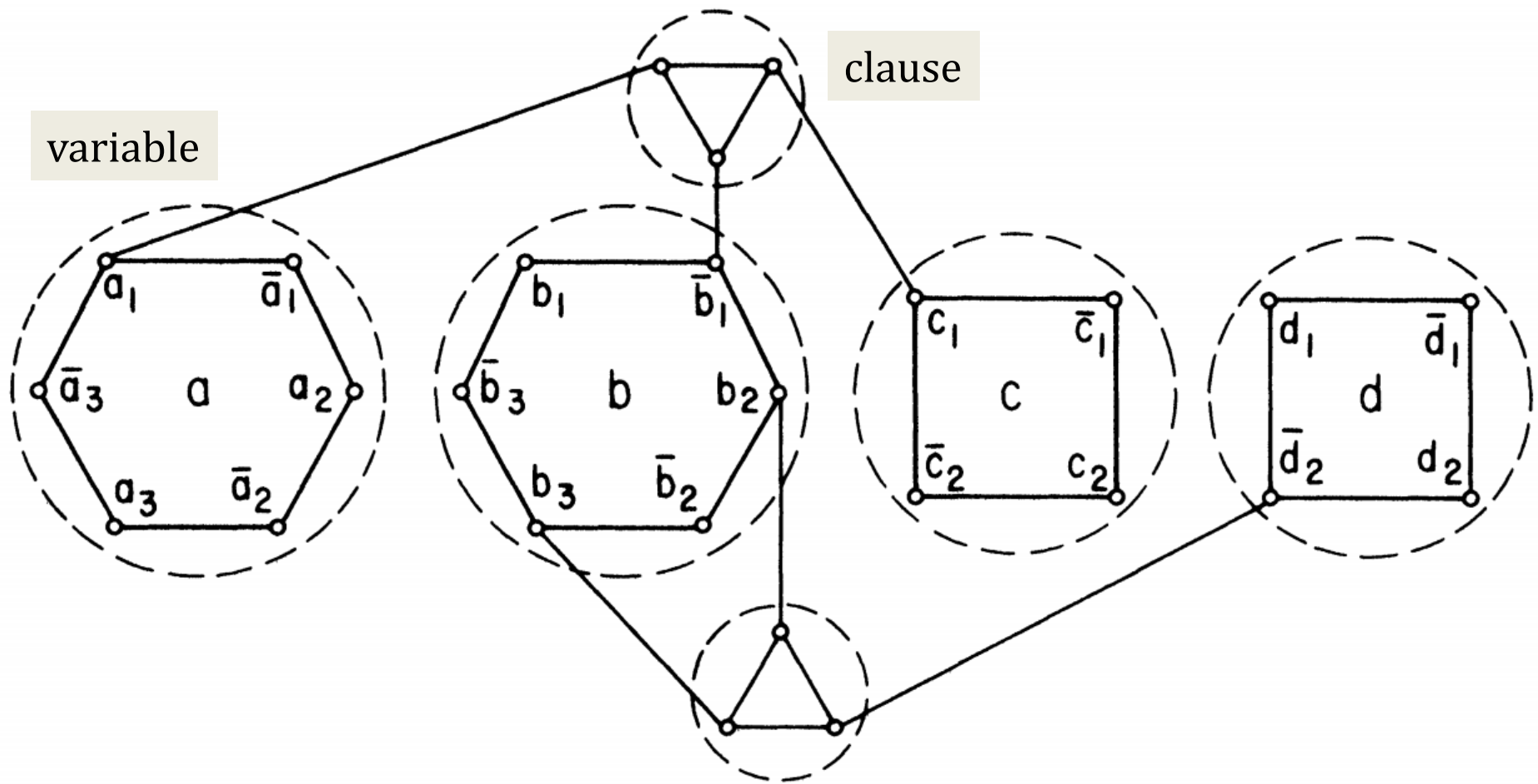


clause gadget



Planar Vertex Cover

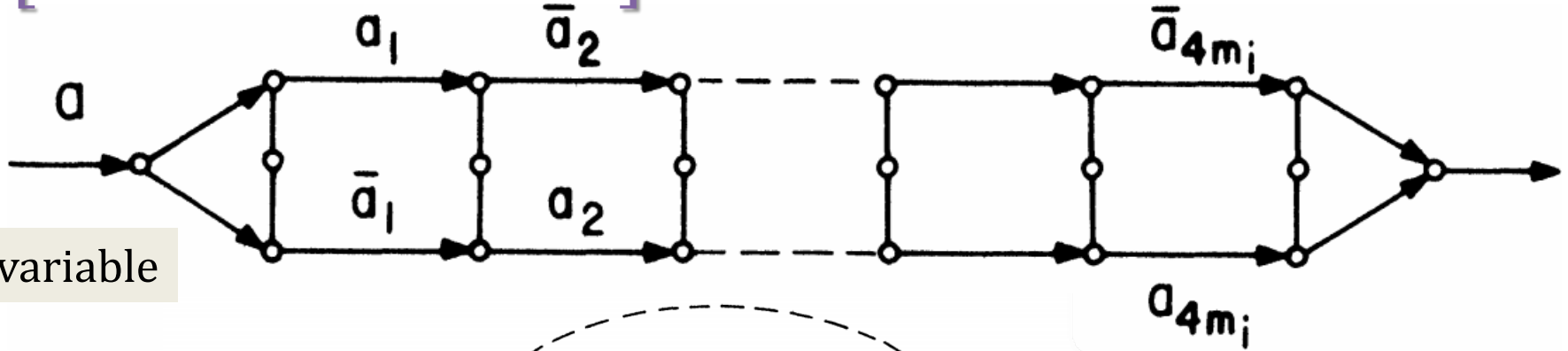
[Lichtenstein 1982]



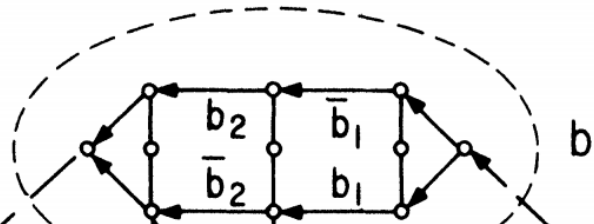
$$\text{Example : } B = (a + \bar{b} + c)(b + b + \bar{d})$$

Planar (Directed) Hamiltonian Cycle

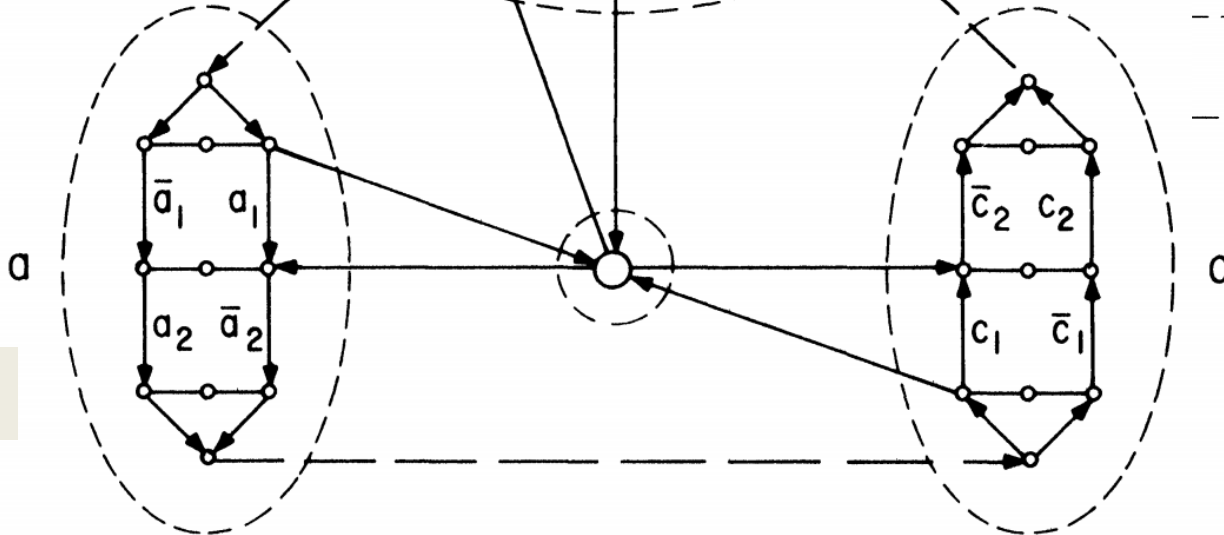
[Lichtenstein 1982]



variable

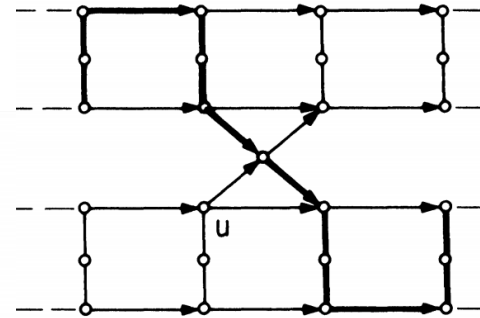


clause



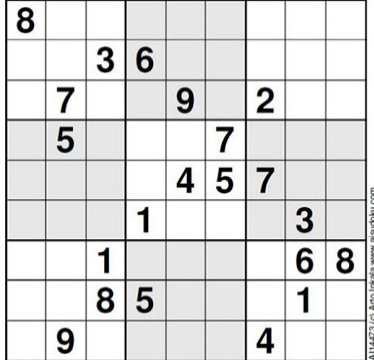
$a \vee \bar{b} \vee c$

a_{4m_i}



c

u



☆☆☆☆☆☆☆☆☆☆
Sudoku

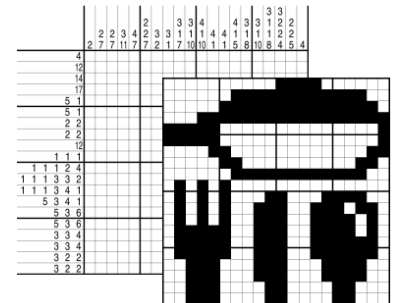


Lits

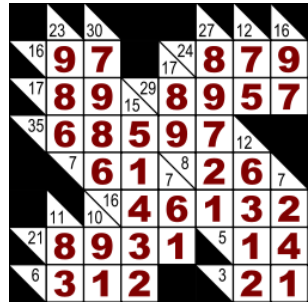


Light Up

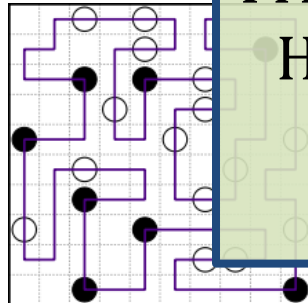
All NP-complete
[Yato & Seta 2003;
Seta 2003; McPhail 2005;
Ueda & Nagao 1996;
Friedman 2002; Hearn 2008;
Holzer, Klein, Kutrib 2004;
Andersson 2007;
Holzer & Ruepp 2007]



Nonogram
(Paint By Numbers)



Kakuro
(Cross Sum)



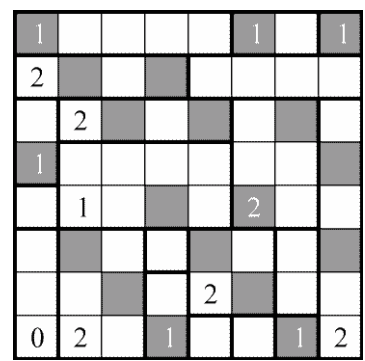
Masyu
(Pearl Puzzle)



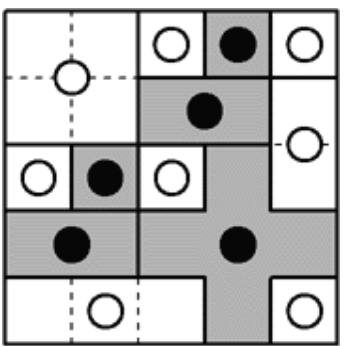
Hitori



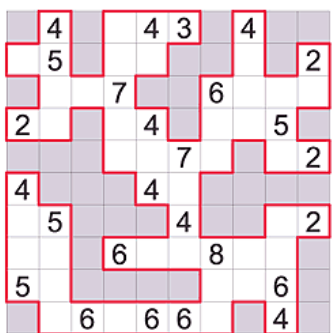
Slitherlink
(Fences)



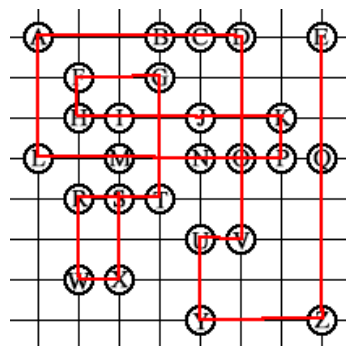
Heyawake



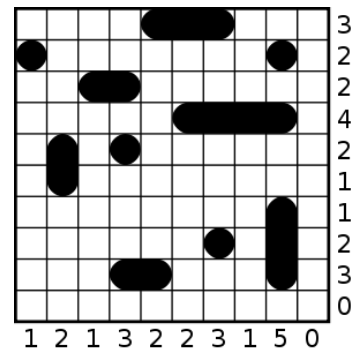
Tentai Show
(Spiral Galaxies)



Bag
(Corral Puzzle)

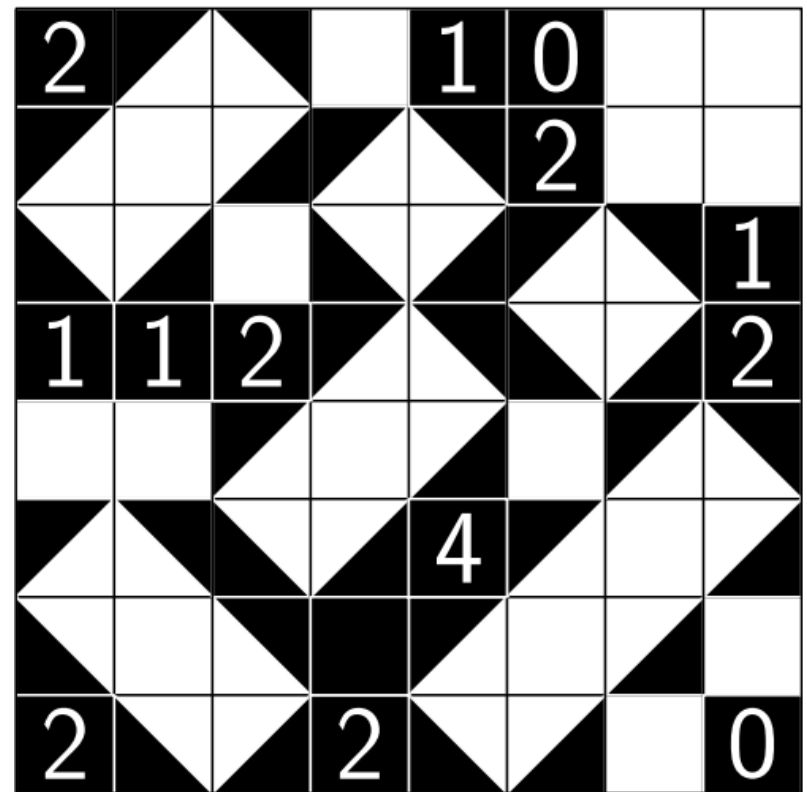


Hiroimono
(Goishi Hiroi)



Battleships

Shakashaka [Guten 2008; Nikoli 2012-]

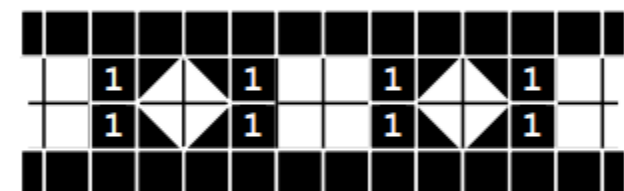
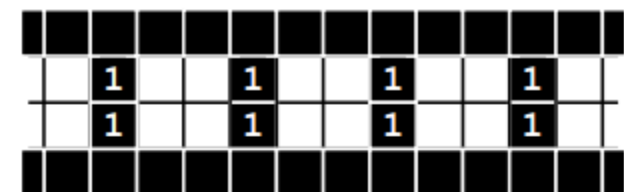


- White squares
- Black squares
 - Labeled 0, 1, 2, 3, 4, or nothing

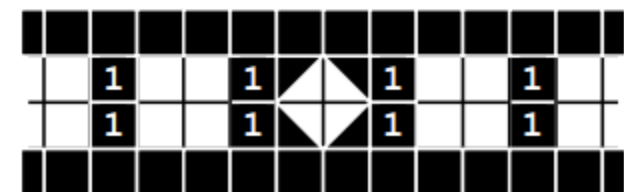
- Half-fill some white squares
- Labels specify number of half-filled neighbors
- Rectangular white regions

Shakashaka is NP-complete

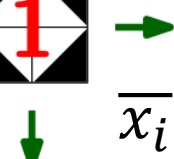
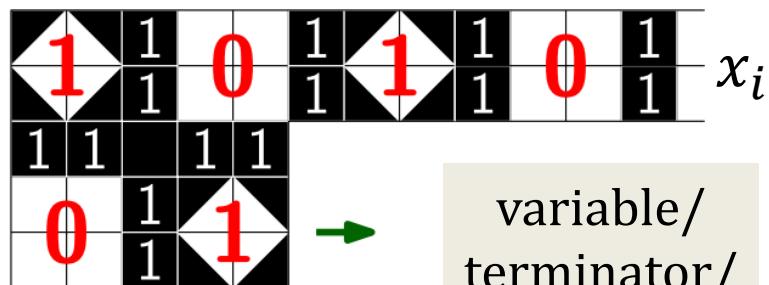
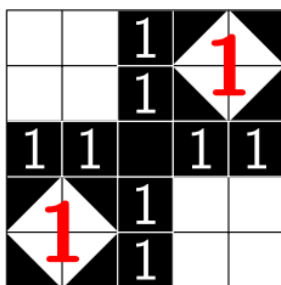
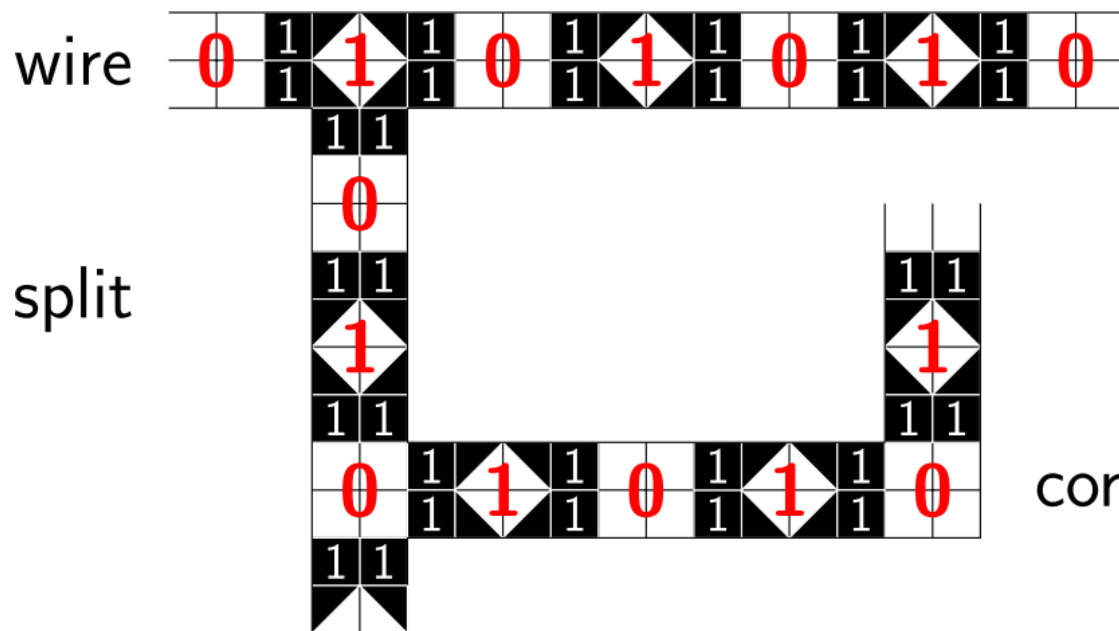
[Demaine, Okamoto, Uehara, Uno 2013]



or



wire



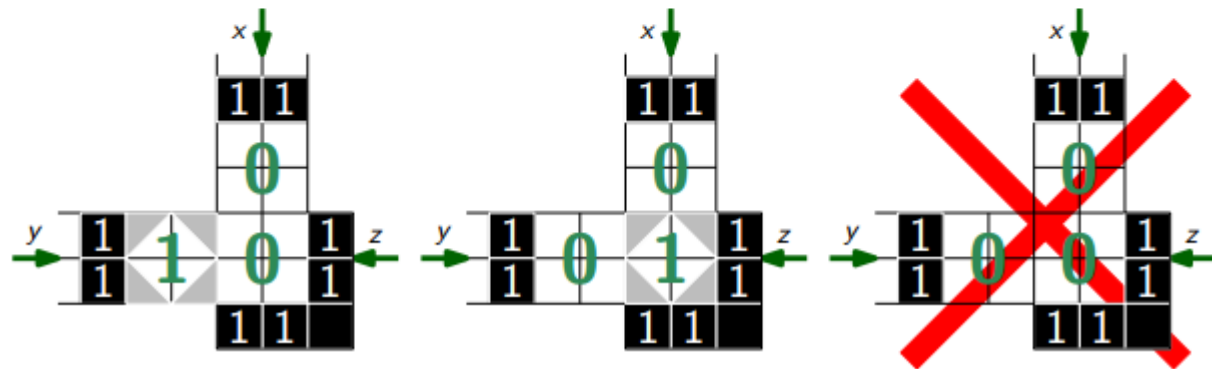
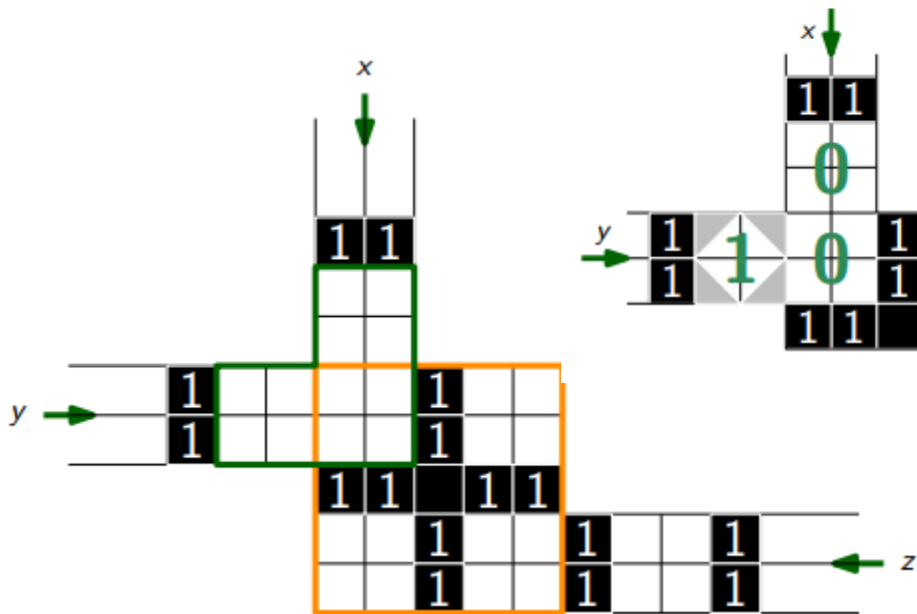
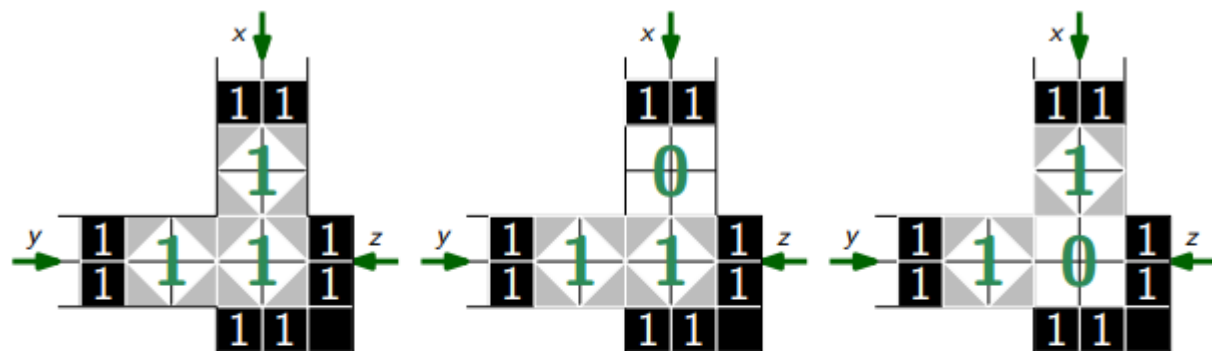
variable/
terminator/
negator

Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]

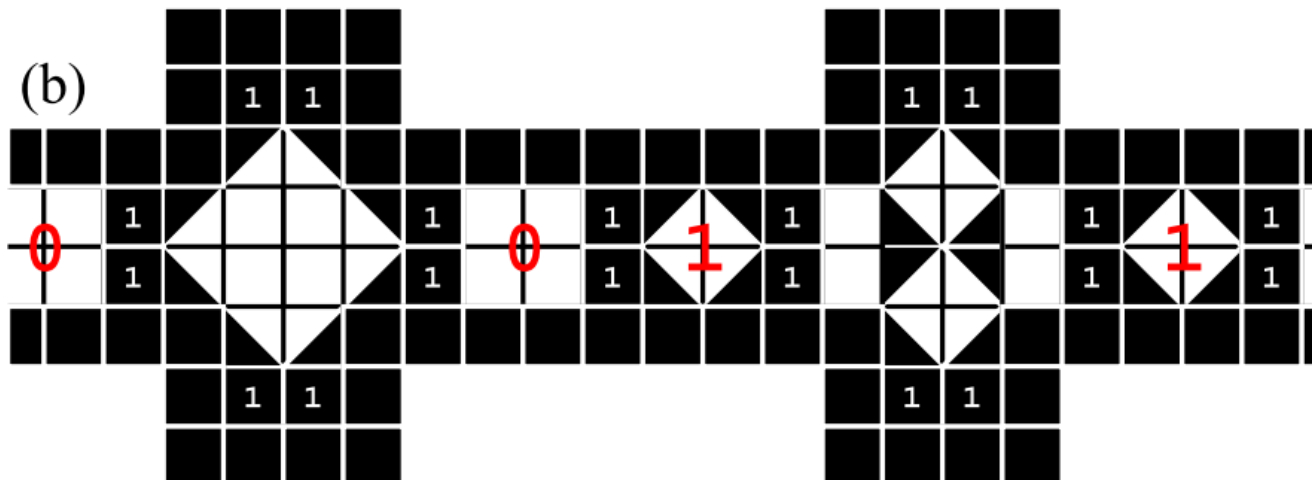
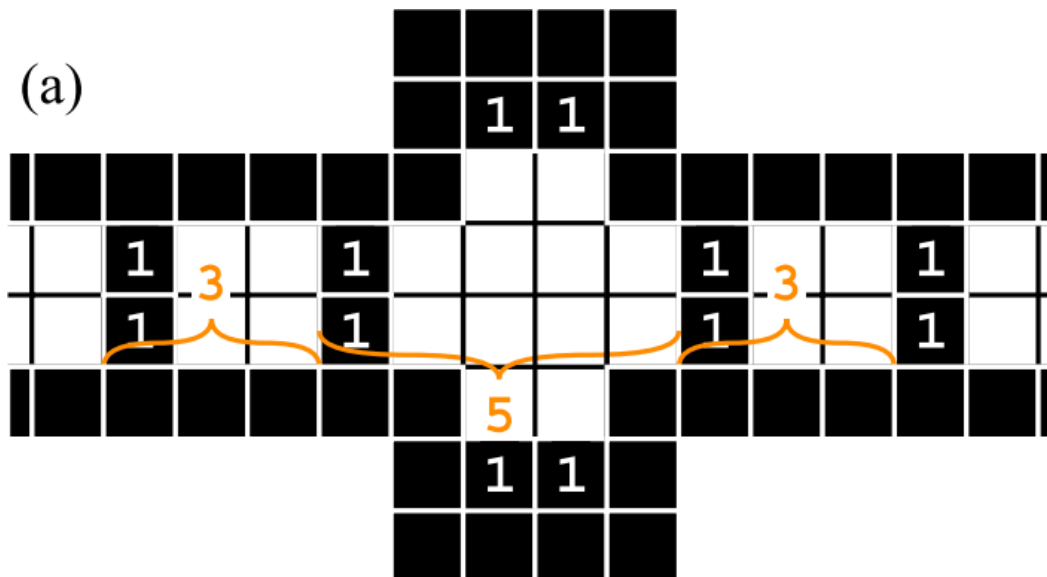
clause

$x \vee y \vee z$



Shakashaka is NP-complete

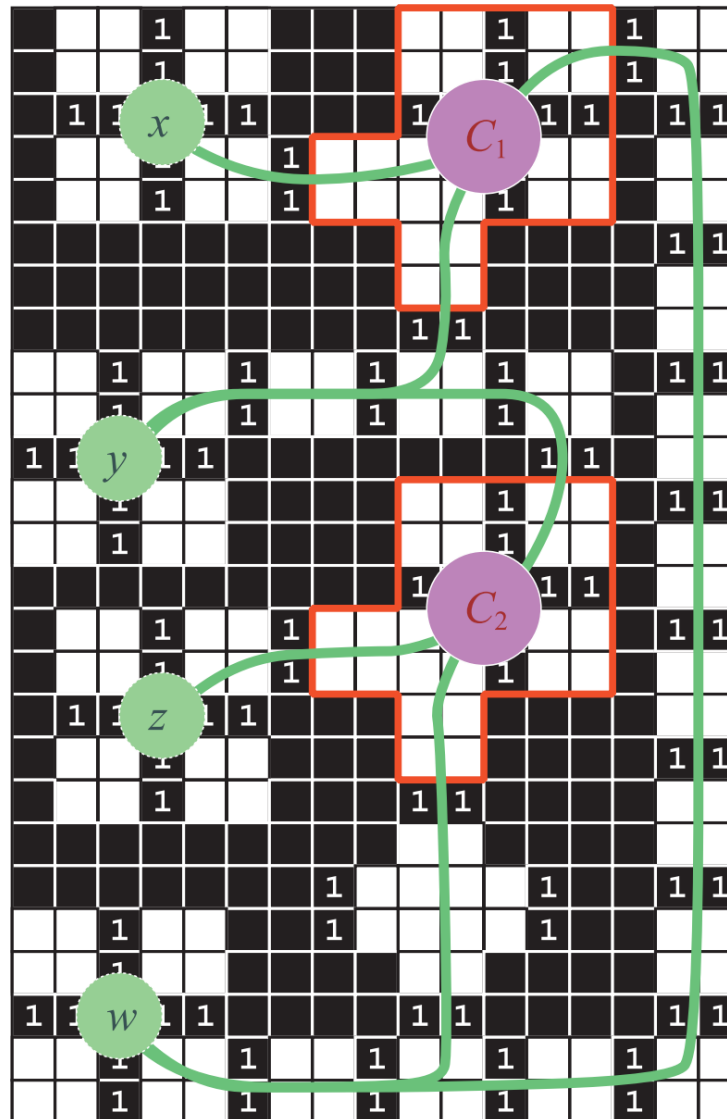
[Demaine, Okamoto, Uehara, Uno 2013]





Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]



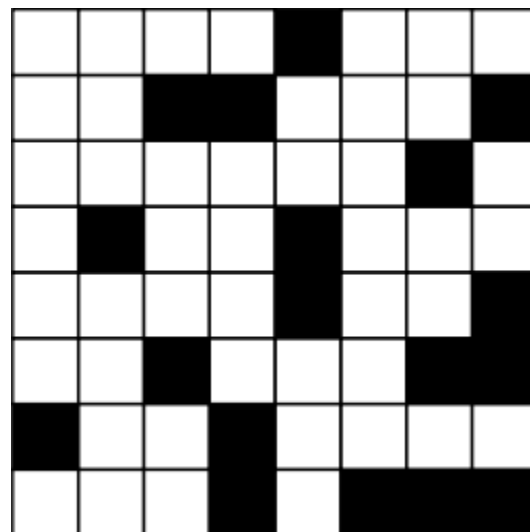
Shakashaka is NP-complete

[Demaine, Okamoto, Uehara, Uno 2013]

- Integer programming can solve small examples

Problem	Size	Level	# of white squares	Time (sec)
1	10 × 10	Easy	76	0.02
2	10 × 10	Easy	77	0.03
3	10 × 10	Easy	82	0.03
4	10 × 18	Easy	131	0.07
5	10 × 18	Medium	156	0.09
6	10 × 18	Medium	144	0.07
7	14 × 24	Medium	297	0.21
8	14 × 24	Hard	295	0.19
9	20 × 36	Hard	645	0.84
10	20 × 36	Hard	632	0.91

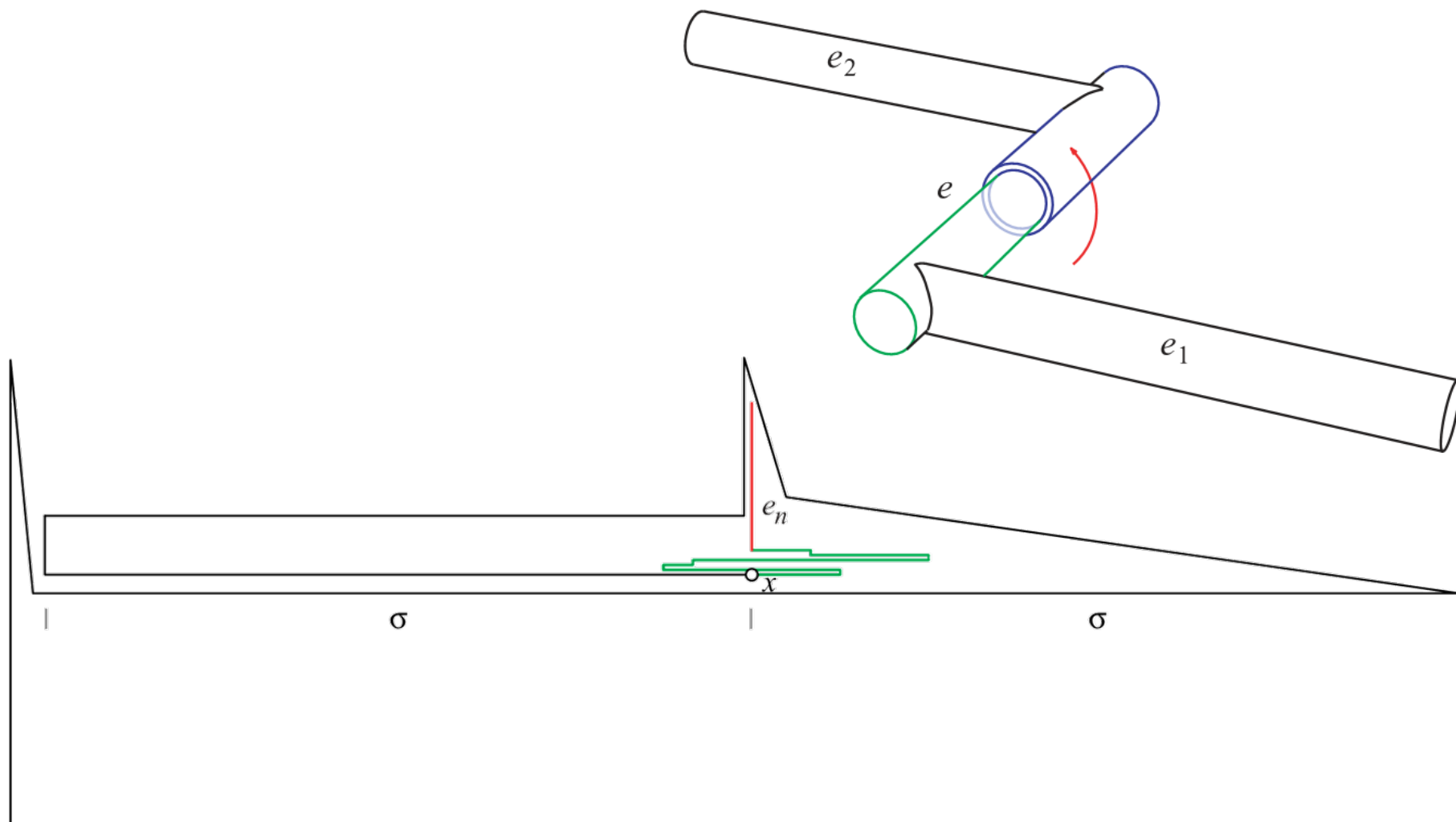
- Open: No labels





Flattening Fixed-Angle Chains

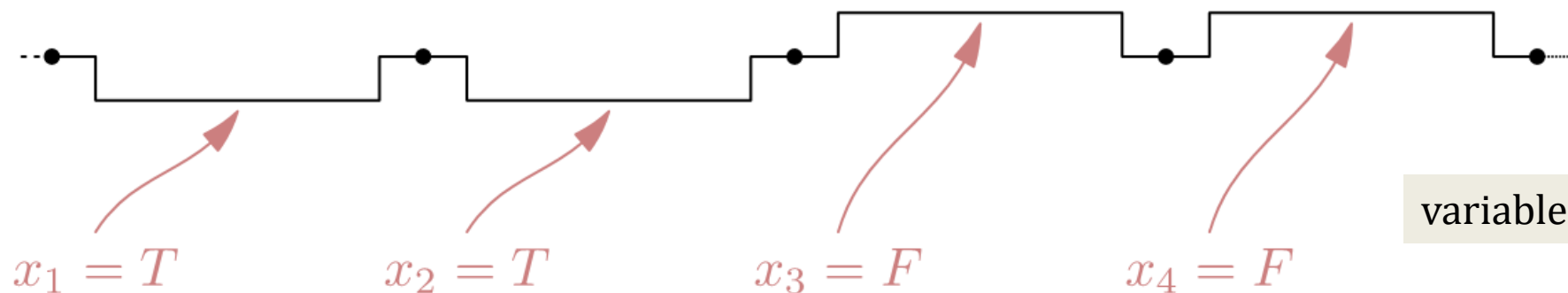
[Soss & Toussaint 2000]





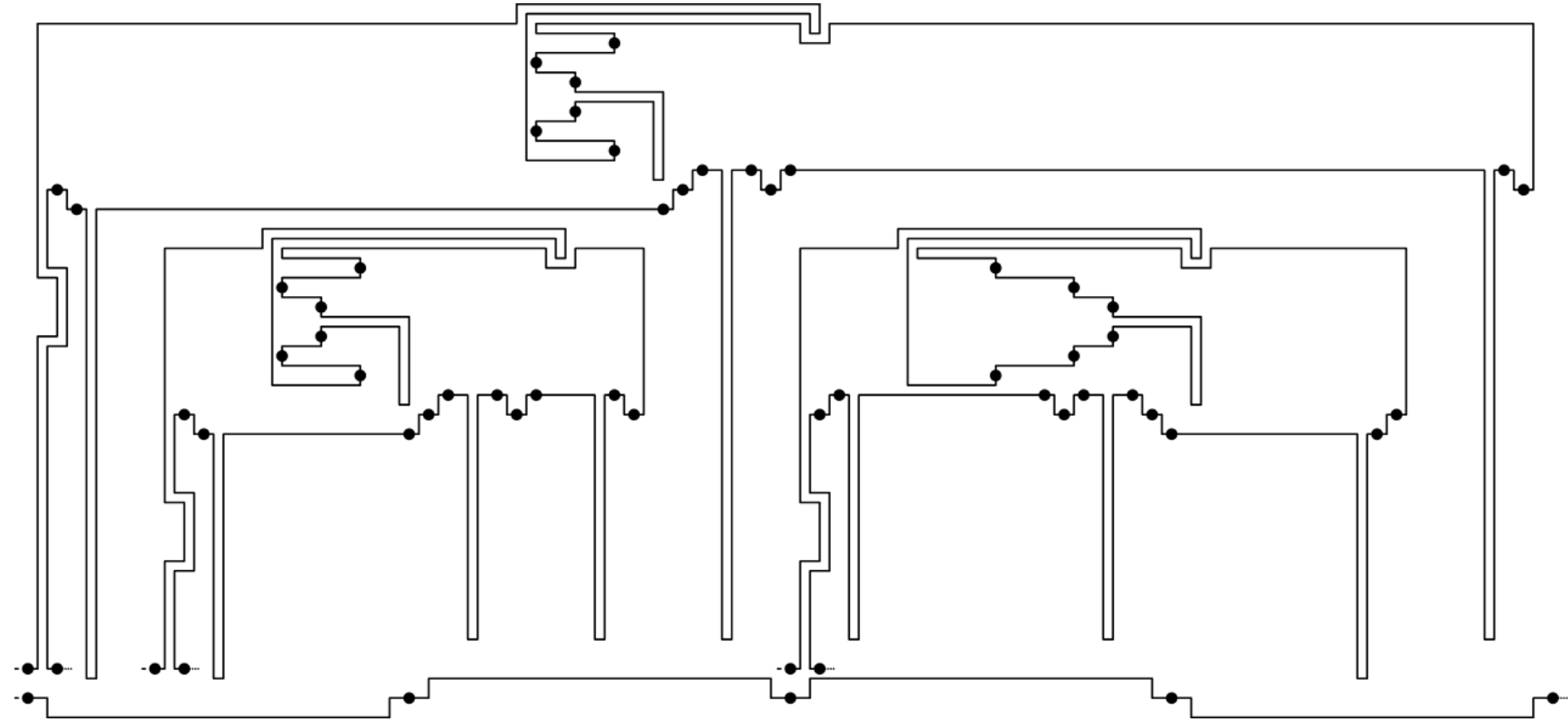
Flattening Fixed-Angle Chains

[Demaine & Eisenstat 2011]



Flat Folding of Fixed-Angle Chains

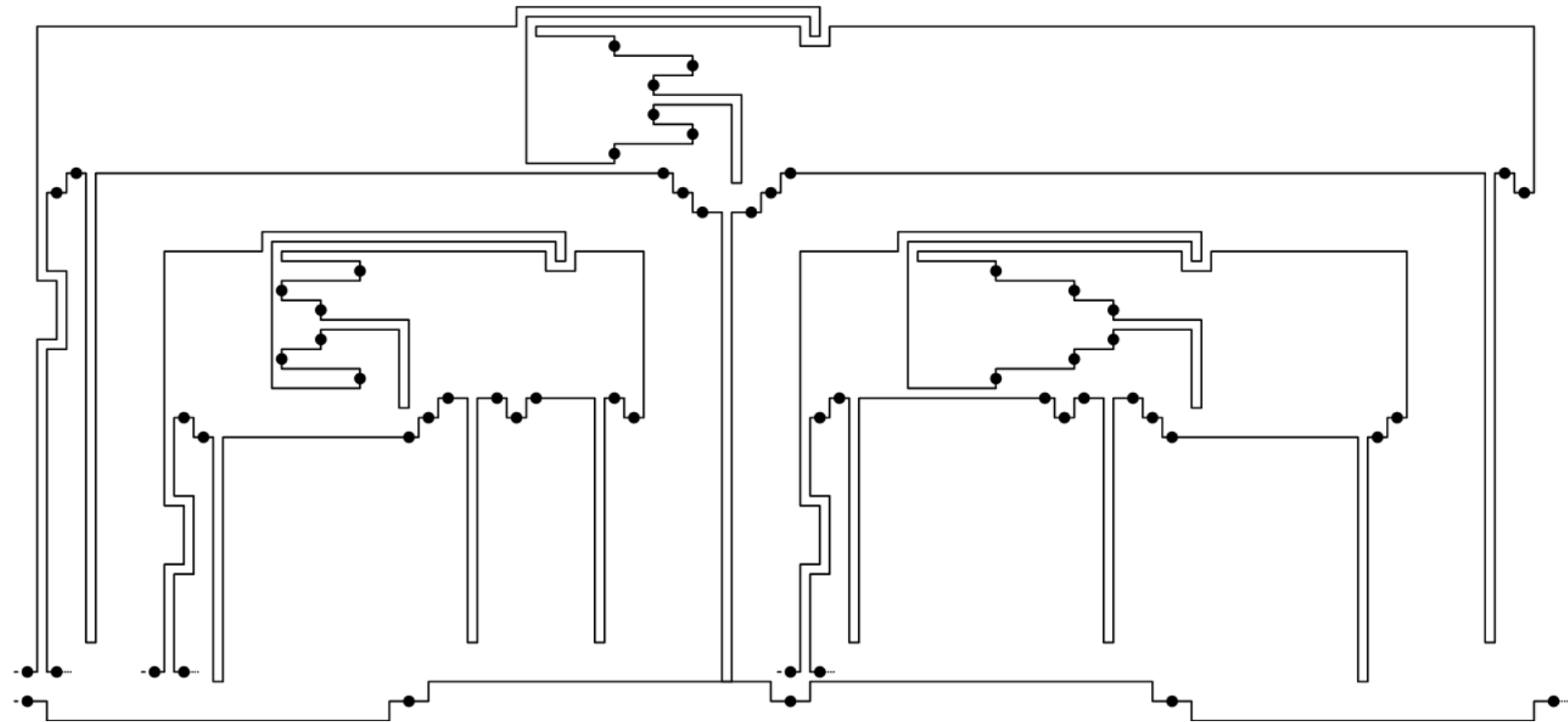
[Demaine & Eisenstat 2011]





Flat Folding of Fixed-Angle Chains

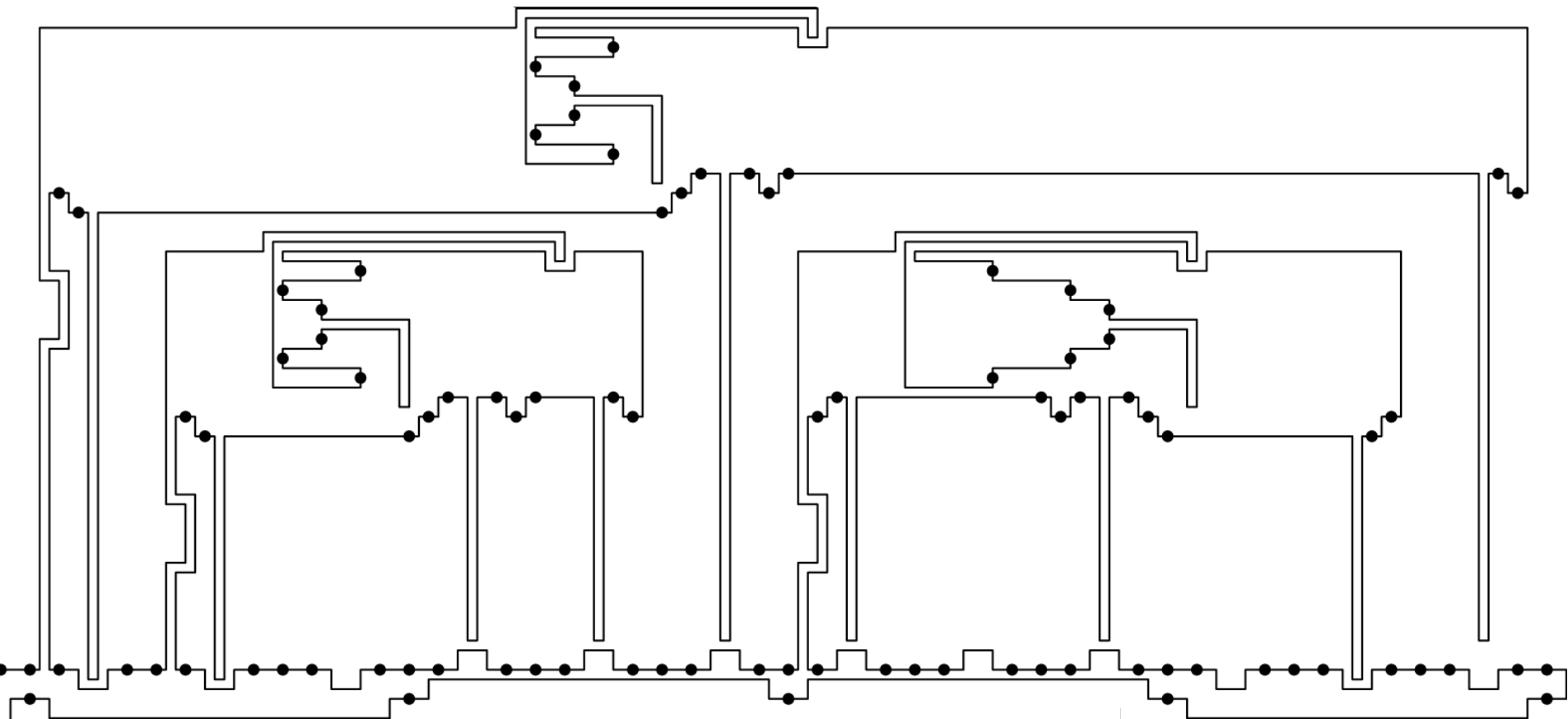
[Demaine & Eisenstat 2011]





Flat Folding of Fixed-Angle Chains

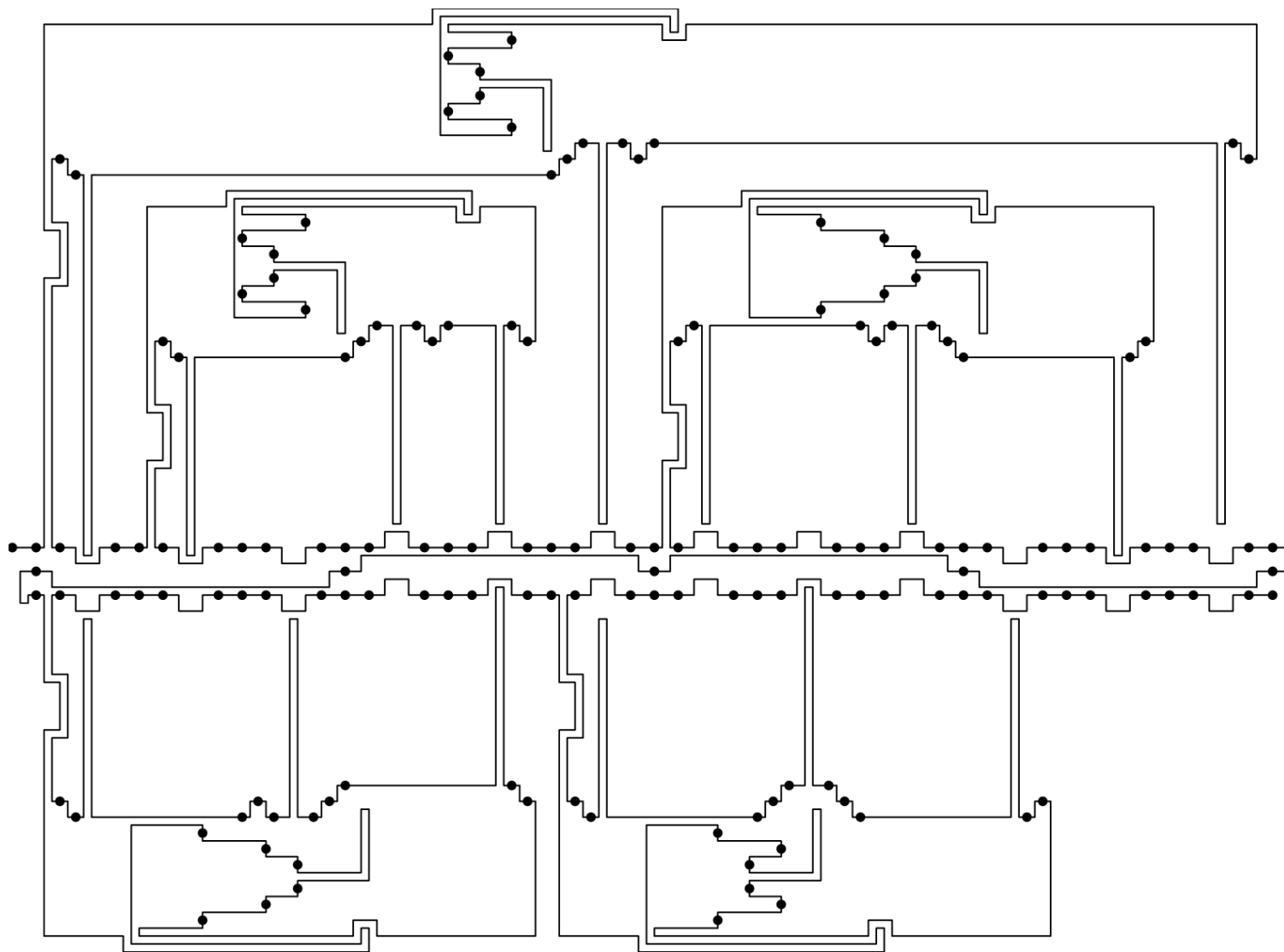
[Demaine & Eisenstat 2011]





Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]



Flat Folding of Fixed-Angle Chains

[Demaine & Eisenstat 2011]

What We
Just Proved

Variant Considered

Linkage Type	Edge Lengths	Angle Range	Result
Fixed-angle chain , but only some edges can spin	Equilateral	$\{90^\circ, 180^\circ\}$	Strongly NP-hard
Fixed-angle chain (all edges can spin)	Equilateral	$[16.26^\circ, 180^\circ]$	Strongly NP-hard
Fixed-angle chain (all edges can spin)	$\Theta(1)$	$[60 - \varepsilon, 180^\circ]$	Strongly NP-hard
Fixed-angle caterpillar tree	Equilateral	$\{90^\circ, 180^\circ\}$	Strongly NP-hard