

2-Colorable Perfect Matching is NP-complete [Schaefer 1978]

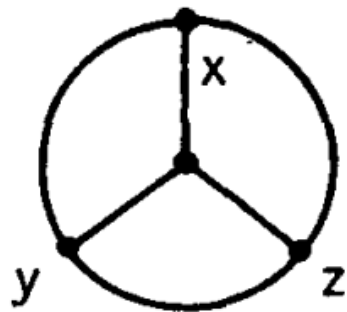


Figure 1(a)

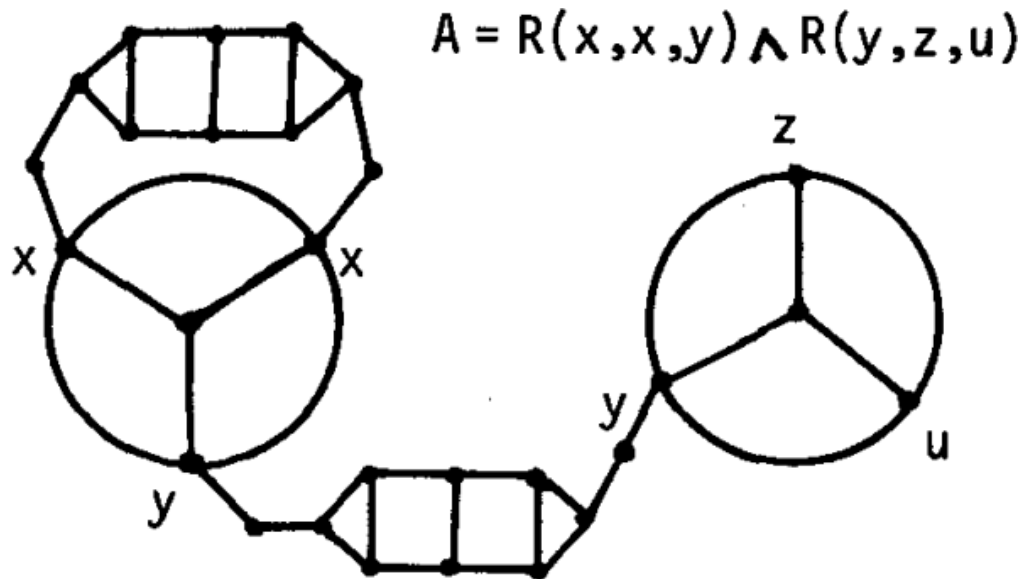


Figure 1(c)

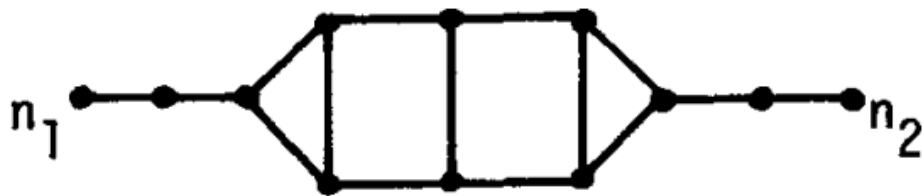
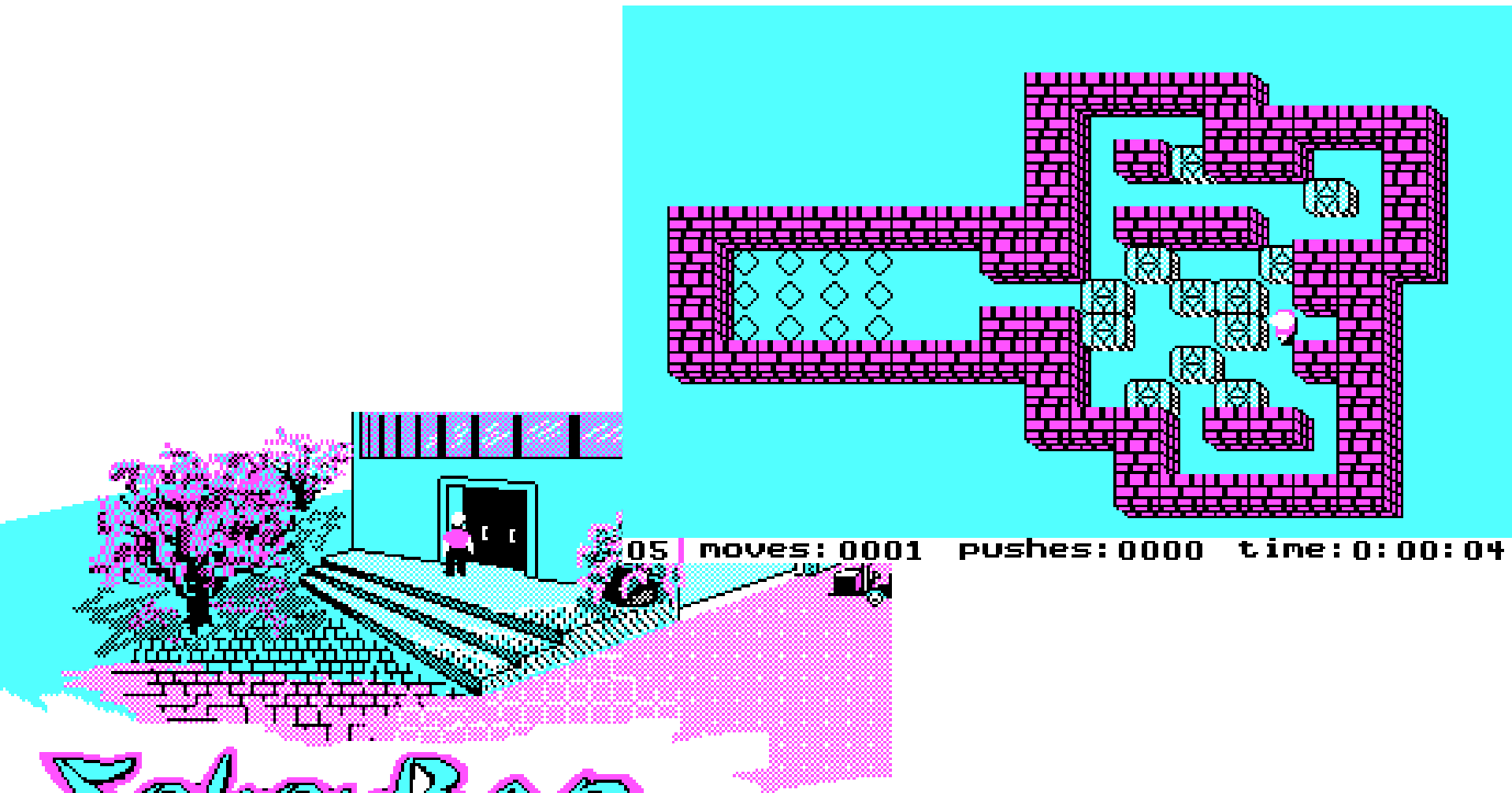


Figure 1(b)

Pushing 1 × 1 Blocks



Sokoban

by *Spectrum HoLoByte*™ a division of Sphere Inc.

Designed by Khaled Bentebal

Programmed by Farah Soebrata

Graphics by Jody Sather

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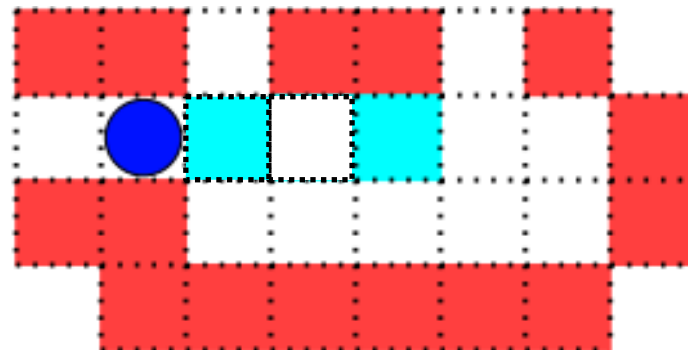
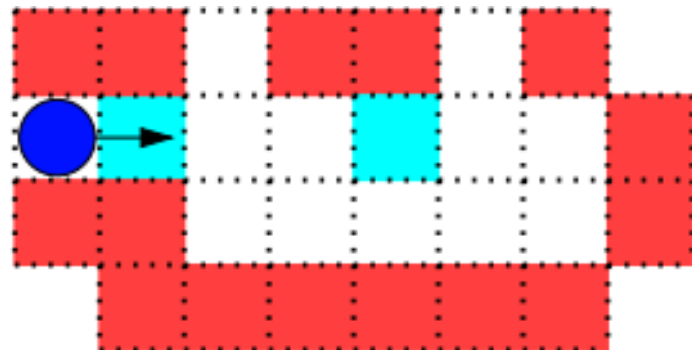
Pushing 1 × 1 Blocks



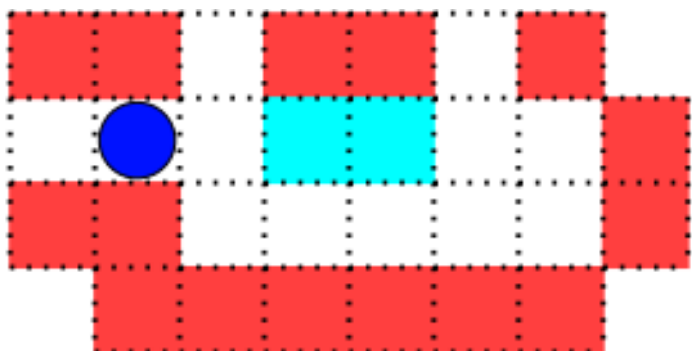
Legend of Zelda: The Minish Cap



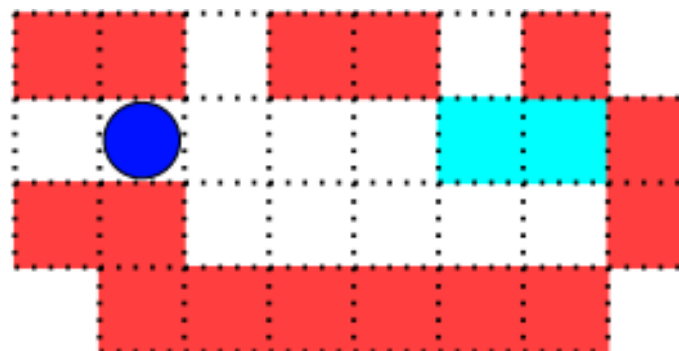
Pushing 1 × 1 Blocks



PUSH-2



PUSH PUSH-2



PUSH PUSH PUSH-2

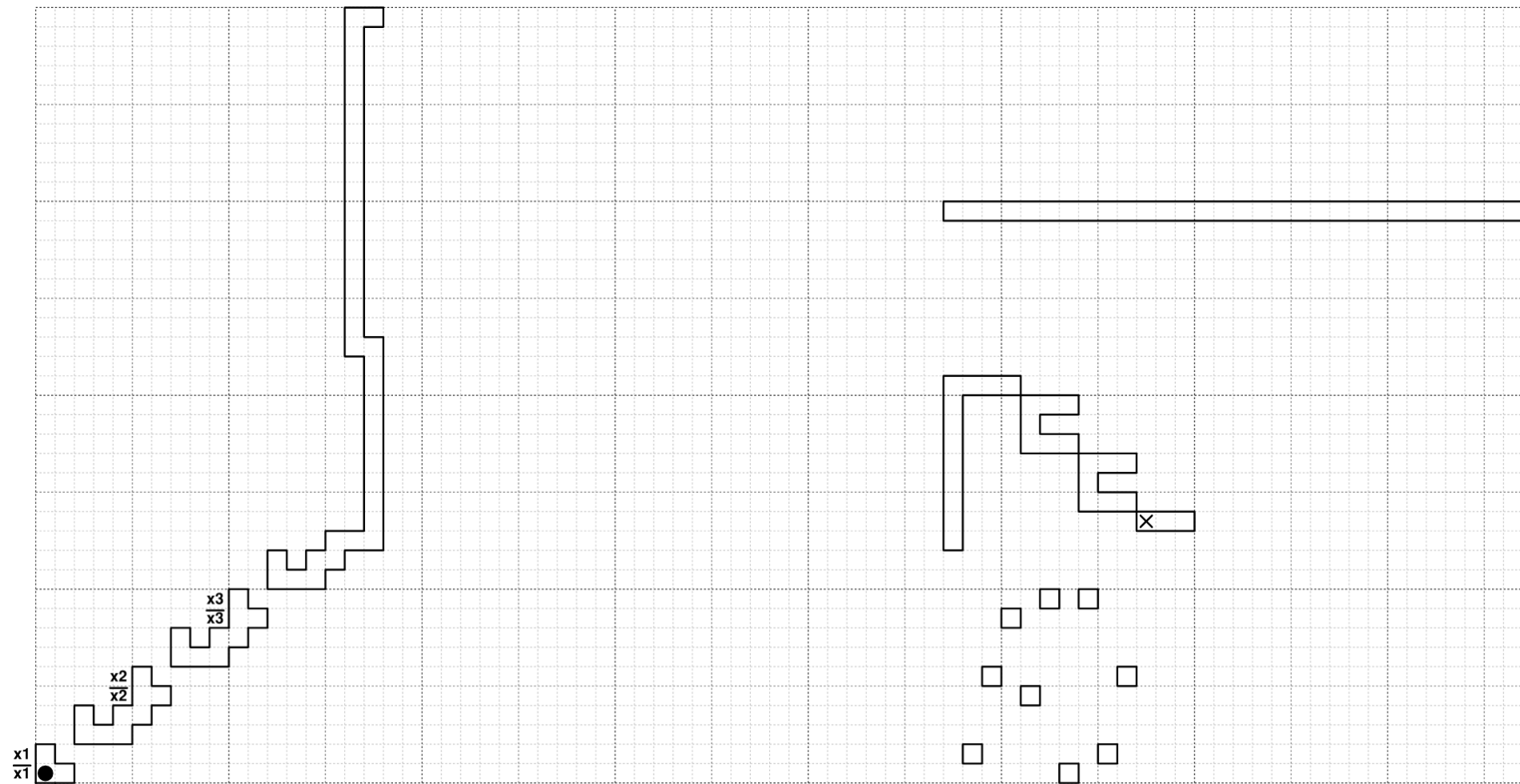
Pushing 1×1 Blocks Complexity

Name	Push	Fixed	Slide	Goal	Complexity	Reference
Push- k	$k \geq 1$	no	min	path	NP-hard	D, D, O'Rourke 2000
Push-*	∞	no	min	path	NP-hard	Hoffmann 2000
PushPush- k	$k \geq 1$	no	max	path	PSPACE-complete	D, Hoffmann, Holzer 2004
PushPush-*	∞	no	max	path	NP-hard	Hoffmann 2000
Push-1F	1	yes	min	path	NP-hard	DDO 2000
Push- k F	$k \geq 2$	yes	min	path	PSPACE-complete	D, Hearn, Hoffmann 2002
Push- $*$ F	∞	yes	min	path	PSPACE-complete	Bremner, O'Rourke, Shermer 1994
Push- k X	$k \geq 1$	no	min	simple path	NP-complete	D, Hoffmann 2001
Push- $*$ X	∞	no	min	simple path	NP-complete	Hoffmann 2000
Sokoban	1	yes	min	storage	PSPACE-complete	Culberson 1998



Push-* is NP-hard

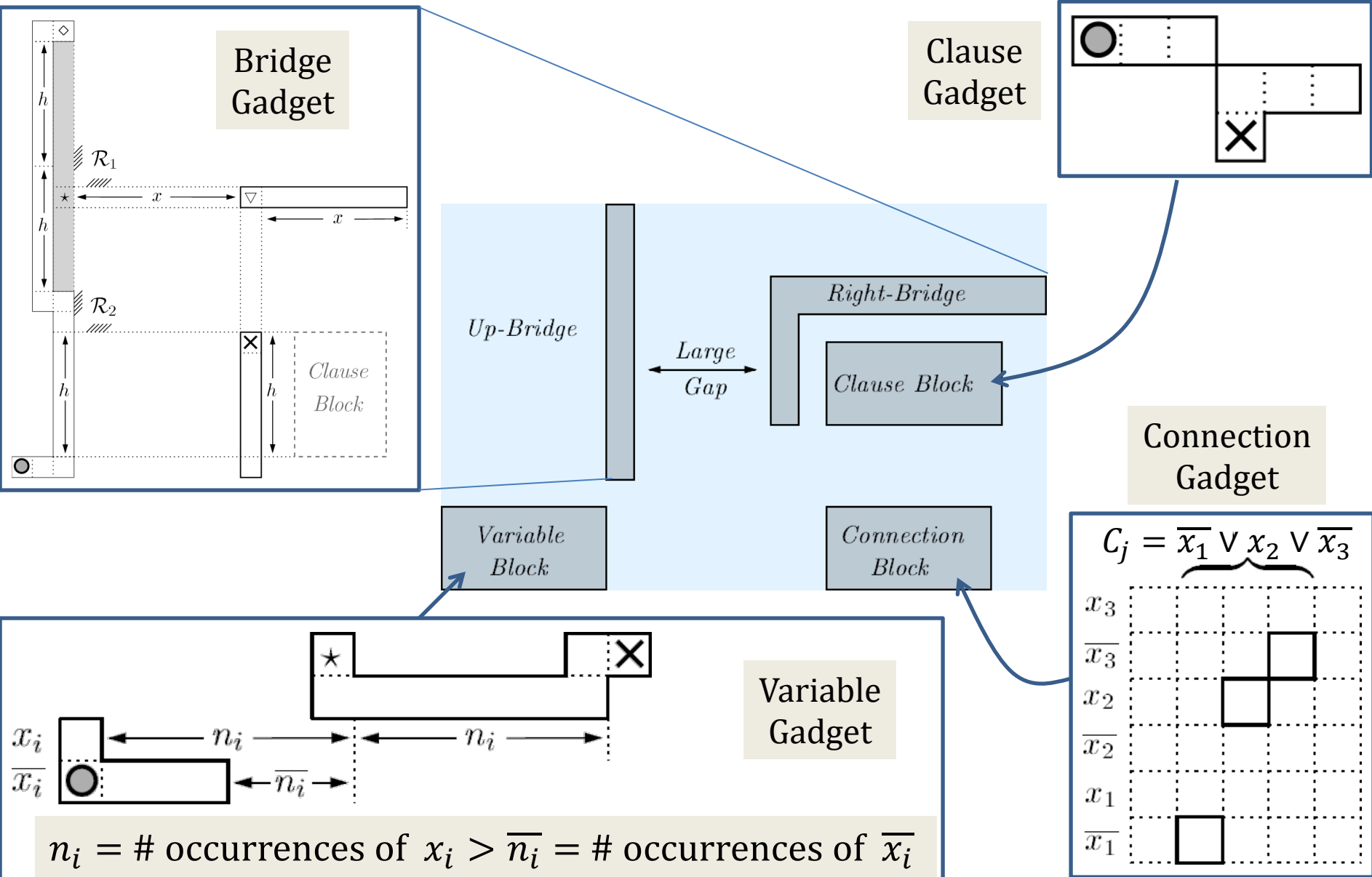
[Hoffmann 2000]



$$(x1 \vee x2 \vee \bar{x3}) \wedge (\bar{x2} \vee x3 \vee \bar{x1}) \wedge (x3 \vee x1 \vee x2)$$

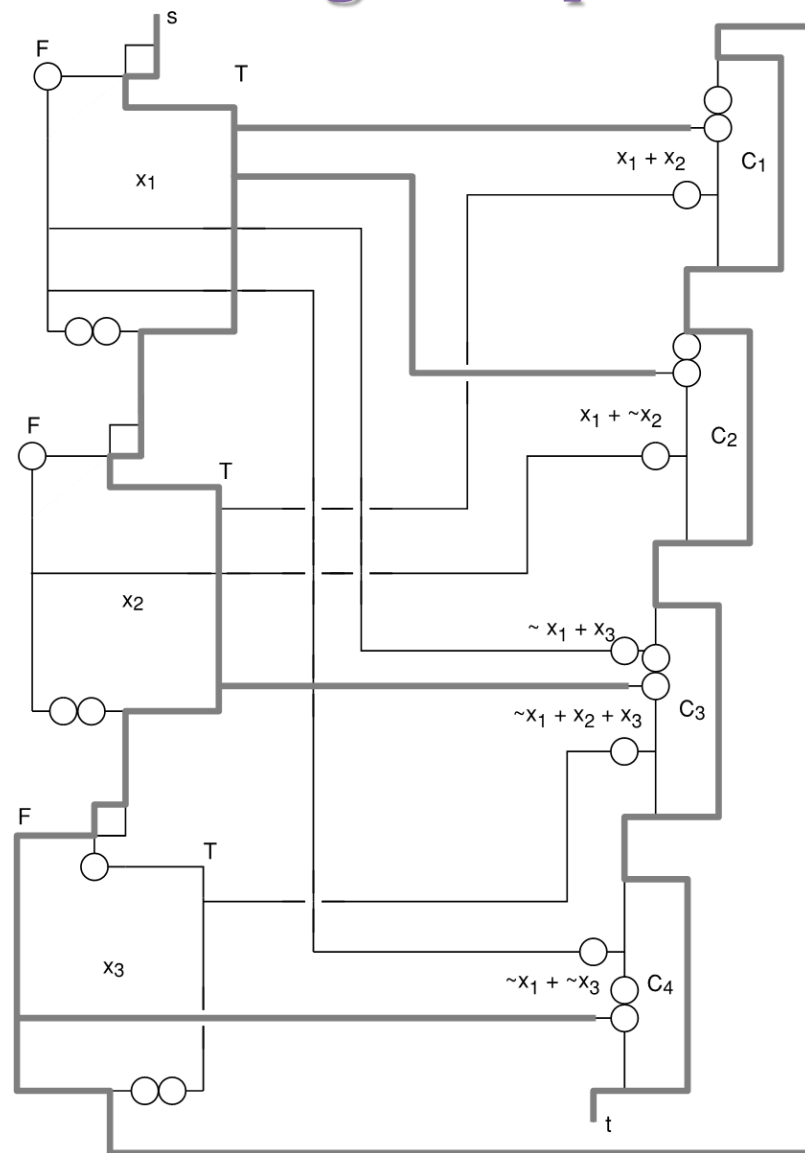
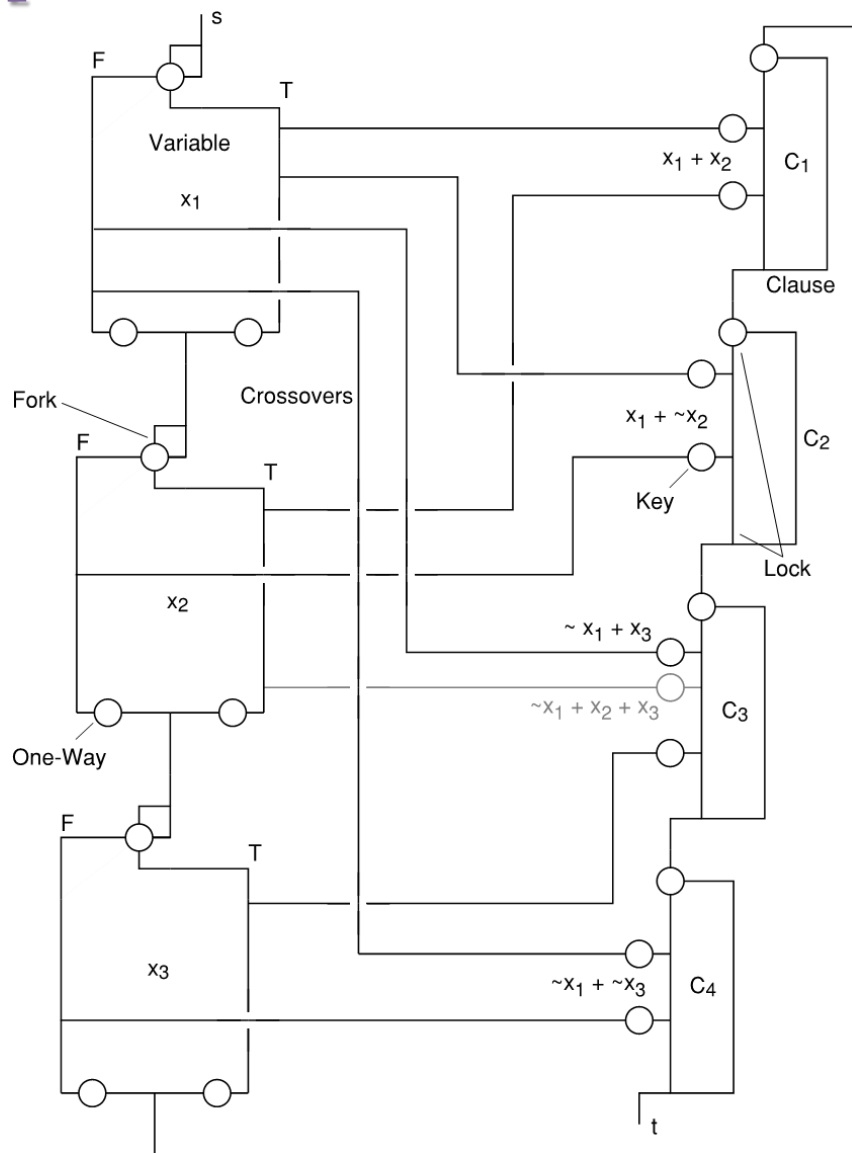
Push-* is NP-hard

[Hoffmann 2000]



PushPush-1 is NP-hard in 3D

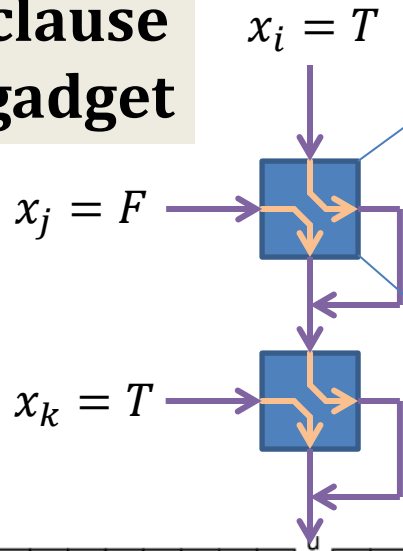
[O'Rourke & Smith Problem Solving Group 1999]



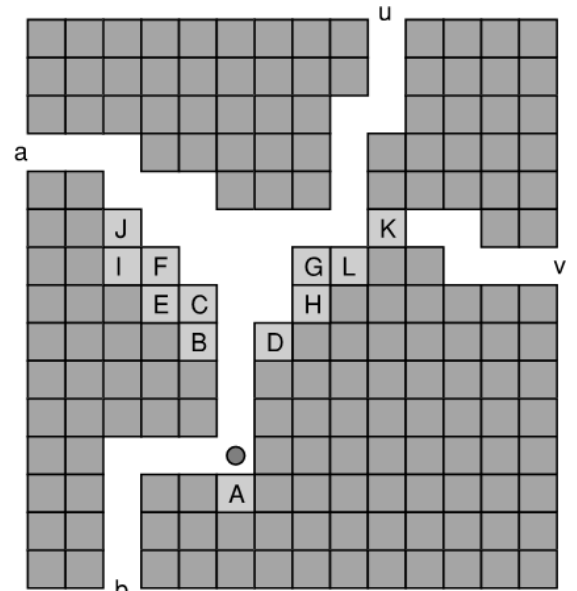
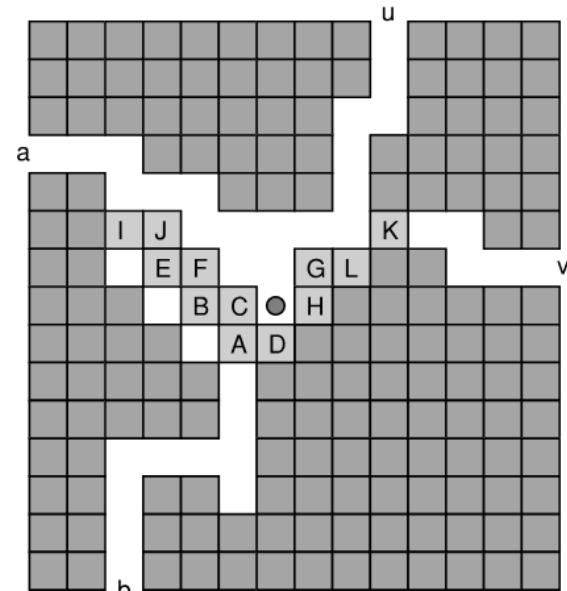
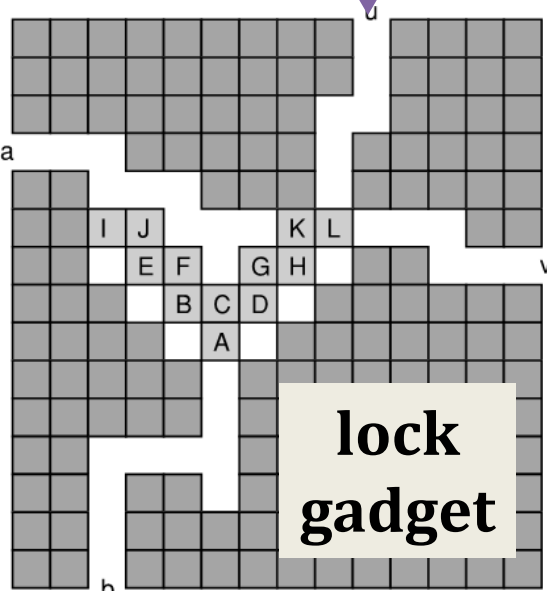
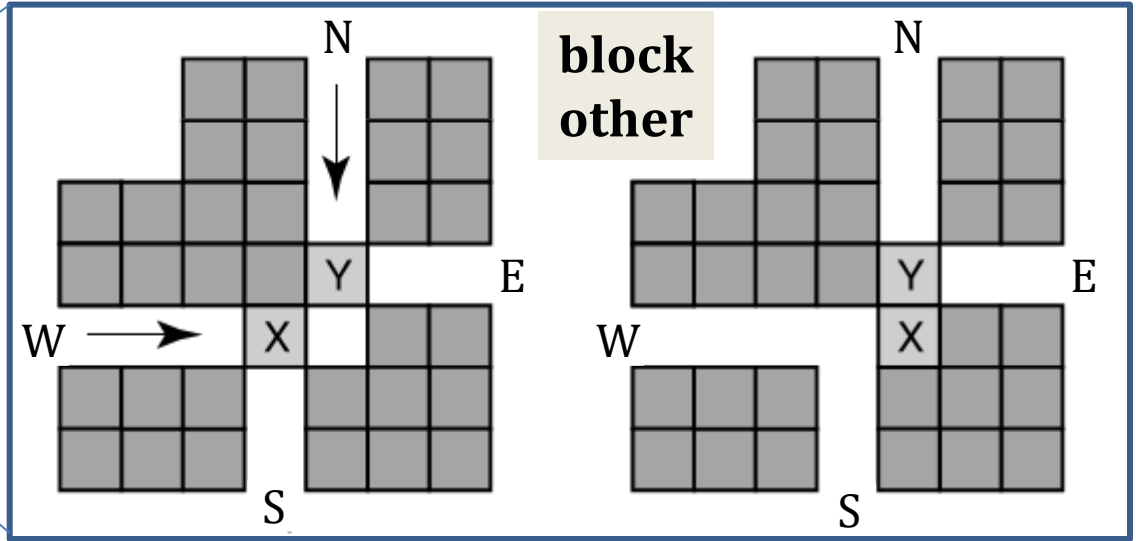
(Push)Push-1 is NP-hard in 2D

[Demaine, Demaine, O'Rourke 2000]

clause gadget



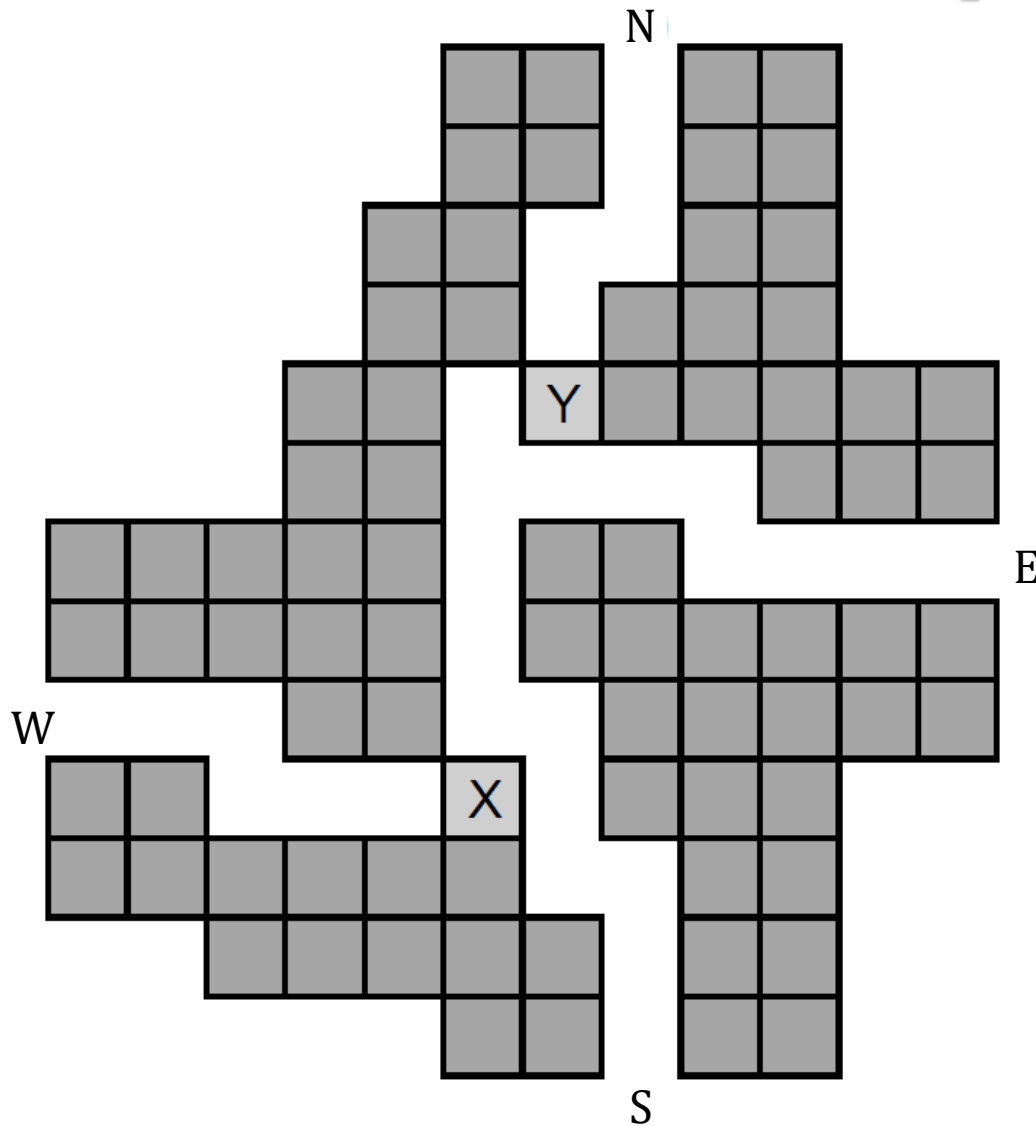
block other





(Push)Push-1 is NP-hard in 2D

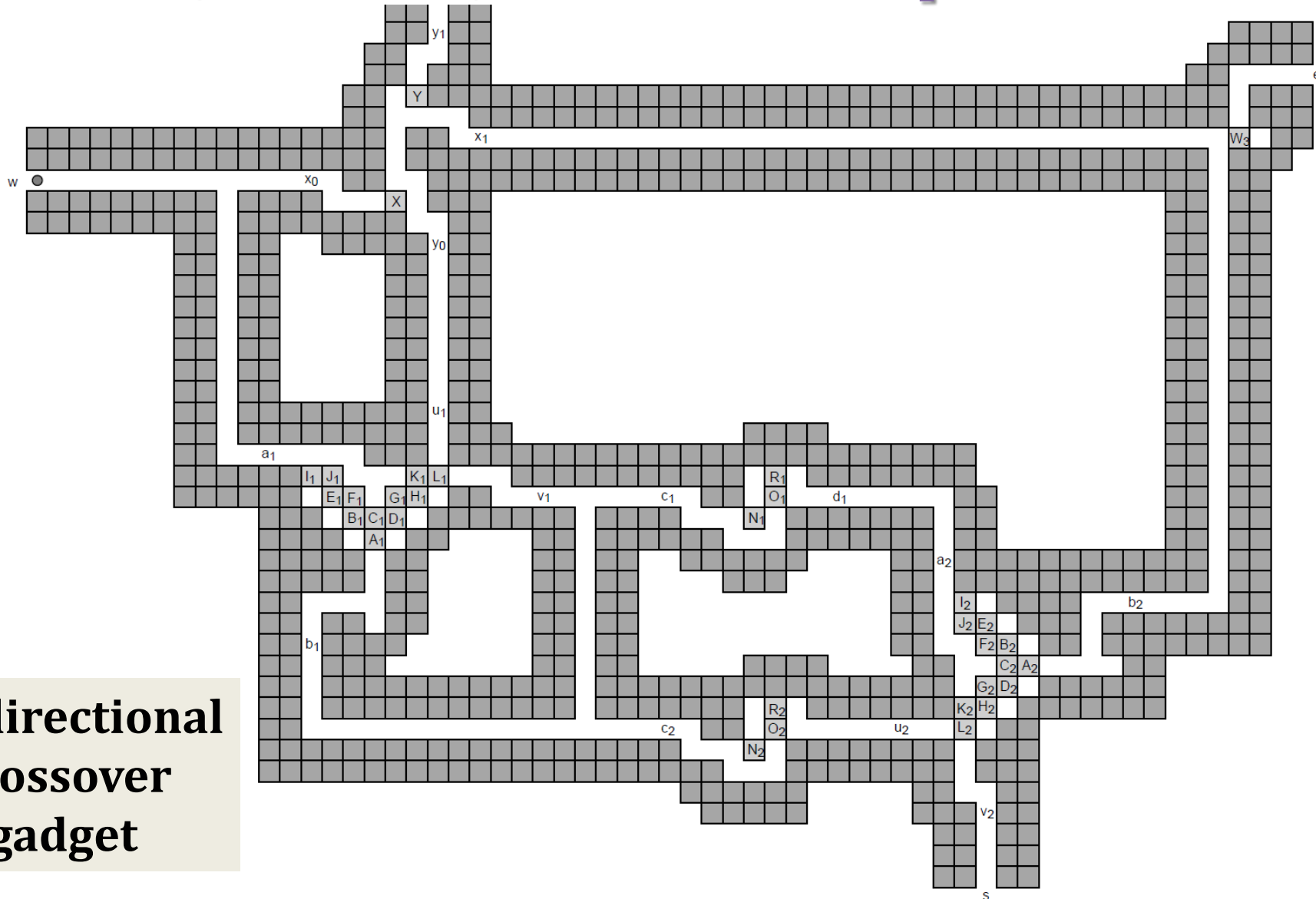
[Demaine, Demaine, O'Rourke 2000]



**NAND
crossover
gadget**

(Push)Push-1 is NP-hard in 2D

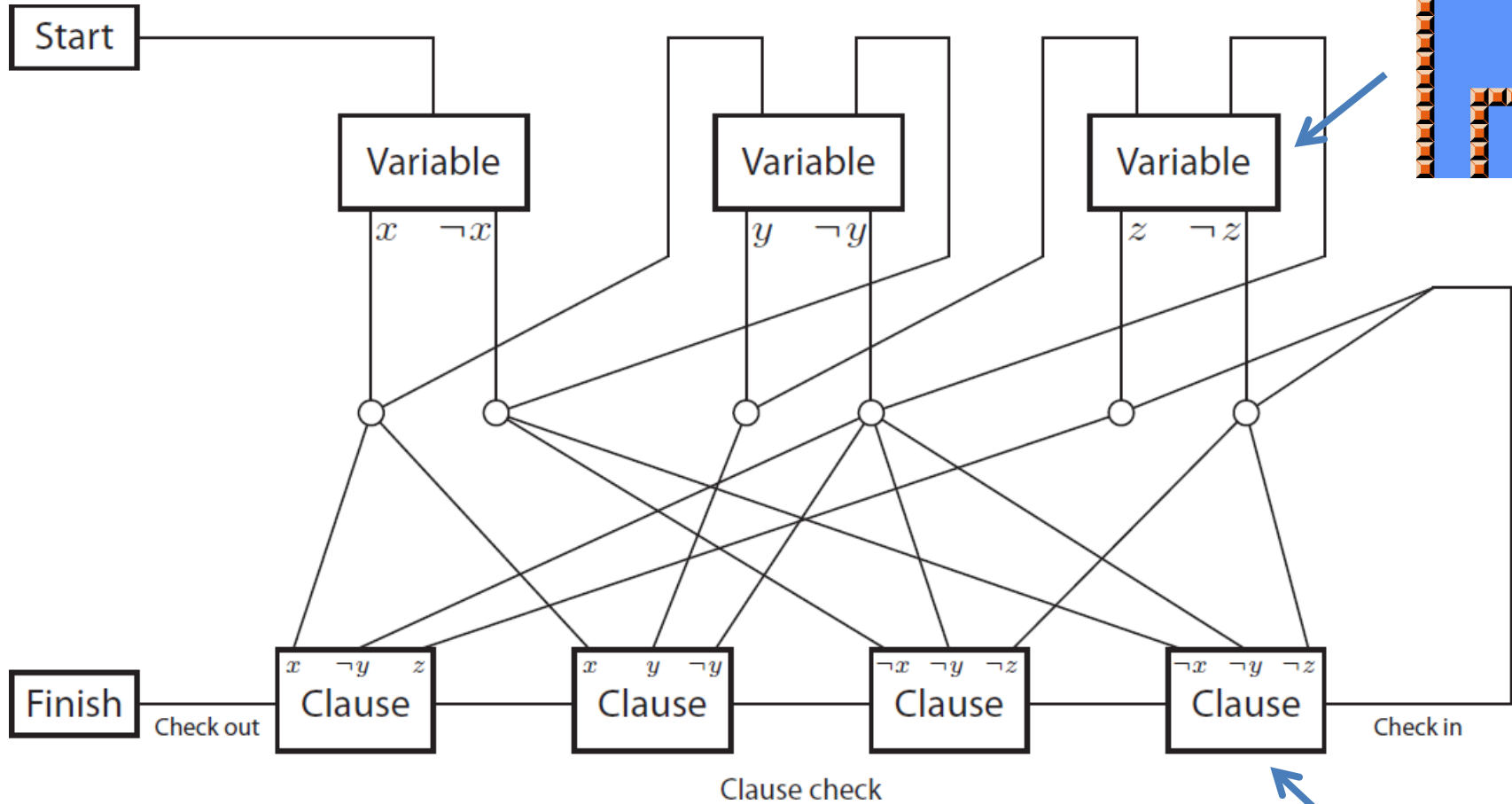
[Demaine, Demaine, O'Rourke 2000]



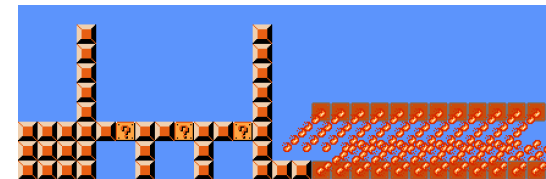
**unidirectional
crossover
gadget**

Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]



$$(x \text{ OR } \neg y \text{ OR } z) \& (x \text{ OR } y \text{ OR } \neg y) \& (\neg x \text{ OR } \neg y \text{ OR } \neg z) \& (\neg x \text{ OR } \neg y \text{ OR } \neg z)$$



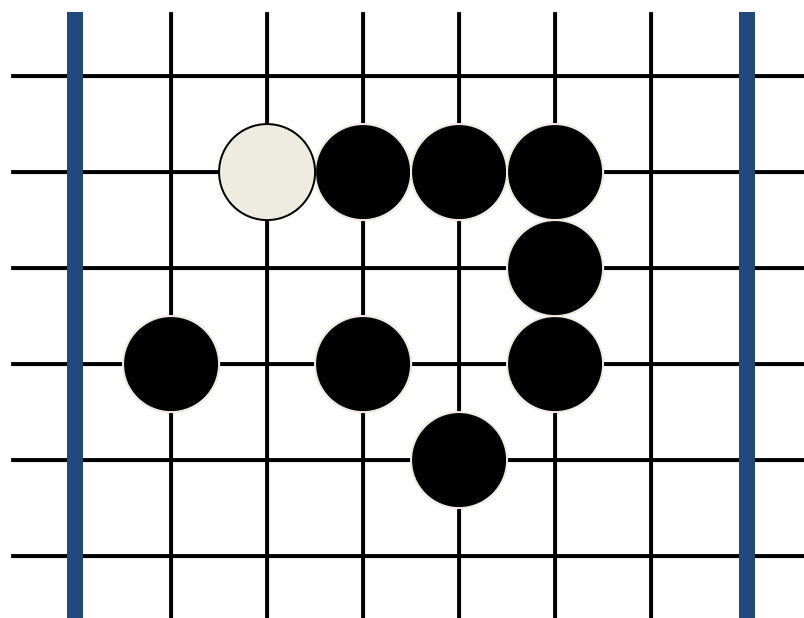
Conway's Phutball (Philosopher's Football)



photo by Rina Sergeeva, 2012

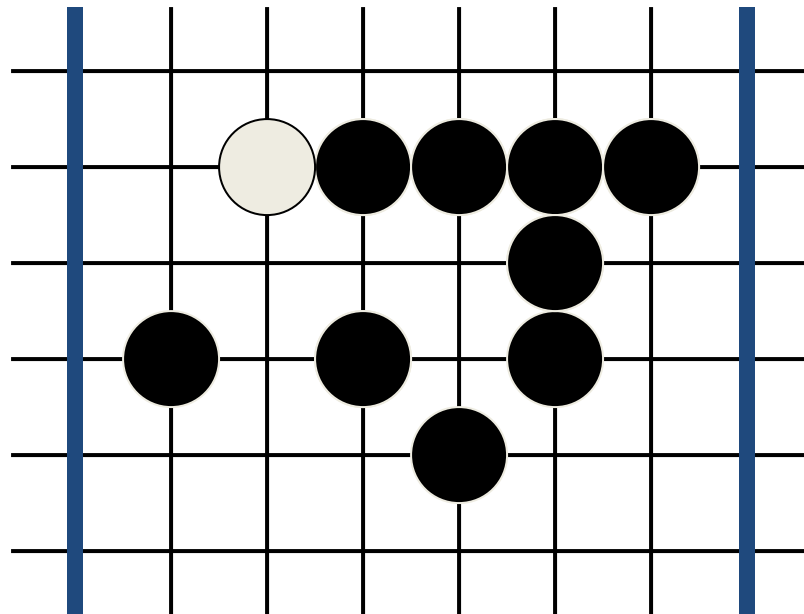


Conway's Phutball (Philosopher's Football)





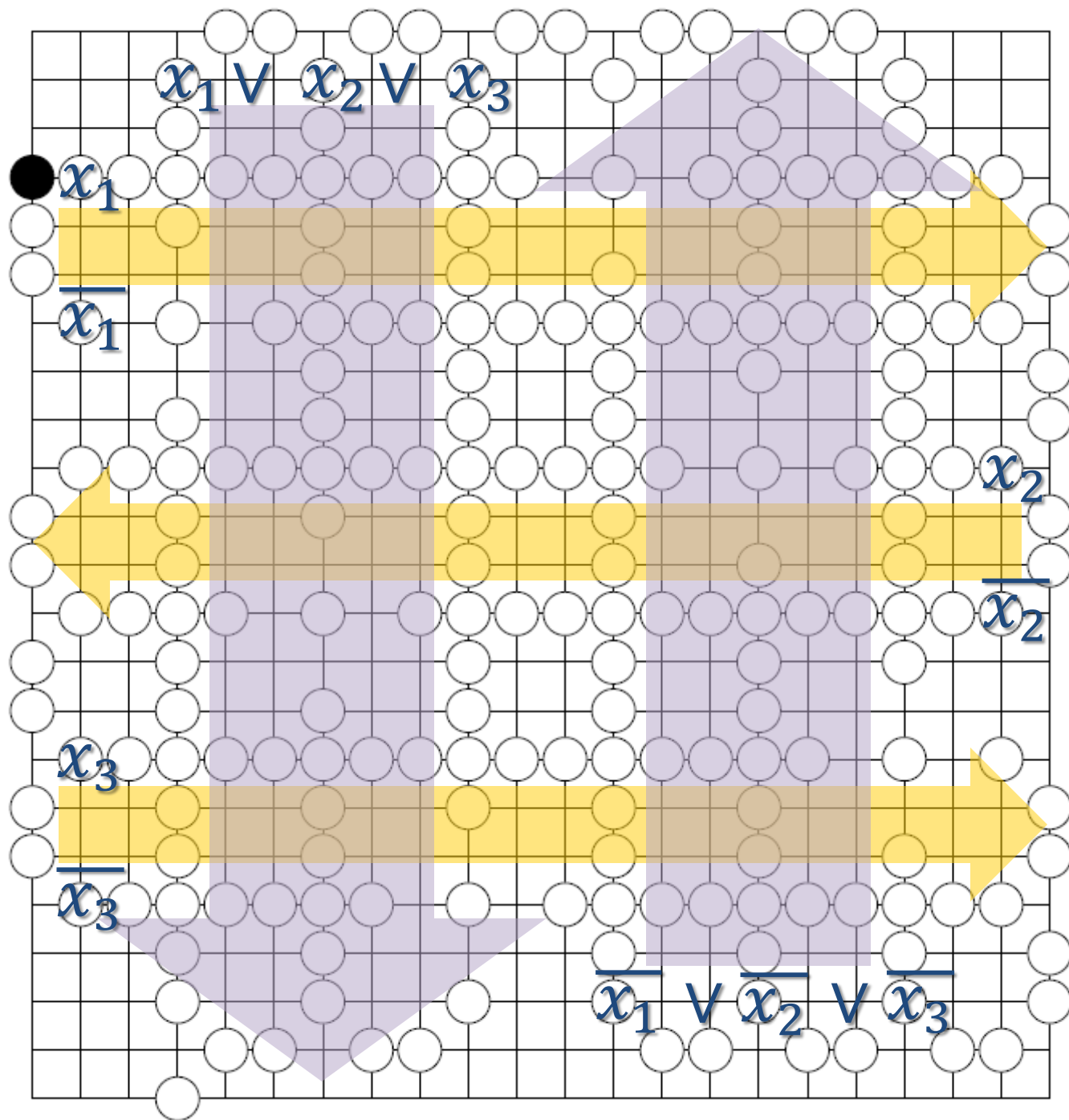
Conway's Phutball (Philosopher's Football)



Phutball

[Conway]

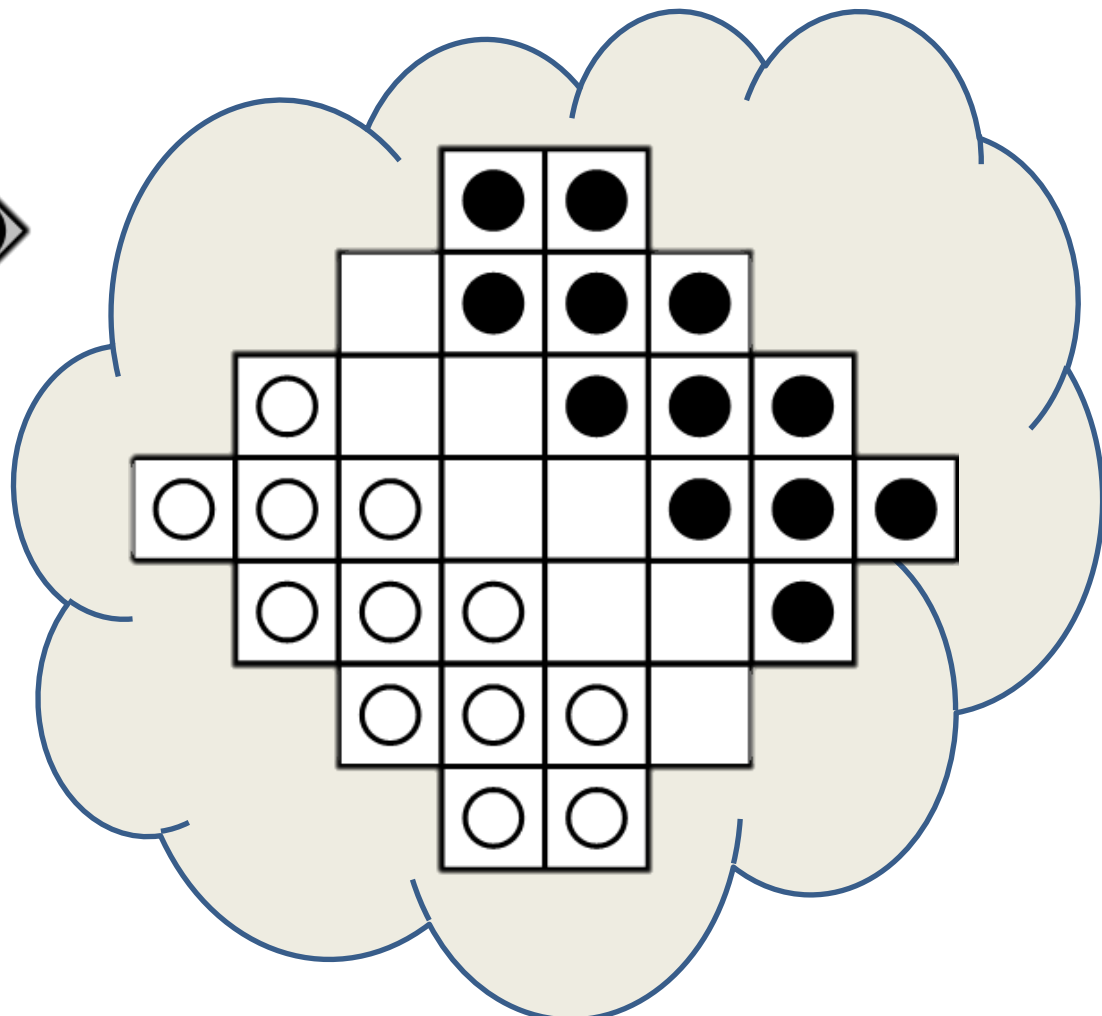
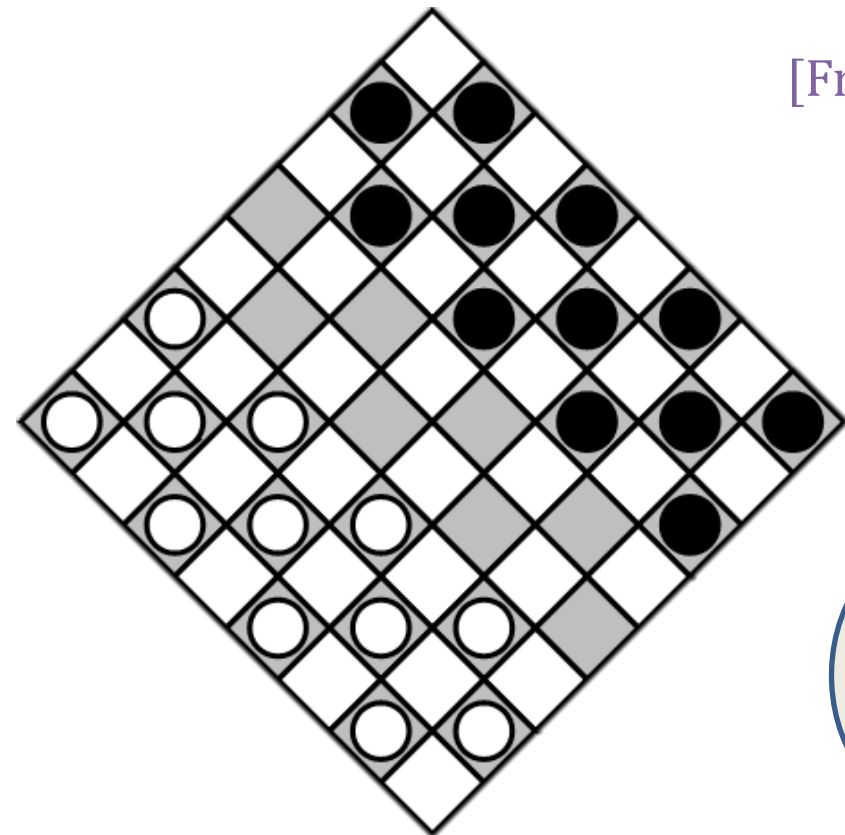
- Mate in 1 is NP-complete
[Demaine, Demaine, Eppstein 2000]
- PSPACE-hard
[Dereniowski 2009]
- EXPTIME-complete?





Checkers Mate-in-1 is Polynomial

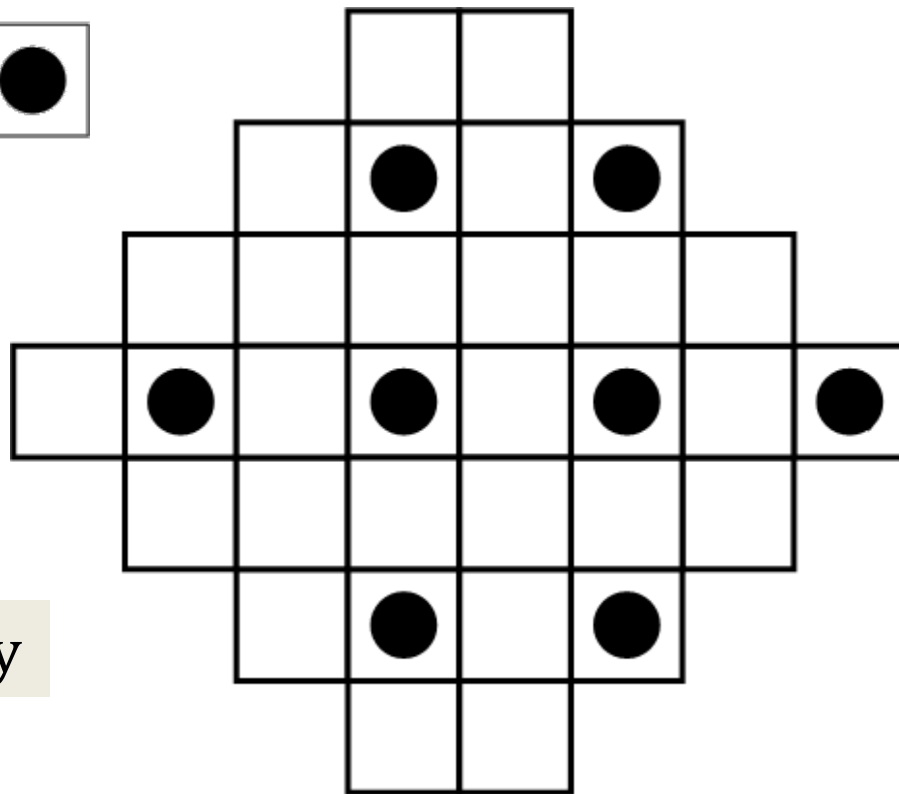
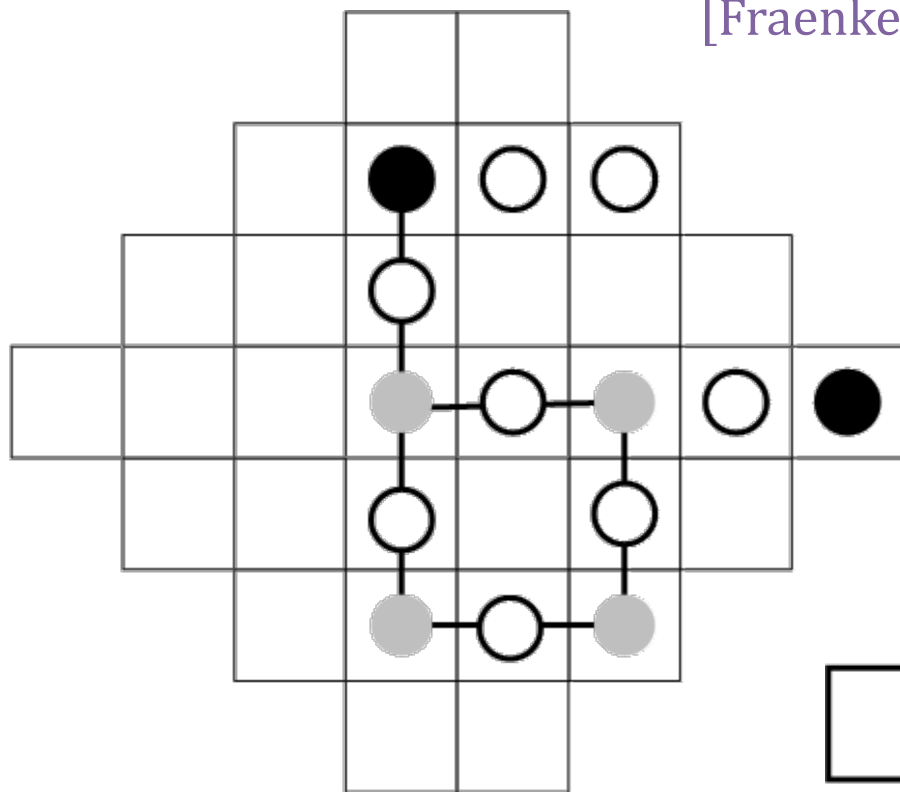
[Fraenkel, Garey, Johnson, Schaefer, Yesha 1978]





Checkers Mate-in-1 is Polynomial

[Fraenkel, Garey, Johnson, Schaefer, Yesha 1978]



jumps preserve x & y parity