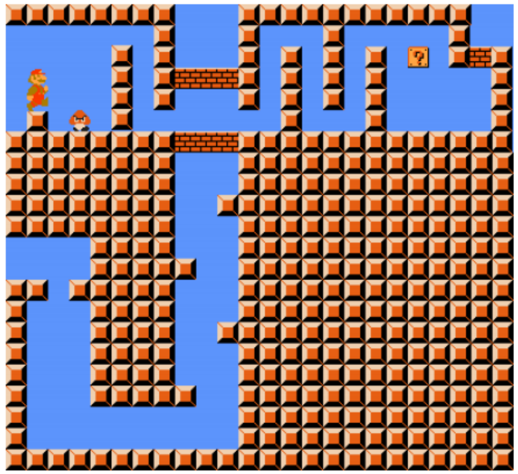
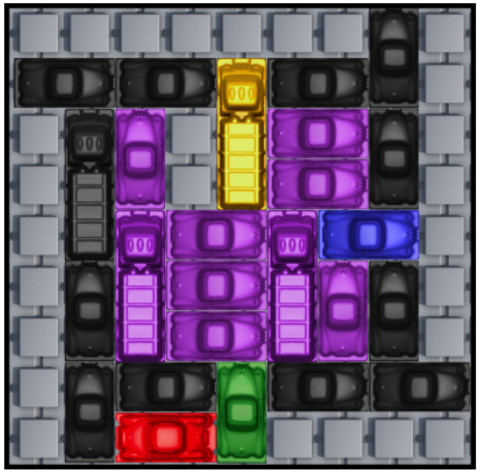
Algorithmic Lower Bounds: Fun with Hardness Proofs

Super Mario Bros.



Crossover gadget for NP-hardness

Rush Hour



AND gadget for PSPACE-hardness

Minesweeper

OR gadget for NP-hardness

Hardness Made Easy*

Learn **when to give up** the search for efficient algorithms; see **connections** between computational problems; **solve puzzles** to prove theorems, solve **open problems,** and write papers.

Topics: NP, PSPACE, EXPTIME, EXPTIME, EXPSPACE, 3SUM, approximation, fixed parameter, games & puzzles, 3SUM, key problems, gadgets, and proof styles.

Fall 2014

6.890 taught by Professor Erik Demaine
Grad H, AUS, and Theoretical CS Concentration
Tuesday & Thursday 3:30-5:00pm in room 2-105

http://courses.csail.mit.edu/6.890/
sign up for our mailing list to join the class

** Easiness not guaranteed. Side effects such as open problems and a heightened sense of complexity may occur. Ask your advisor if 6.890 is right for you!*

6.890	Lecture 1	Sept. 4, 2014

6.890: Algorithmic Lower Bounds / Fun with Hardness Proofs — "Hardness made Easy"

Prof. Erik Demaine

TAs: Sarah Eisenstat & Jayson Lynch

http://courses.csail.mit.edu/6.890/fall14/

What is this class?

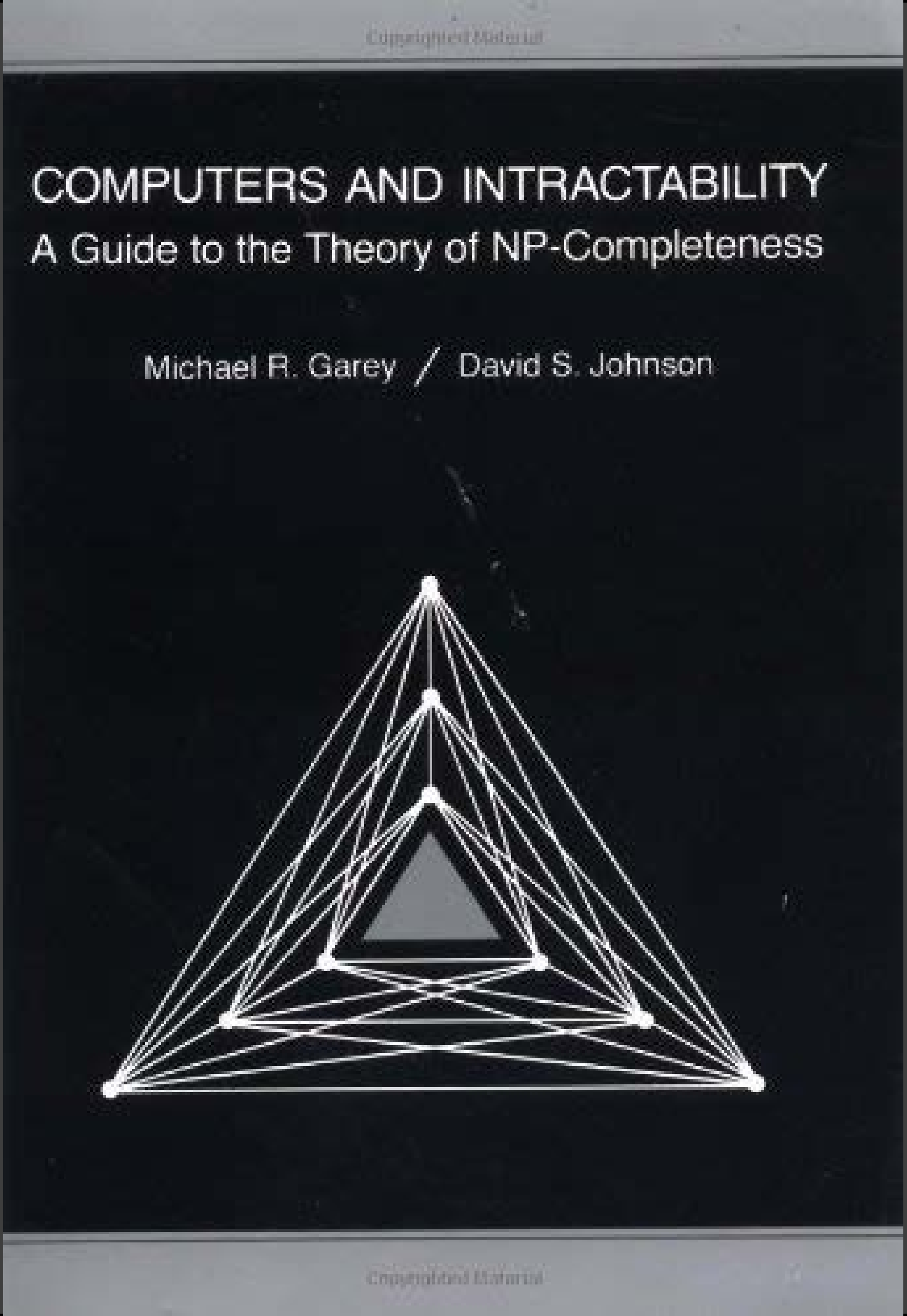
- practical guide to proving computational problems are formally hard / intractable
- NOT a complexity course (but we will use/refer to needed results)
- (anti)algorithmic perspective

Why take this class?

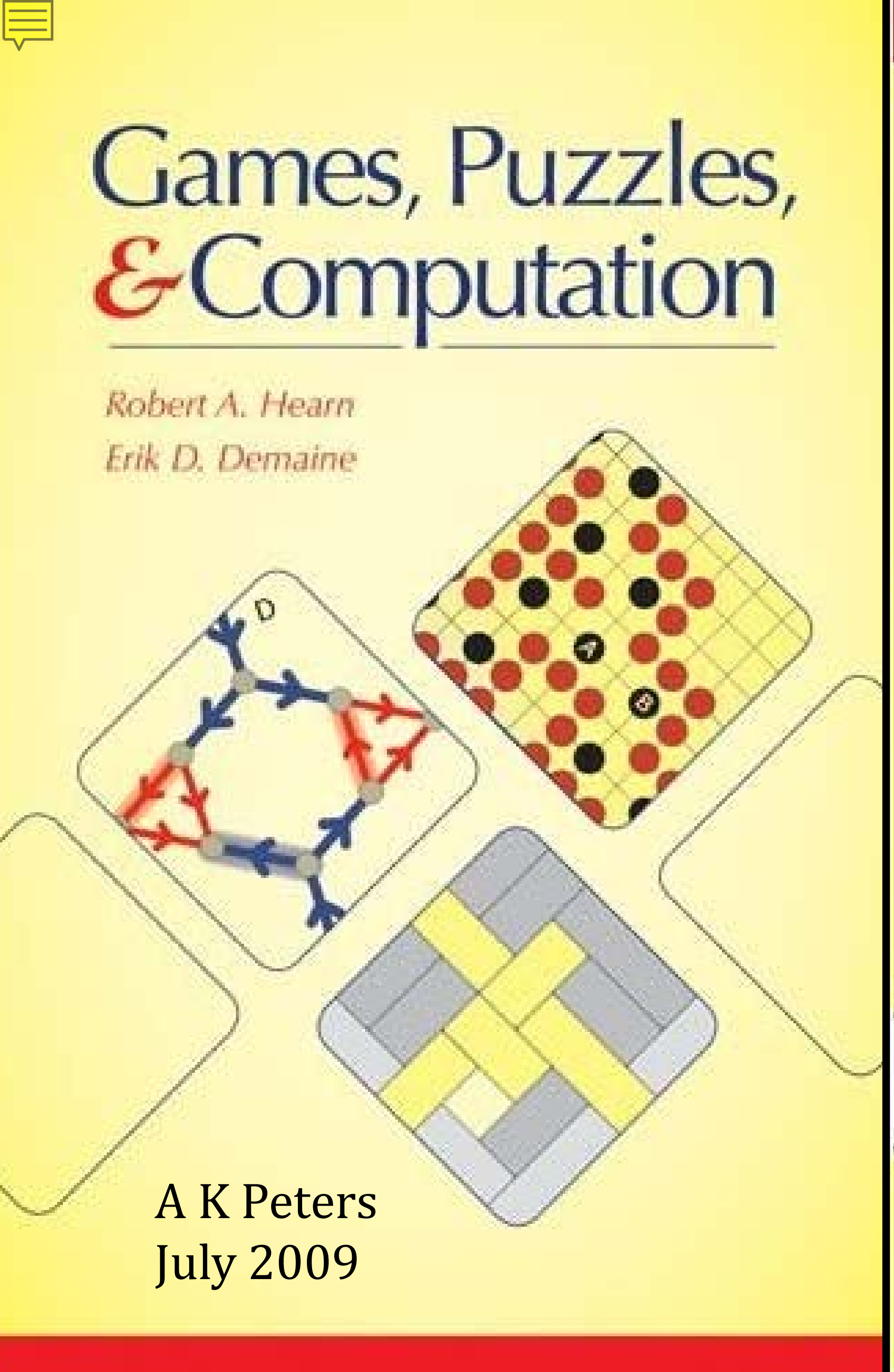
- know your limits in algorithmic design
- master techniques for proving hardness — key problems, proof styles, gadgets
- cool connections between problems
- fun problems like Mario & Tetris (serious problems too)
- solve puzzles → publishable papers

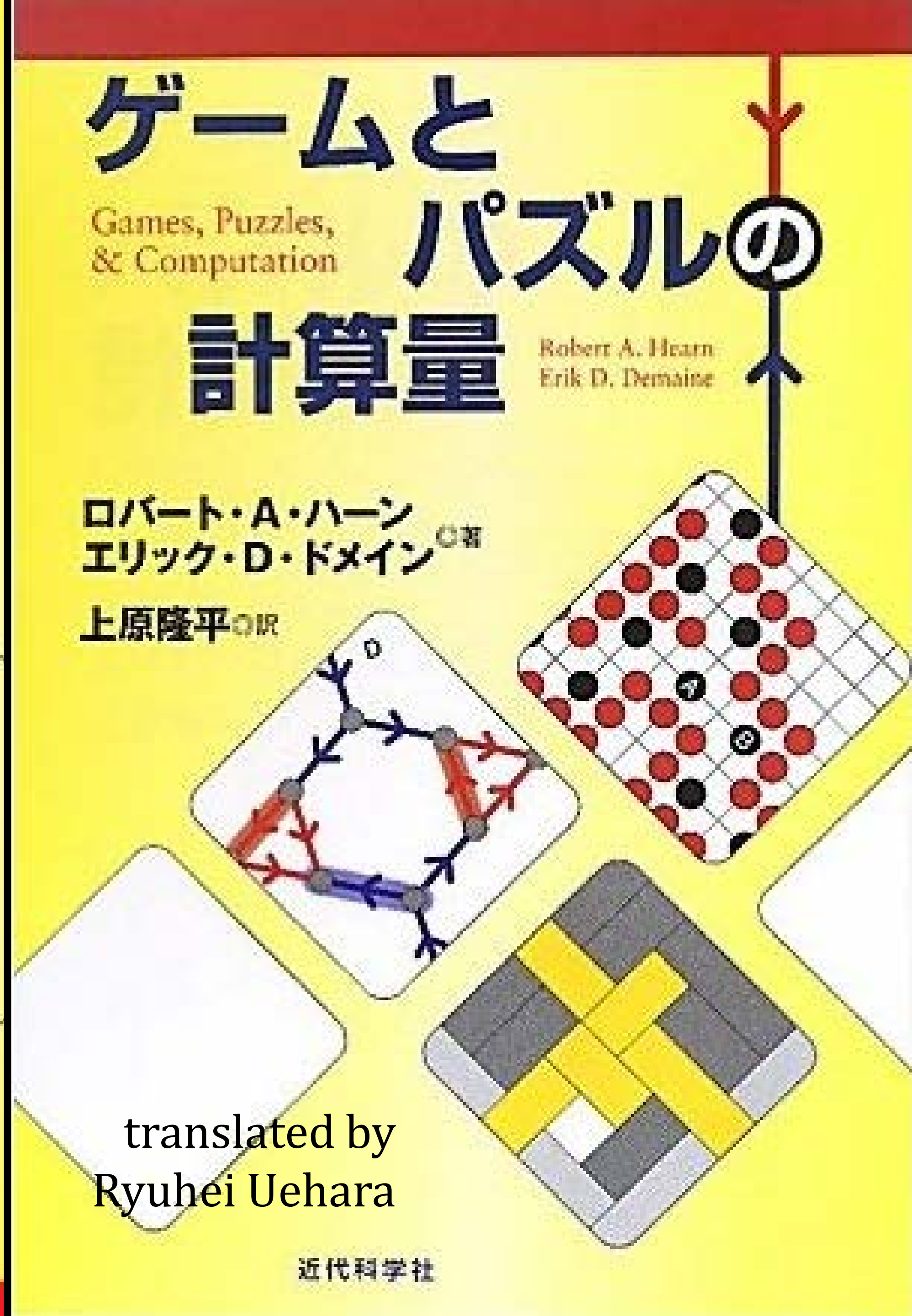
Background: algorithms, asymptotics, combinatorics

- no complexity background needed (but also little overlap with a complexity class)



W. H. Freeman
1979


Games, Puzzles, & Computation
Robert A. Hearn
Erik D. Demaine
D
A
B
A K Peters
July 2009


ゲームとパズルの計算量
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Erik D. Demaine
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エリック・D・ドメイン◎著
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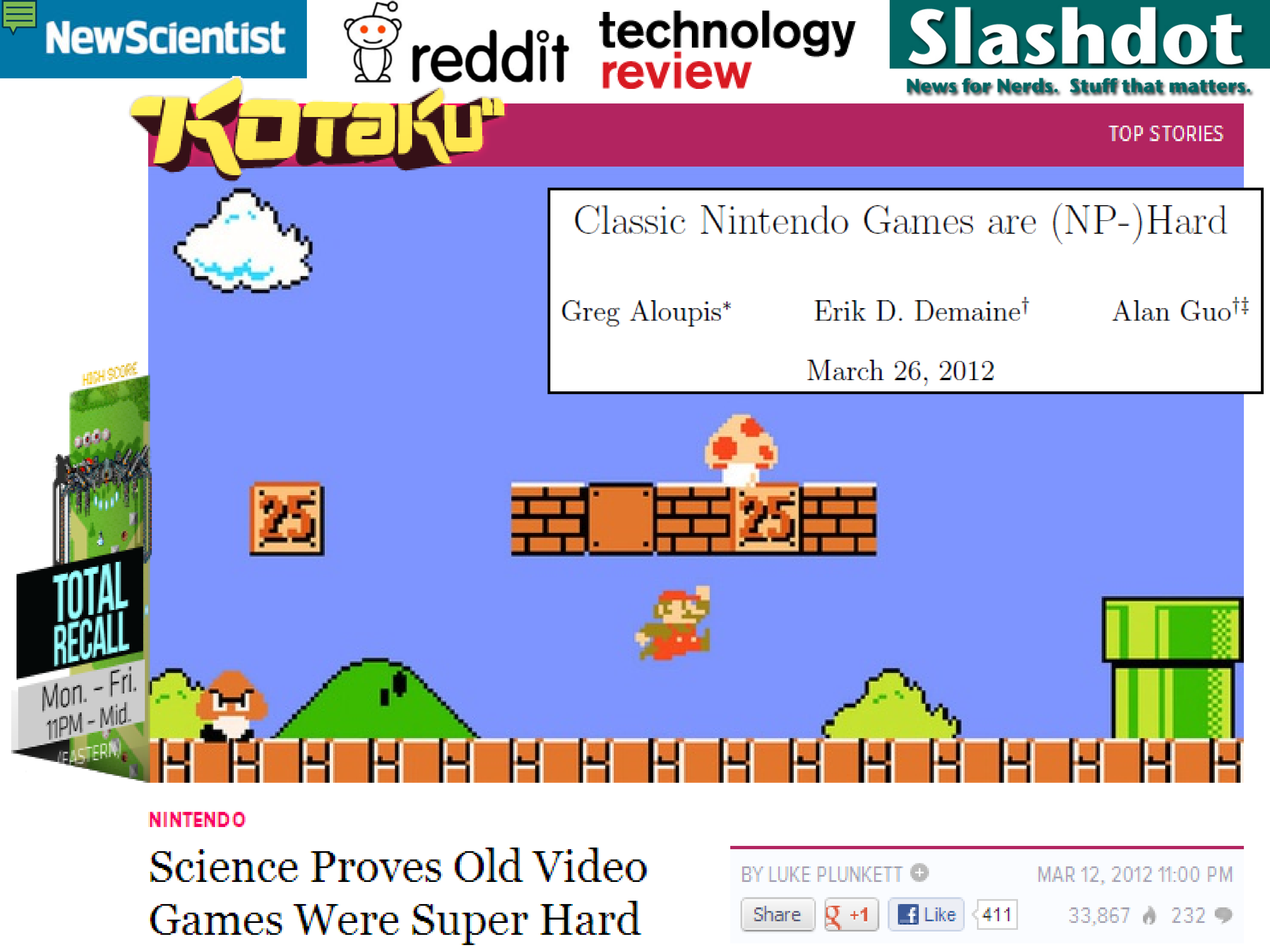
Ryuhei Uehara
近代科学社


NewScientist
reddit
technology review
Slashdot
News for Nerds. Stuff that matters.
Kotaku
TOP STORIES
Classic Nintendo Games are (NP-)Hard
Greg Aloupis*
Erik D. Demaine†
Alan Guo†‡
March 26, 2012
25
25
HIGH SCORE
TOTAL RECALL
Mon. - Fri.
11PM - Mid.
NINTENDO
Science Proves Old Video Games Were Super Hard
BY LUKE PLUNKETT
MAR 12, 2012 11:00 PM
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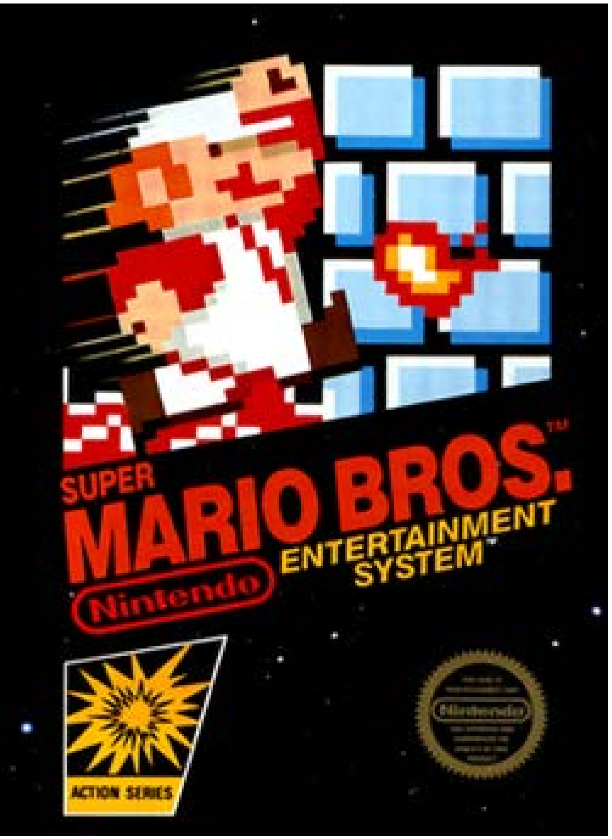
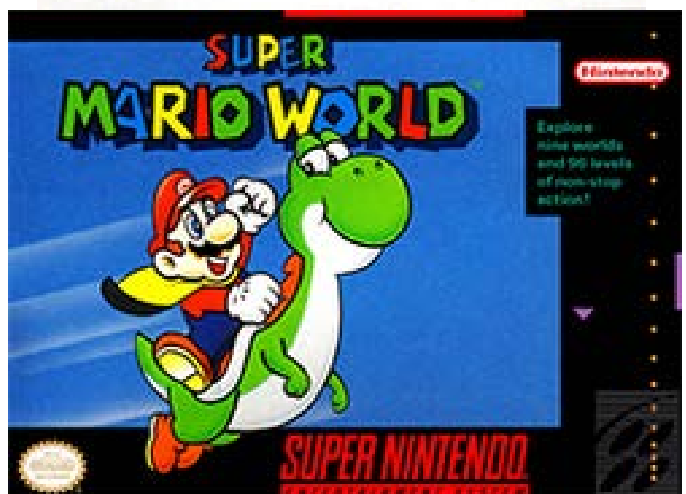
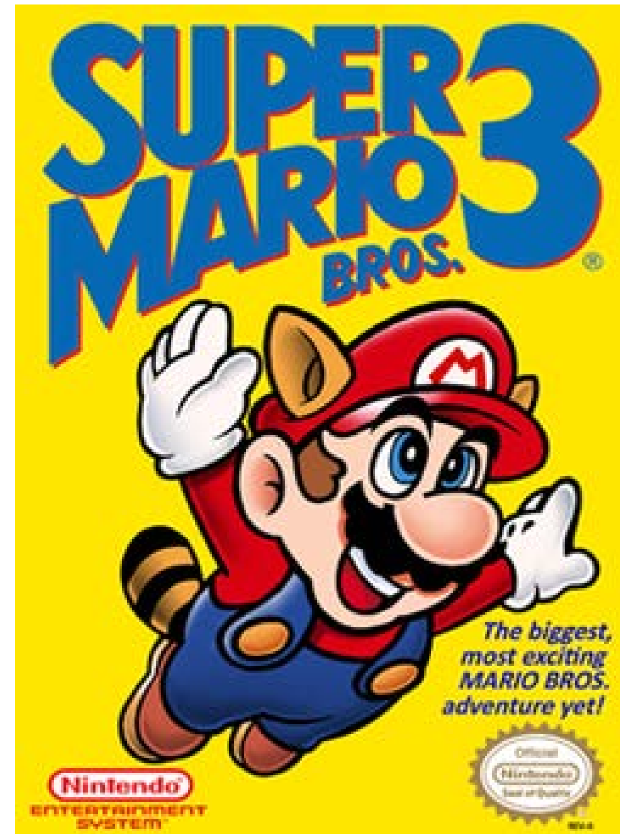
Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo 2012]



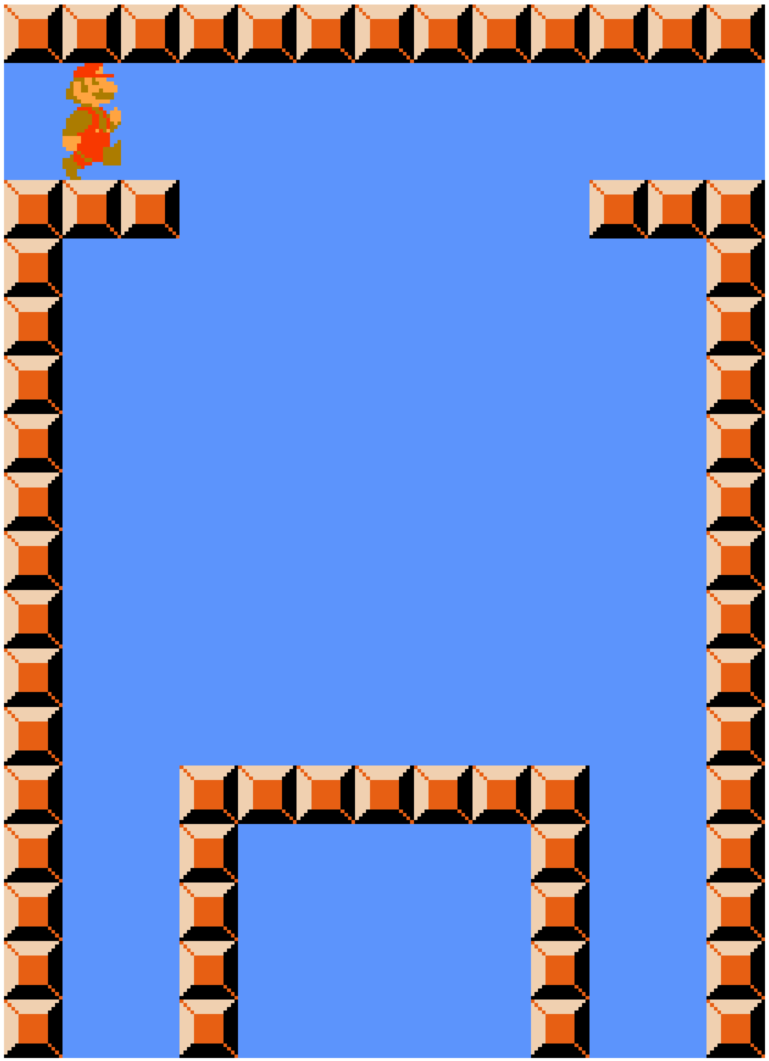
The Lost Levels





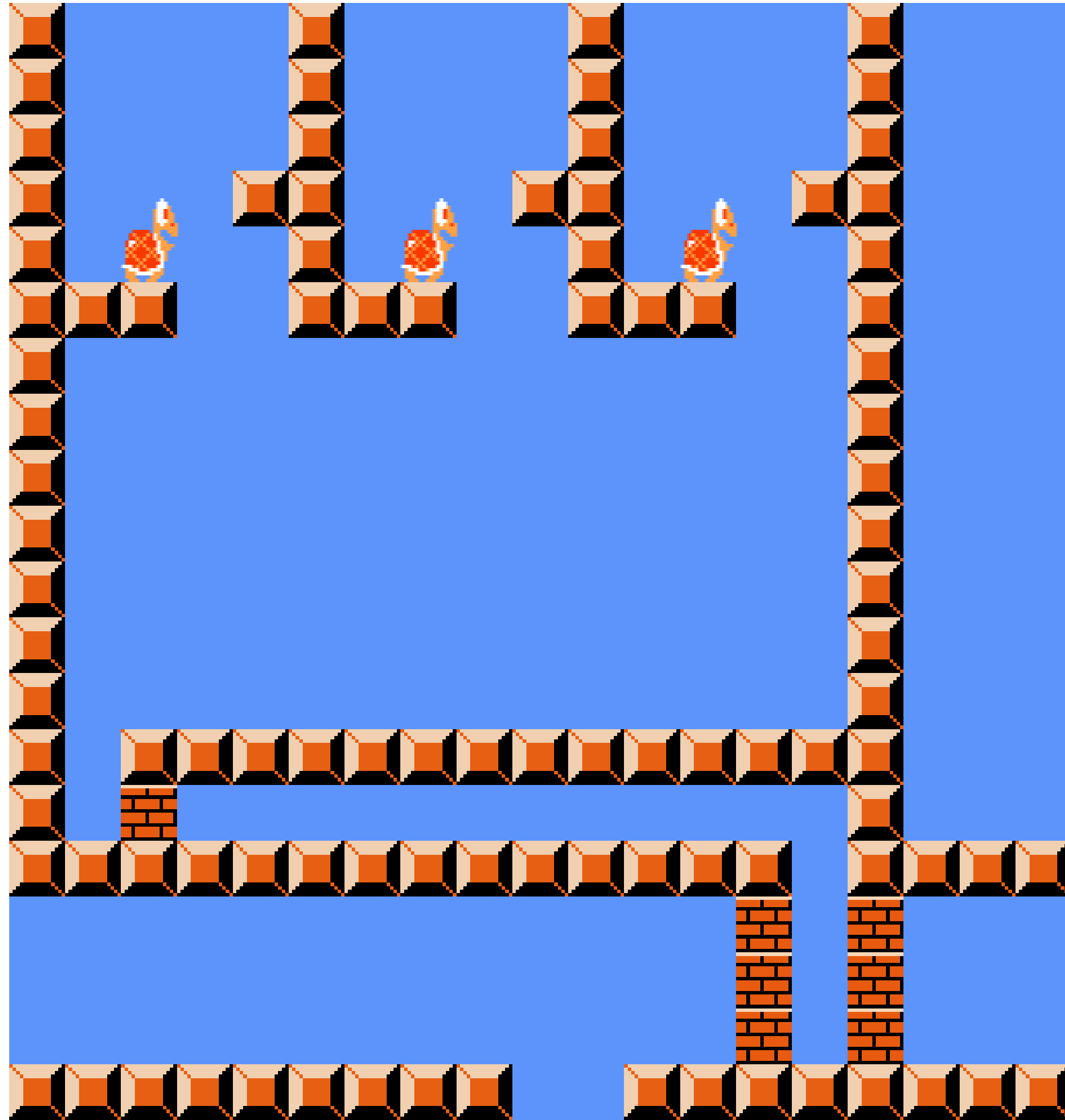
Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo 2012]



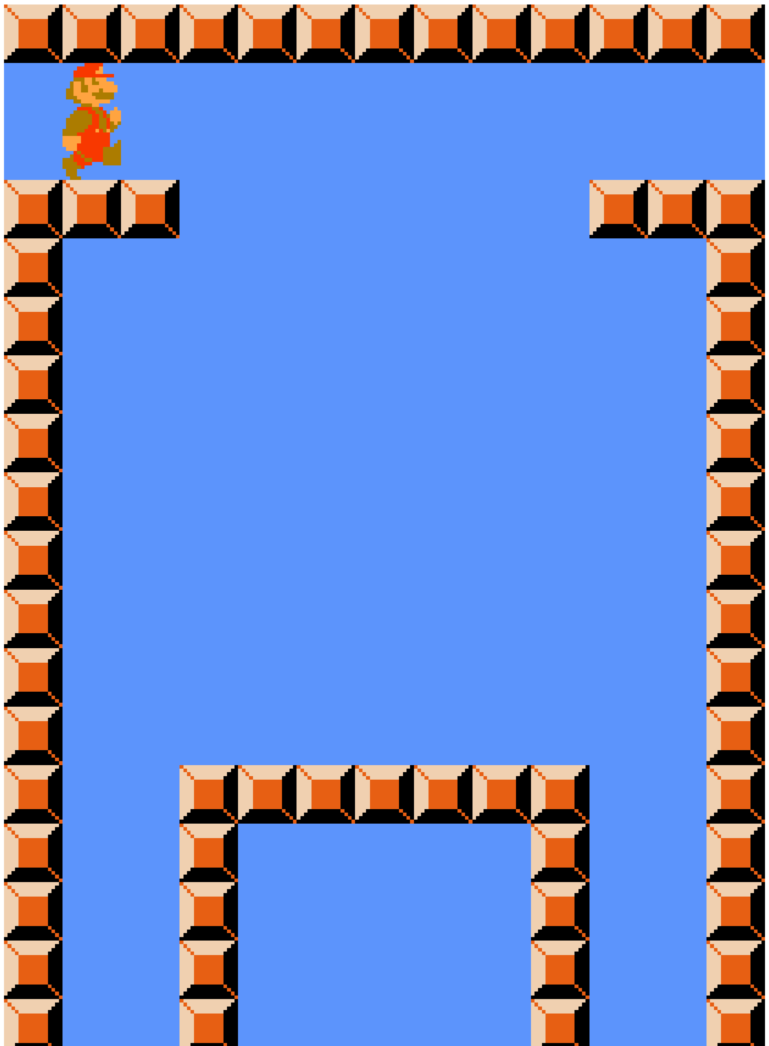
variable

clause



Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo 2012]



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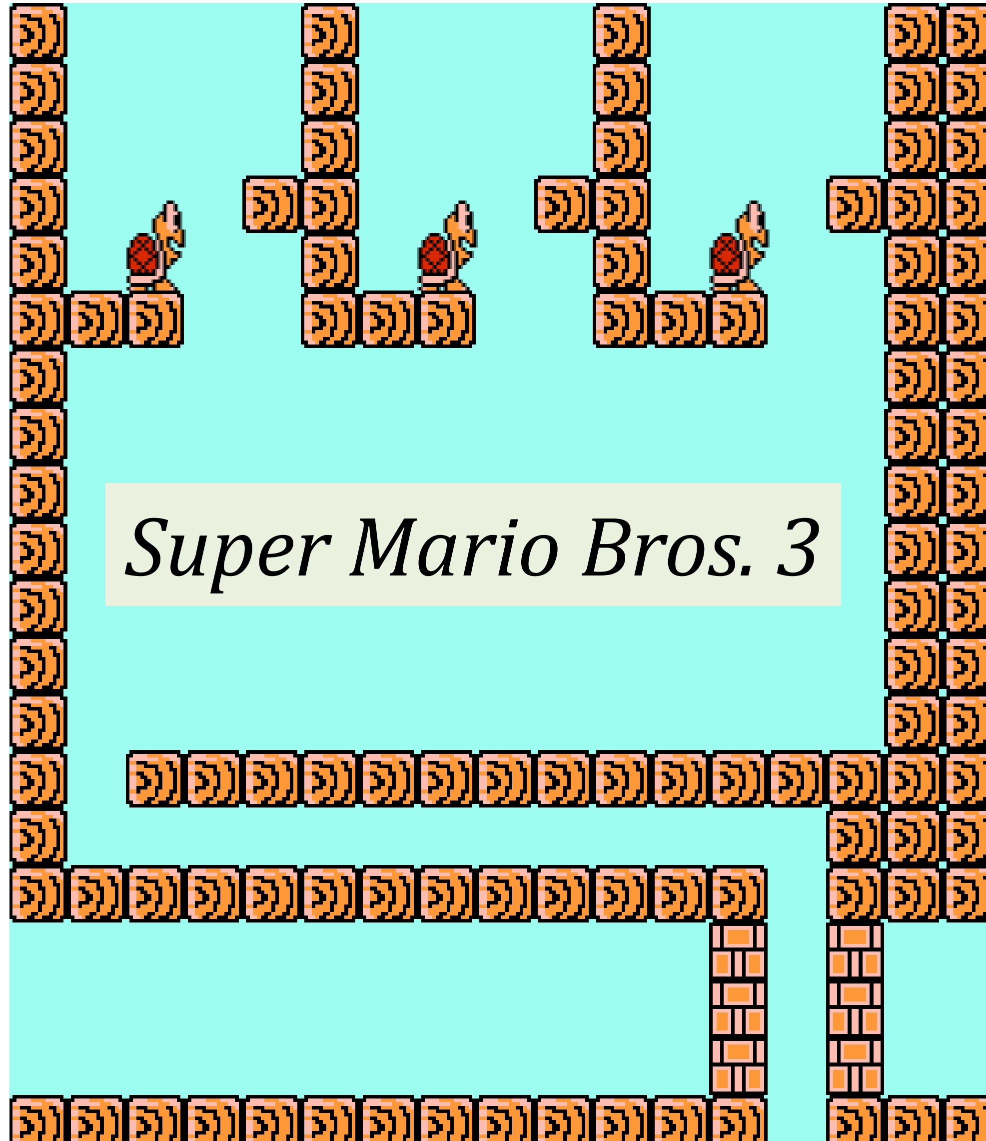
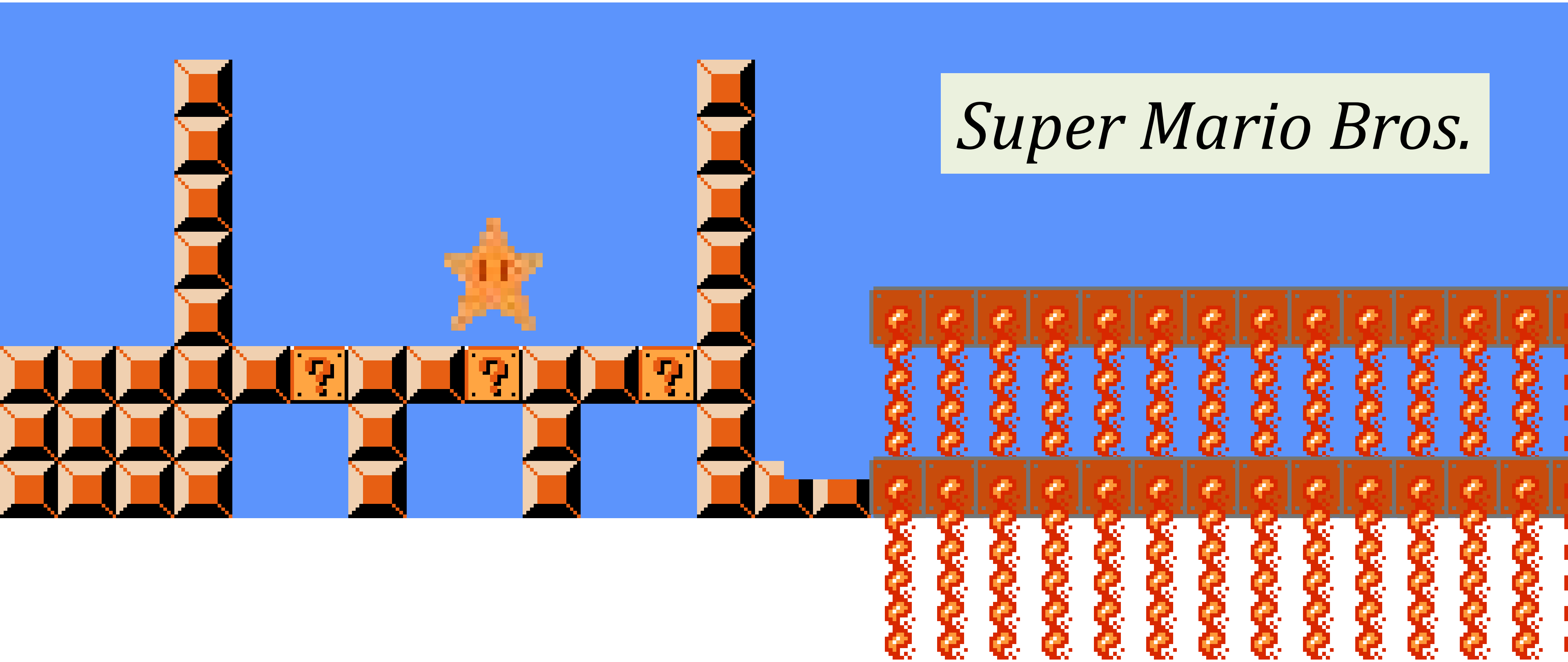


Super Mario Bros. is NP-Hard

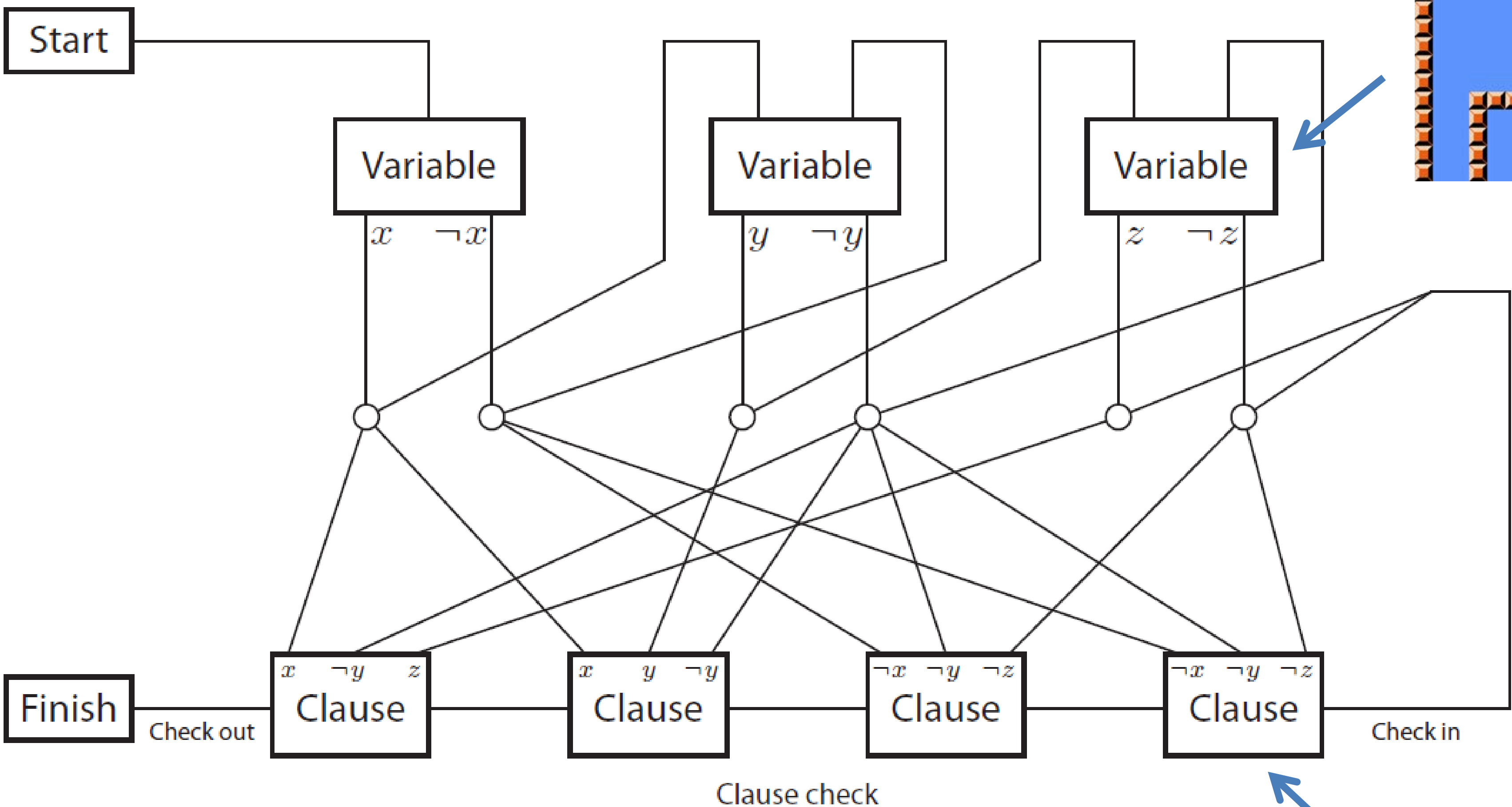
[Aloupis, Demaine, Guo, Viglietta 2014]

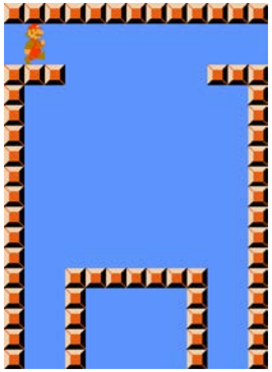
clause



Super Mario Bros. is NP-Hard

[Aloupis, Demaine, Guo, Viglietta 2014]

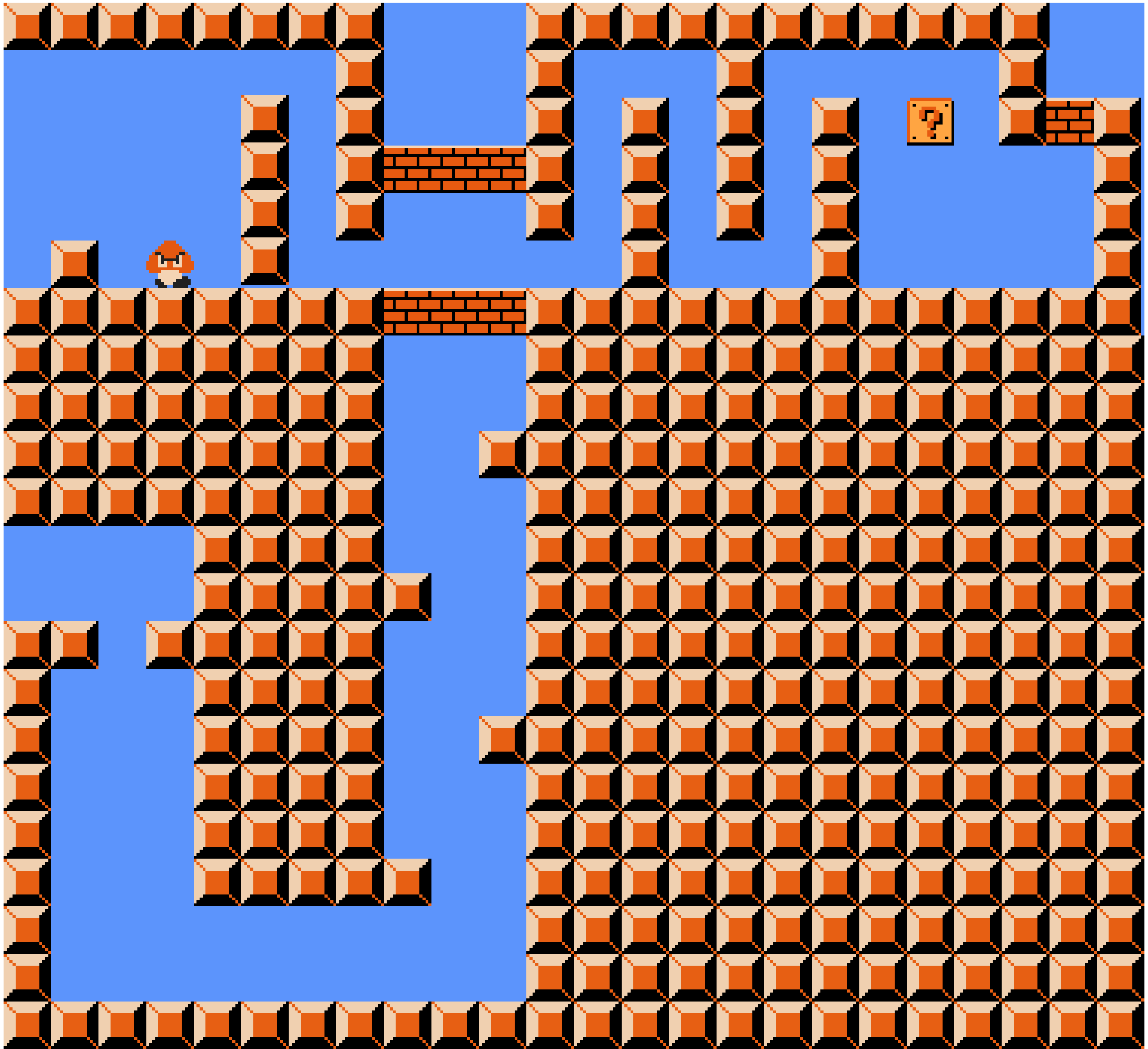




$(x \text{ OR } \neg y \text{ OR } z)\ \&\ (x \text{ OR } y \text{ OR } \neg y)\ \&$
$(\neg x \text{ OR } \neg y \text{ OR } \neg z)\ \&\ (\neg x \text{ OR } \neg y \text{ OR } \neg z)$

Super Mario Bros. is NP-Hard

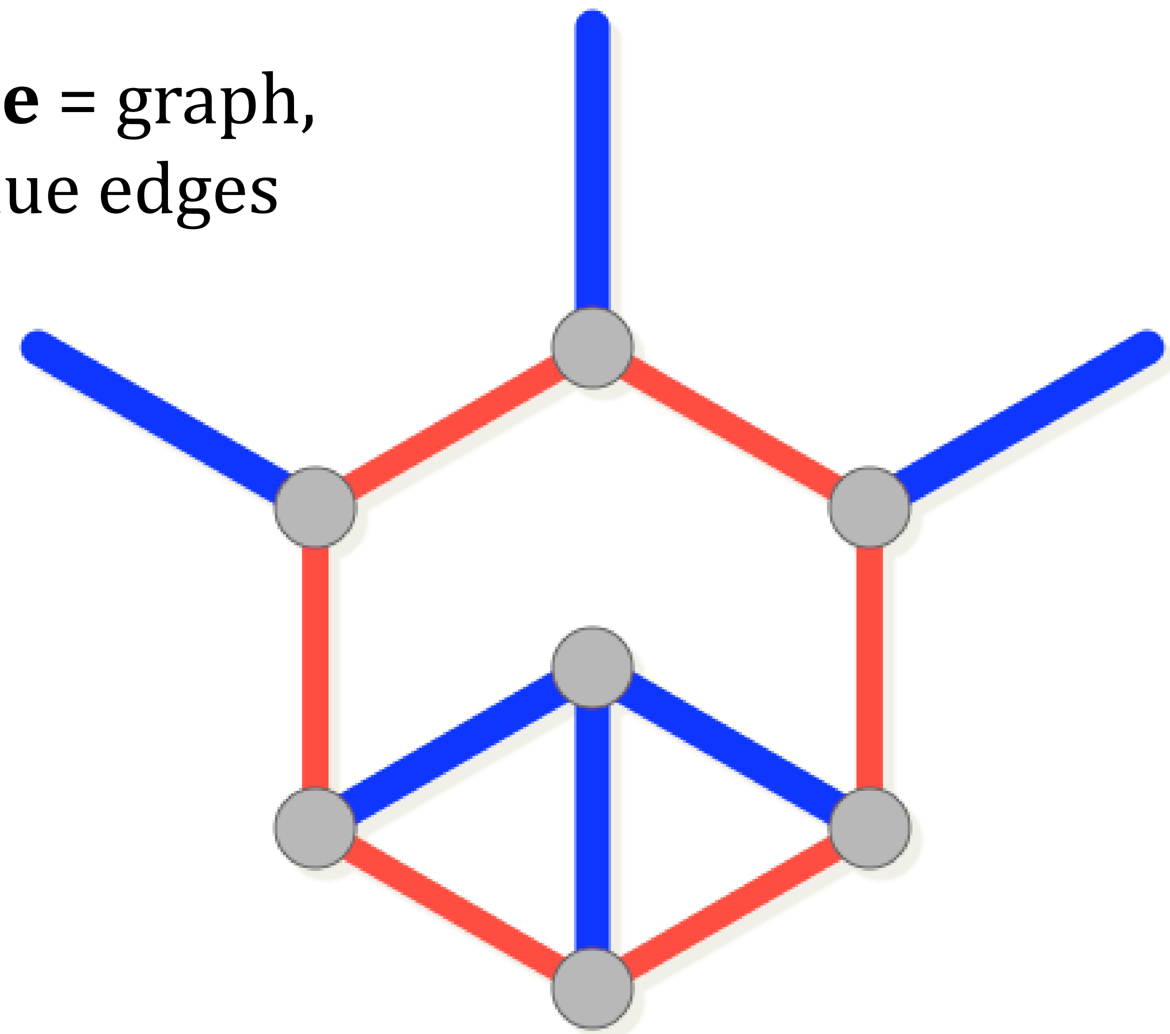
[Aloupis, Demaine, Guo, Viglietta 2014]



crossover

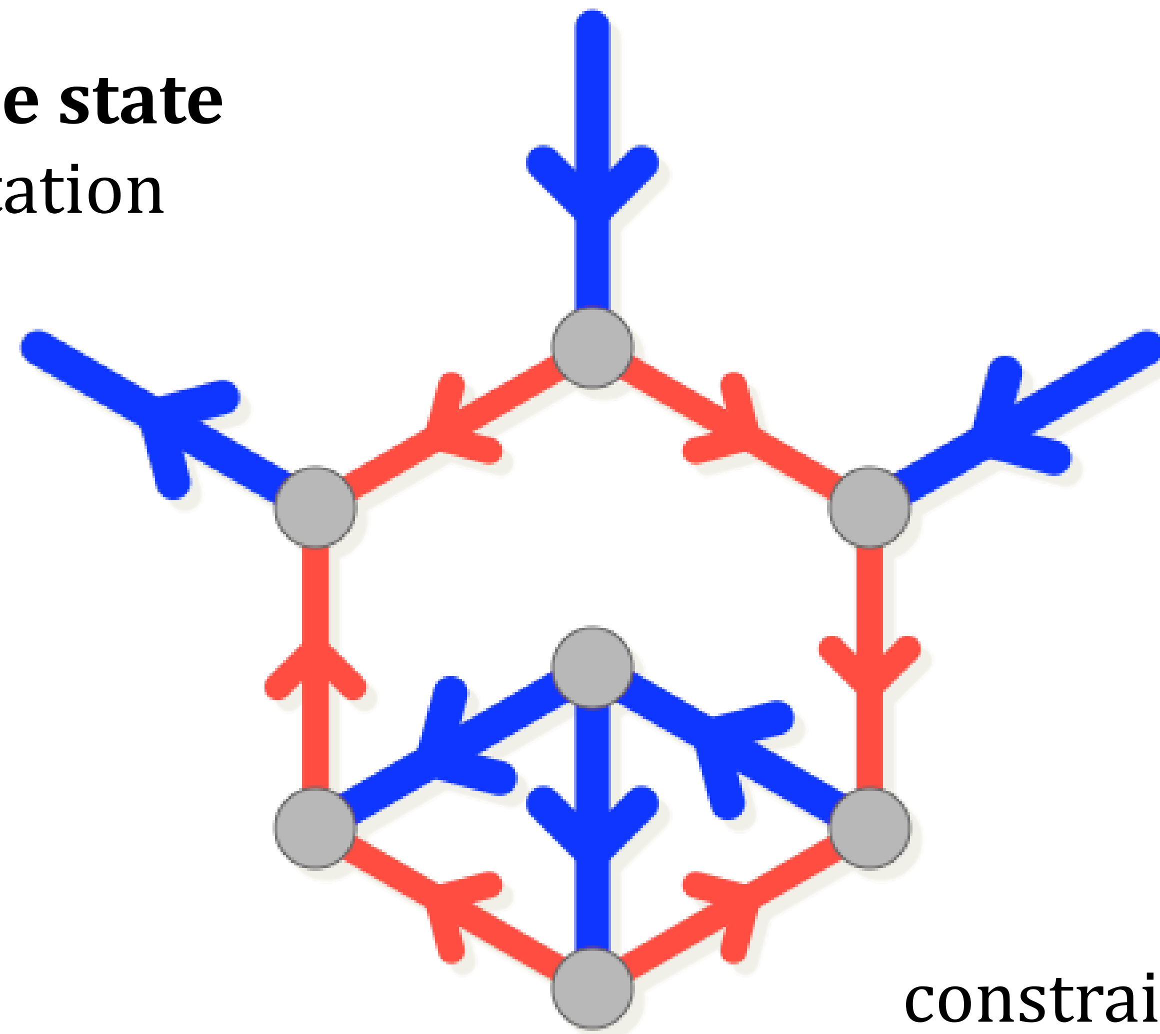
Constraint Graphs

Machine = graph,
red & blue edges



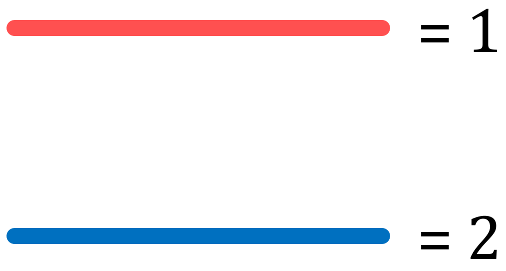
Constraint Graphs

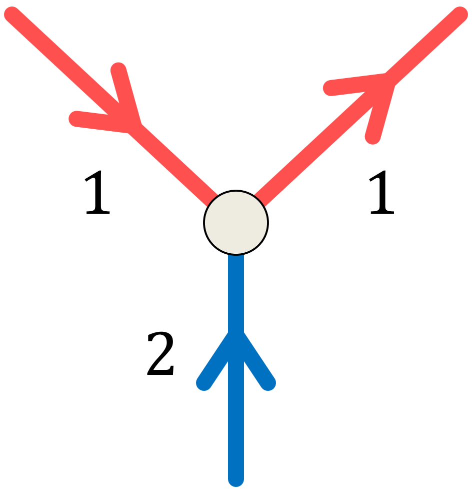
Machine state
= orientation



constraint graph

Constraint Logic

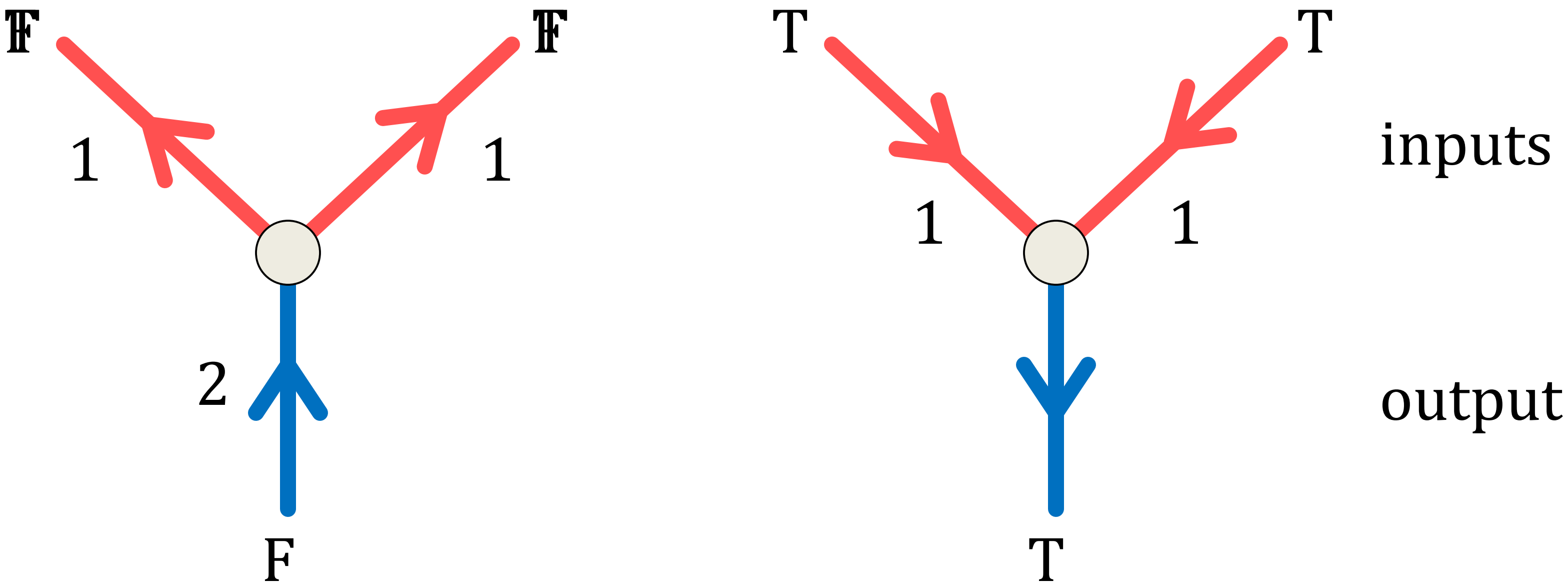




Rule: at least 2 units incoming at a vertex

Move: reverse an edge, preserving Rule

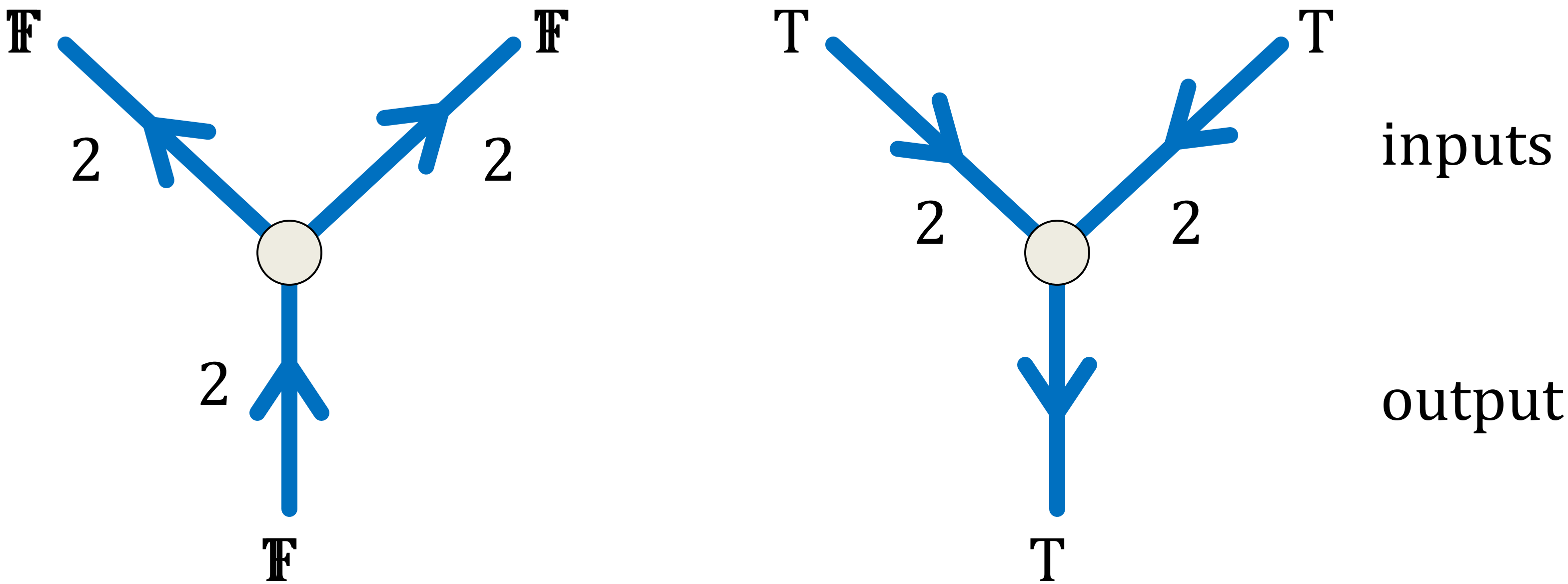
AND vertex



not your usual AND gate!

Rule: at least 2 units incoming at a vertex

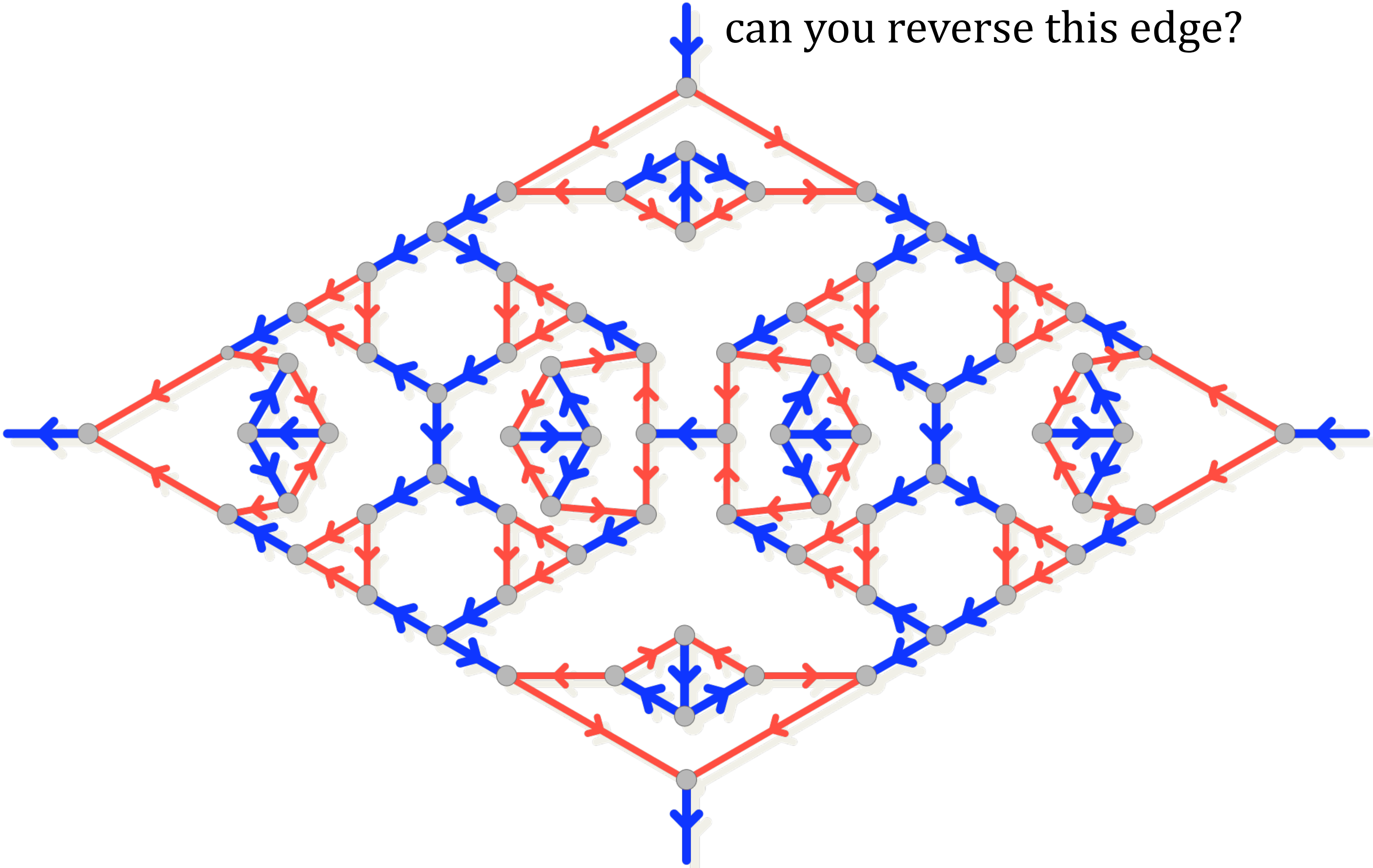
OR vertex



not your usual
OR gate!

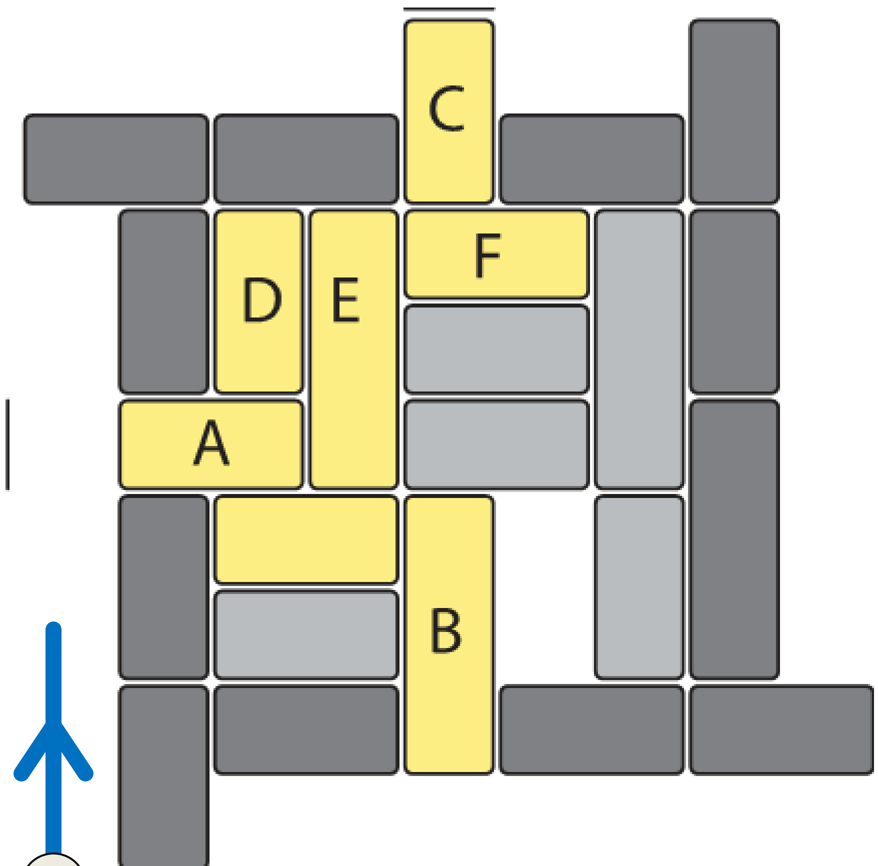
Rule: at least 2 units
incoming at a vertex

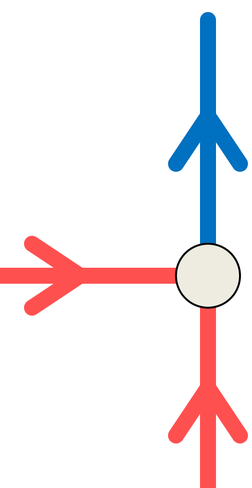
Decision Problem



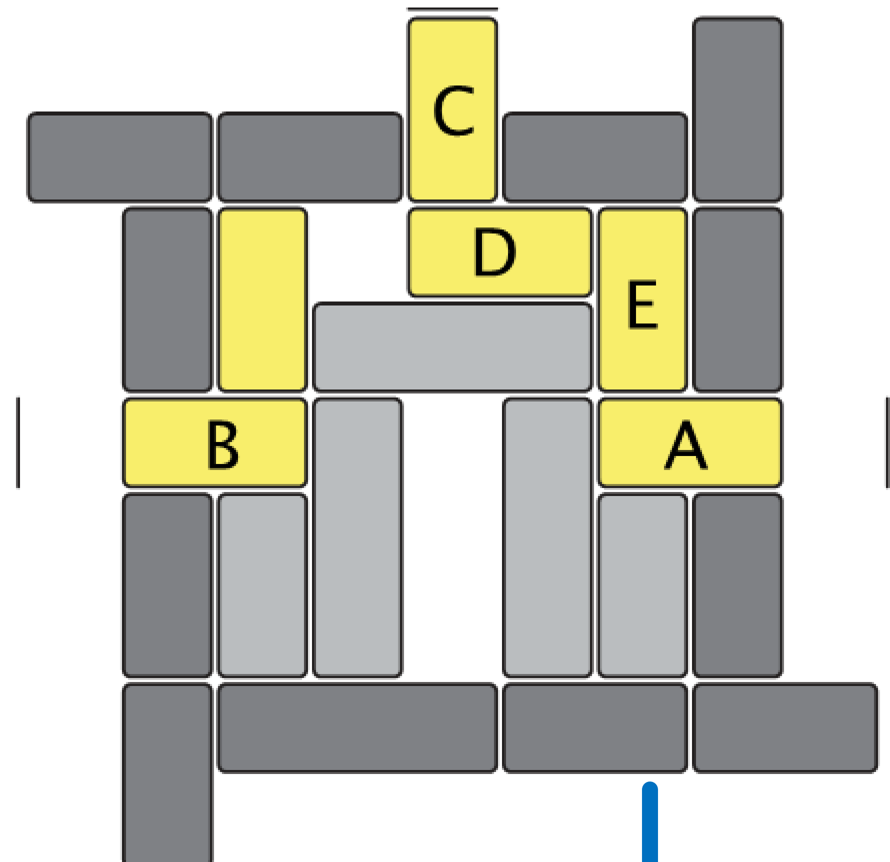
Rush Hour is PSPACE-complete

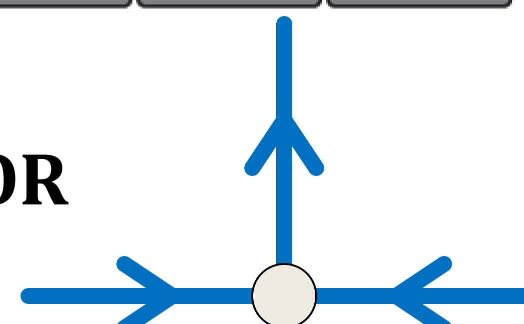
[Flake & Baum 2002; Hearn & Demaine 2002]





AND

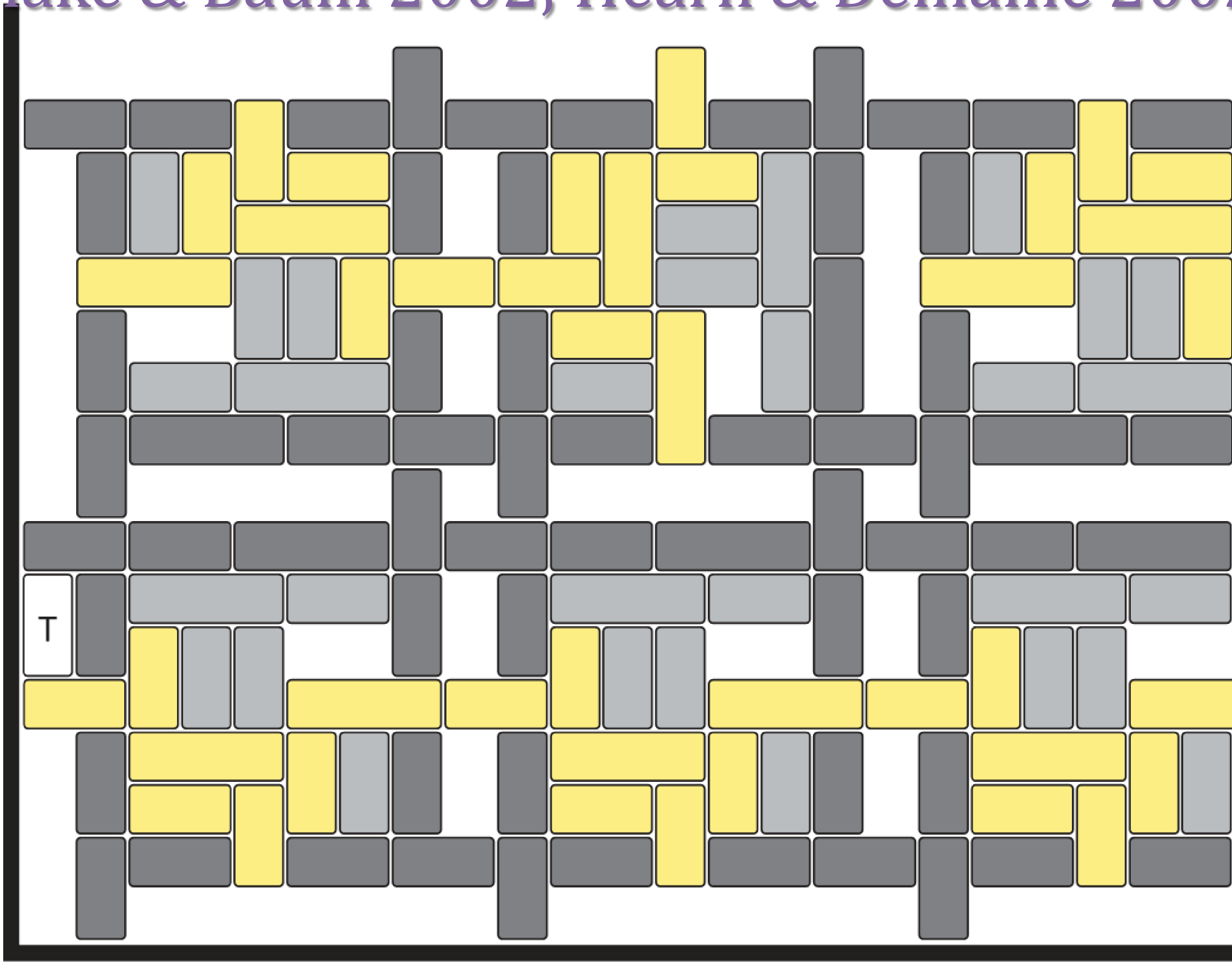




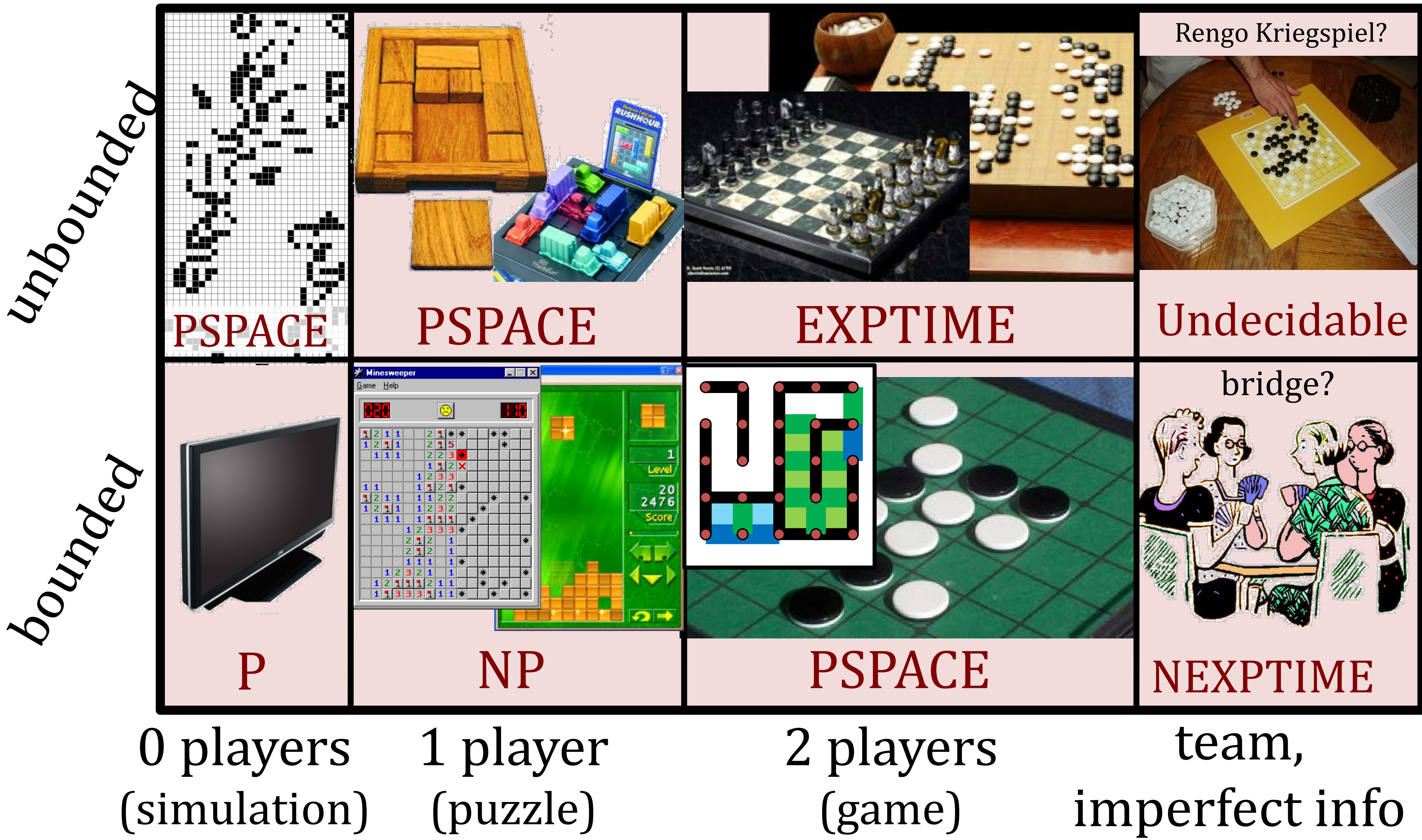
Protected **OR**

Rush Hour is PSPACE-complete

[Flake & Baum 2002; Hearn & Demaine 2002]



Complexity of Games & Puzzles



Constraint Logic

[Hearn & Demaine 2009]