

2-player games naturally make quantifiers alternate  
 $NP \rightarrow PSPACE$

Recall: PH

$$\Sigma_k$$

$$\Pi_k = \text{co}\Sigma_k$$

$$\begin{array}{l} \exists X_1 : \forall X_2 : \dots X_k : \varphi(X_1, \dots, X_k) \\ \forall X_1 : \exists X_2 : \dots X_k : \varphi(X_1, \dots, X_k) \end{array}$$

set of variables

Mate-in-k  $\in \Sigma_{2k-1}$  → # moves by player 1

Lose-in-k  $\in \Sigma_{2k}$

2nd player mate-in-k  $\in \Pi_{2k+1}$

can I force my win?  
 can I force my loss?  
 can they force their win?

Real variant of NP & PSPACE: → polynomials

$$\text{— } \exists \mathbb{R}: \exists x_1 \in \mathbb{R} : \dots : \exists x_n \in \mathbb{R} : \begin{array}{l} P_1(x_1, \dots, x_n) \geq 0 \\ \wedge \dots \wedge \\ P_m(x_1, \dots, x_n) \geq 0 \end{array}$$

$\subseteq PSPACE$  [Canny 1988]

e.g.: art gallery problem (k guards to see polygon)  
 unit-disk graph recognition  
 drawing k planar graphs on same vertices  
 Nash equilibria of multiplayer games  
 linkage flexibility are  $\exists \mathbb{R}$ -complete

— First-Order Theory of Reals:  $\exists : \forall : \exists : \forall : \dots$

$\subseteq 2EXPTIME$

— k alternations  $\Rightarrow 2^{2^{O(k)}} \cdot n^{O(1)}$

[Renegar 1989]

## 2-player motion planning through gadgets:

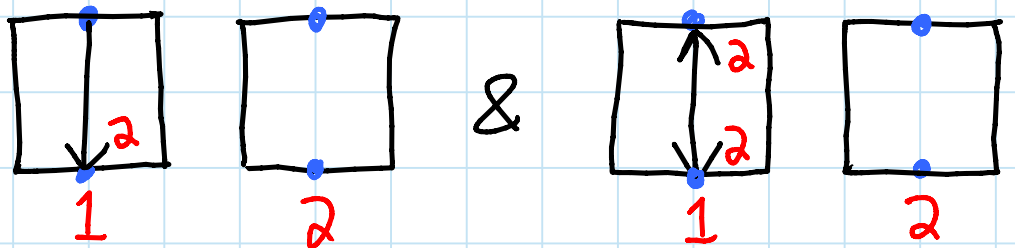
[Demaine, Hendrickson, Lynch - ITC 2020]

- each player has their robot & goal location
- players take turns making gadget transitions  
(connection graph doesn't count as moves)
- first player to reach their goal wins

Bounded characterization: 2-player motion planning with DAG gadgets is PSPACE-complete iff some gadget is nontrivial: has  $\geq 1$  transition

[Demaine, Hendrickson, Lynch - arXiv 2018]

Examples:



single-use 1-way

single-use 2-way

Bounded

Team characterization: <sup>can see state of only incident gadgets</sup> team motion planning with DAG gadgets is NEXPTIME-complete iff some gadget is nontrivial: has  $\geq 1$  transition

[Demaine, Hendrickson, Lynch - arXiv 2018]

Unbounded: 2-player/team motion planning

is EXPTIME/RE-complete if some gadget is reversible deterministic with interacting tunnels (like 1-player unbounded)