

ClassComplete SAT Problem

$\Sigma_1 = NP$

$\Pi_1 = coNP$

Σ_2

$\Pi_2 = co\Sigma_2$

Σ_k

$\Pi_k = co\Sigma_k$

PSPACE

$\exists X_1 : \varphi(X_1)$

$\forall X_1 : \varphi(X_1)$

$\exists X_1 : \forall X_2 : \varphi(X_1, X_2)$

$\forall X_1 : \exists X_2 : \varphi(X_1, X_2)$

$\exists X_1 : \forall X_2 : \dots X_k : \varphi(X_1, \dots, X_k)$

$\forall X_1 : \exists X_2 : \dots X_k : \varphi(X_1, \dots, X_k)$

$\exists X_1 : \forall X_2 : \dots X_n : \varphi(X_1, \dots, X_n)$

or $\forall X_1 : \exists X_2 : \dots X_n : \varphi(X_1, \dots, X_n)$

set of variables

SAT

UNSAT

PH

QSAT

Complements: $\exists \equiv \neg \forall \neg$ & $\forall = \neg \exists \neg$

$\Rightarrow \neg \exists X_1 : \varphi(X_1)$

UNSAT

$\equiv \forall X_1 : \neg \varphi(X_1)$

 Σ_2 -complete: [Schaefer & Umans - SIGACT News 2002 + www]

L1, L2

- $\exists : \forall : 3SAT$ - $\exists : \forall : NAE 3SAT$

SP5

- $\exists S_1 \subseteq M_1 : \forall S_2 \subseteq M_2 : S_1 \cup S_2$ is a 3DM

L13

- is circuit A = circuit B with some inputs fixed?

GT19

- $\exists k$ -coloring: no maximal clique is monochromatic

L21

- $\exists : \exists !$: planar 3SAT or planar 1-in-3SAT- $\exists k$ "clues" to force unique answer to:

- planar 3SAT - planar 1-in-3SAT

- Akari - Shakashaka - Sudoku

[Demaine, Ma, Schwartzman, Waingarten, Aaronson - TCS 2018]