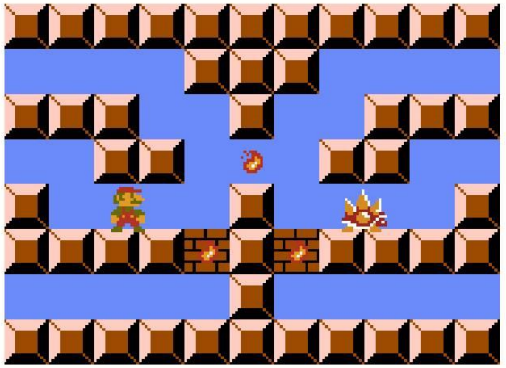
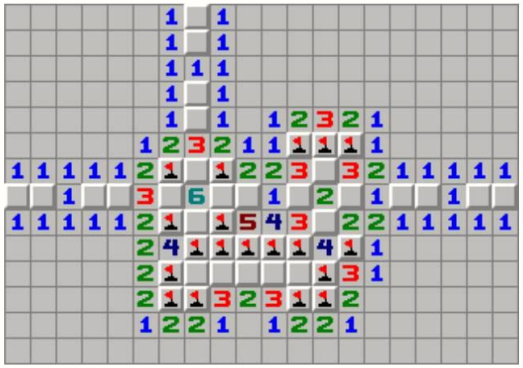
Algorithmic Lower Bounds: Fun with Hardness Proofs

Super Mario Bros.

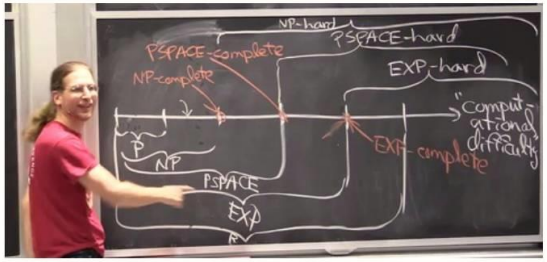


Door gadget for PSPACE-hardness

Minesweeper



OR gadget for NP-hardness



Hardness Made Easy*

Learn **when to give up** the search for efficient algorithms; see **connections** between computational problems; **solve puzzles** to prove theorems, solve **open problems**, and write papers.

Topics: NP, PSPACE, EXPTIME, EXPSPACE, approximation, fixed parameter, counting; games & puzzles, key problems, gadgets, and proof styles.

6.5440 (register for **6.S954**)
taught by Professor Erik Demaine
featuring *supercollaborative* problem solving

Mondays & Wednesdays 3:00-4:30pm
Room 32-082

https://courses.csail.mit.edu/6.5440/fall23/
sign up for our mailing list to join the class
AUS (CS Theory Track), AAGS (Theoretical CS Concentration)

Fall 2023

** Easiness not guaranteed. Side effects such as open problems and a heightened sense of complexity may occur. Ask your advisor if 6.5440 is right for you!*

Computers and Intractability: A Guide to Algorithmic Lower Bounds

by Erik D. Demaine, William Gasarch, and Mohammad Hajiaghayi

MIT Press, 2024

Book Draft

View Book Draft (PDF)

Last updated: Thursday, August 25, 2022

This is a ***draft*** of the book. There will be typos, missing references, rough/missing figures, and other mistakes. Please help us find them!

Send comments and suggestions for improvements to hardness-book@mit.edu.

https://hardness.mit.edu/

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COMPUTERS AND INTRACTABILITY

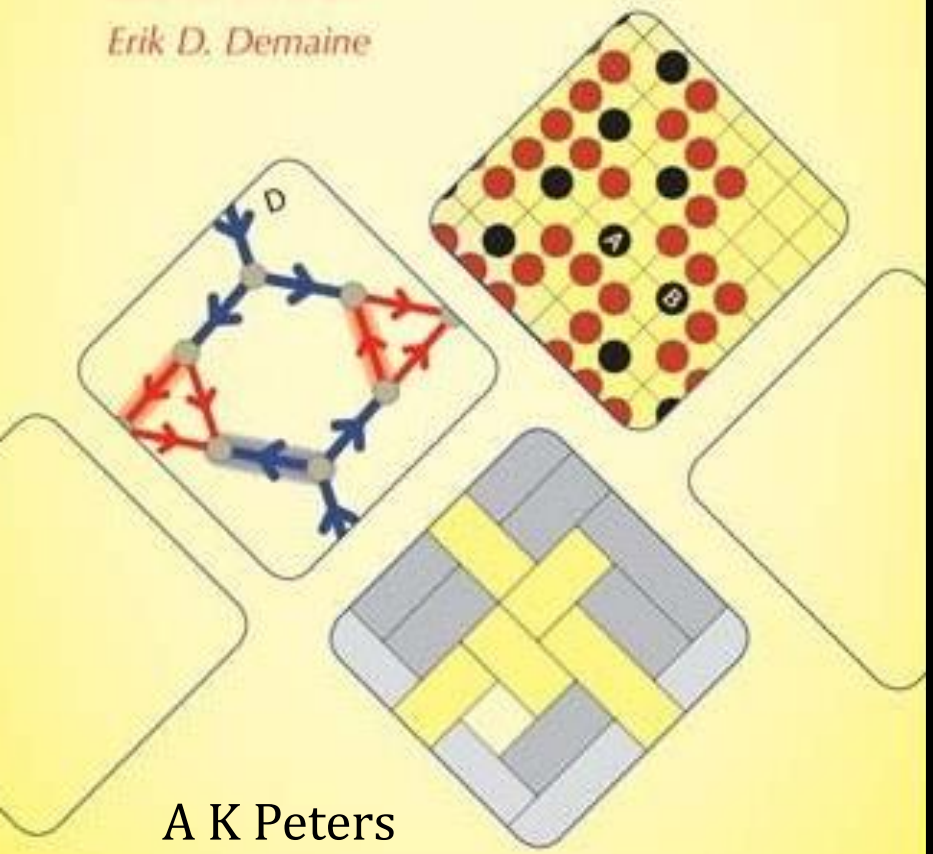
A Guide to the Theory of NP-Completeness

Michael R. Garey / David S. Johnson

W. H. Freeman
1979

Games, Puzzles, & Computation

Robert A. Hearn
Erik D. Demaine



A K Peters
July 2009

ゲームとパズルの計算量

Games, Puzzles, & Computation

Robert A. Hearn
Erik D. Demaine

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エリック・D・ドメイン
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translated by
Ryuhei Uehara

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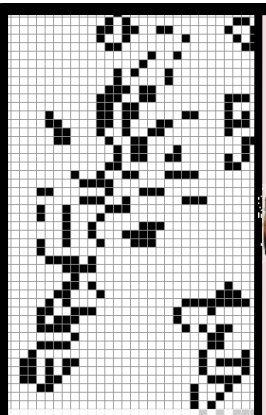
Complexity of Games & Puzzles

unbounded

bounded



PSPACE

PSPACE

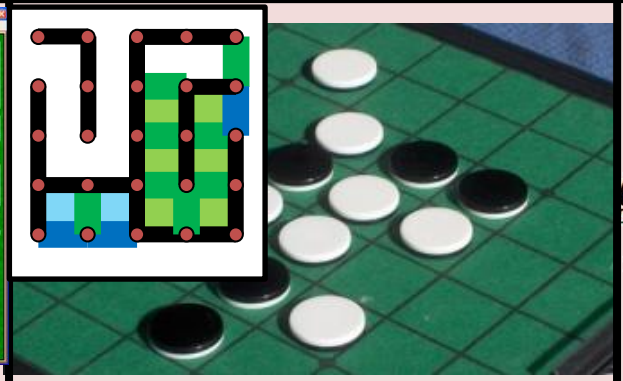
EXPTIME

Rengo Kriegspiel?

Undecidable

P

NP



PSPACE

bridge?

NEXPTIME

0 players (simulation)

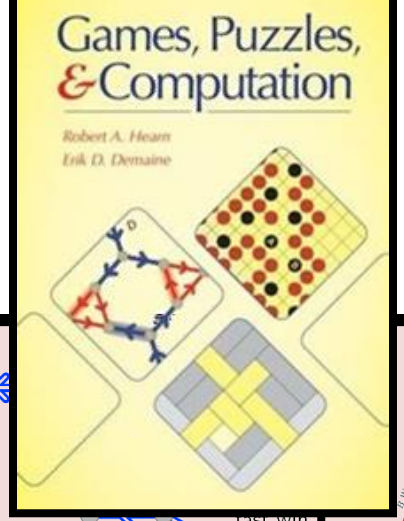
1 player (puzzle)

2 players (game)

team, imperfect info

Constraint Logic

[Hearn & Demaine 2009]



unbounded

bounded

PSPACE

PSPACE

EXPTIME

Undecidable

P

NP

PSPACE

NEXPTIME

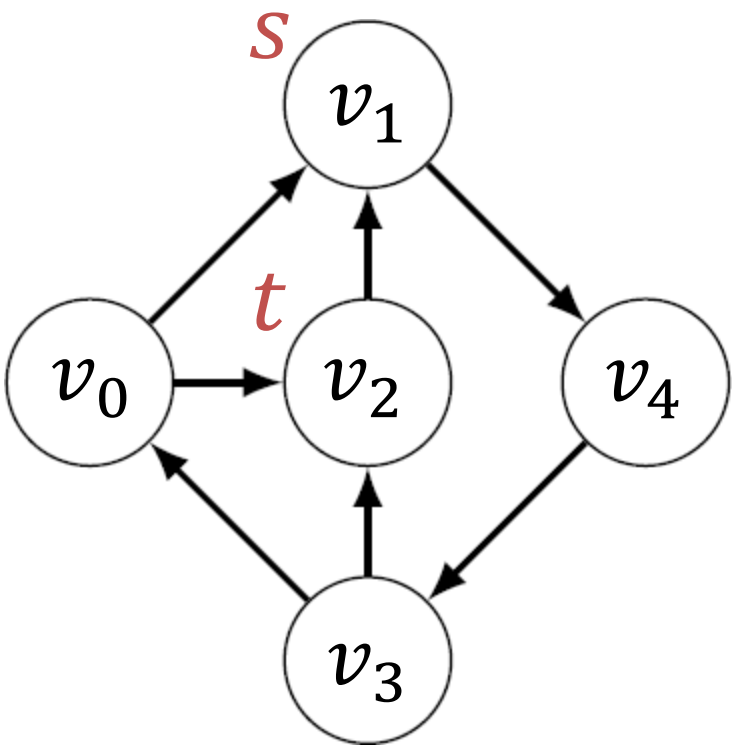
0 players (simulation)

1 player (puzzle)

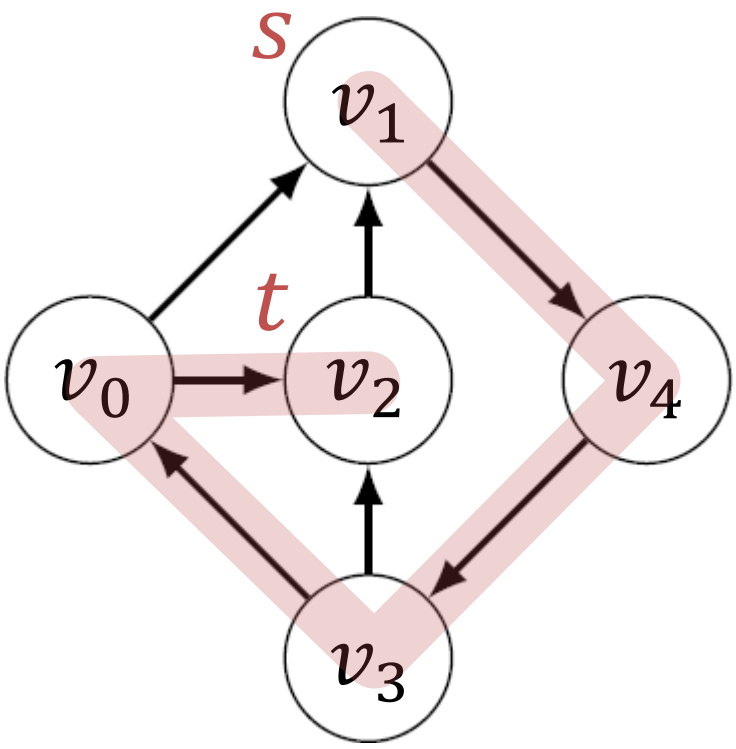
2 players (game)

team, imperfect info

Hamiltonian (s, t)-Path

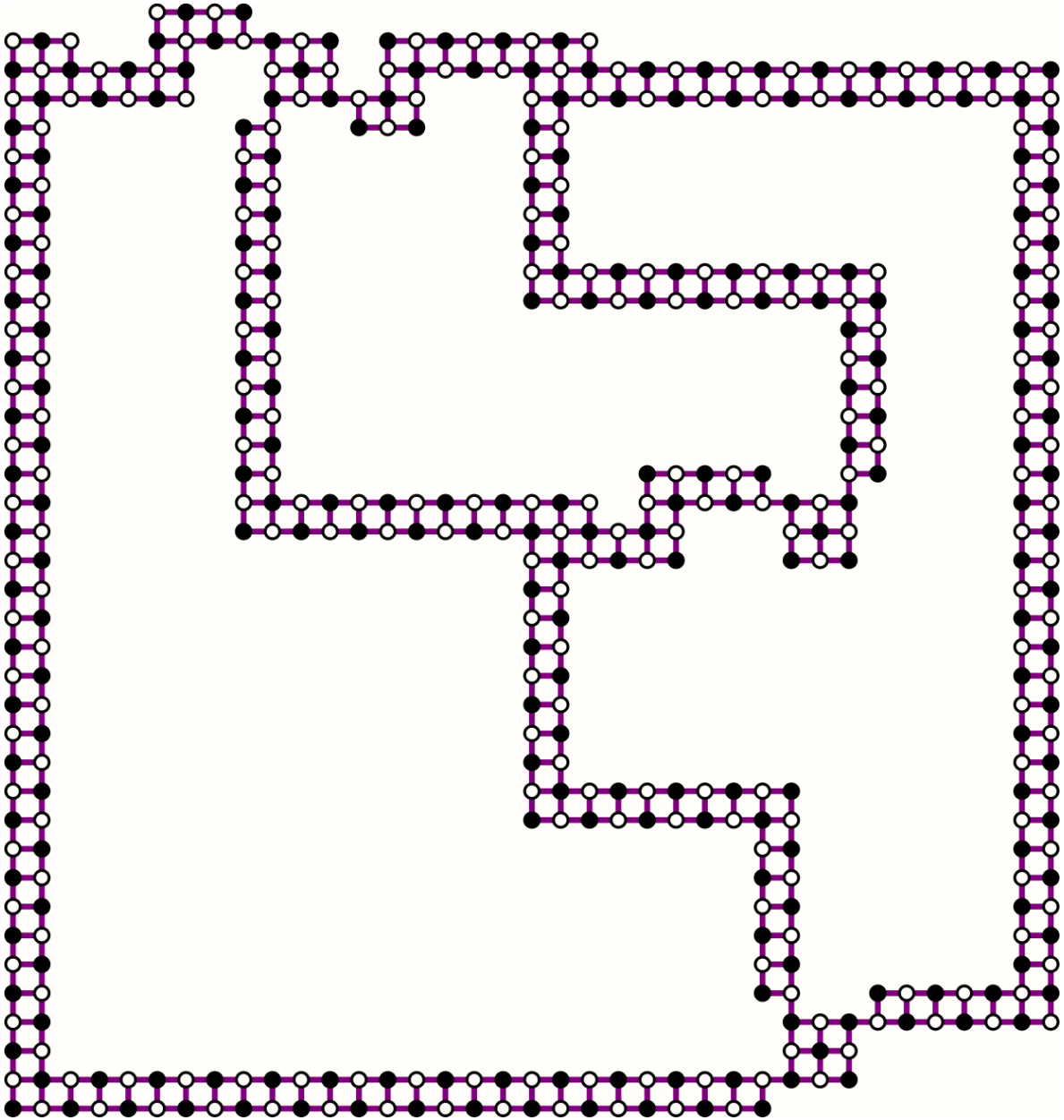


input

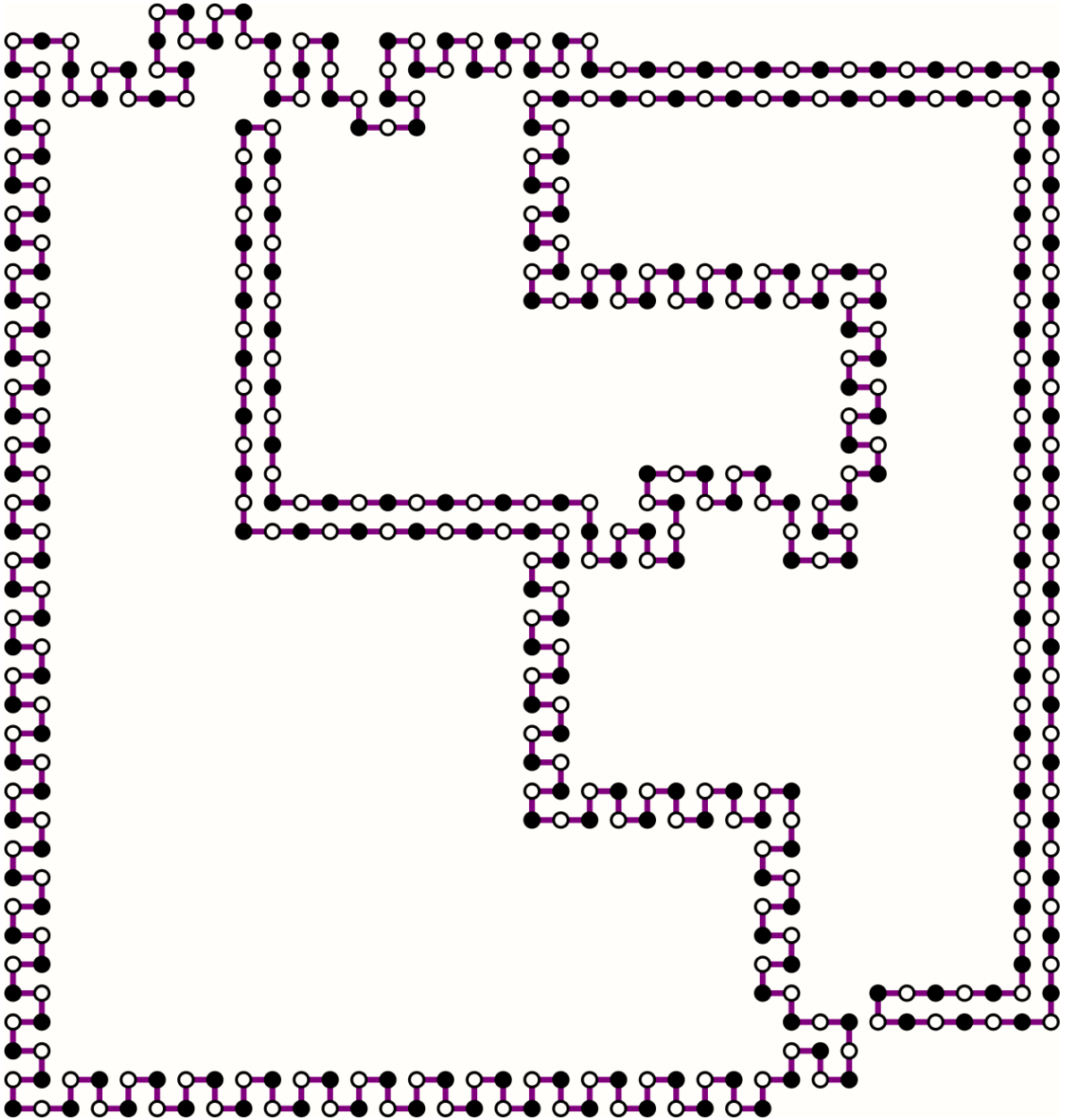


goal

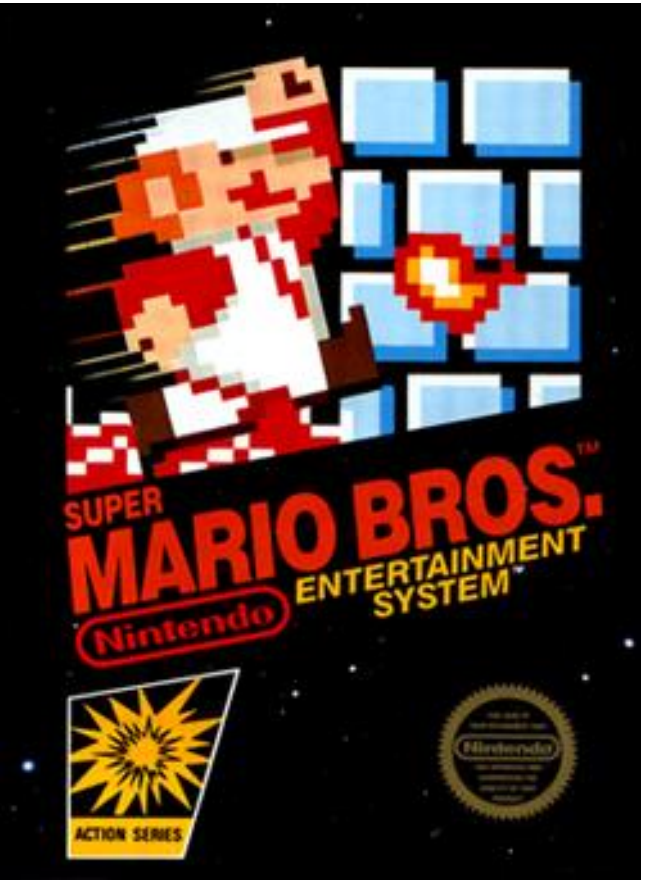
Hamiltonicity in Grid Graphs



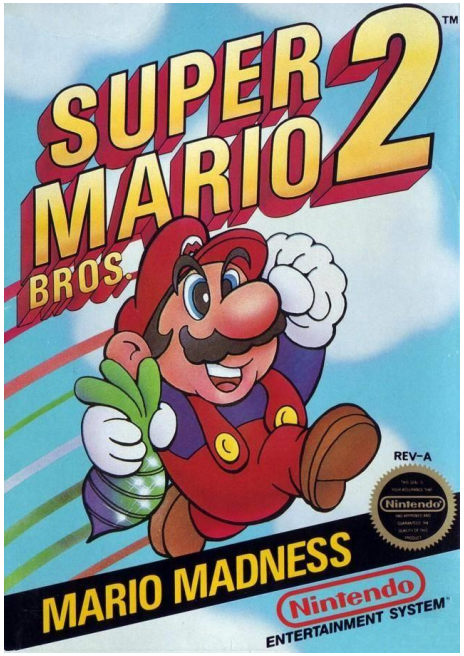
Hamiltonicity in Grid Graphs

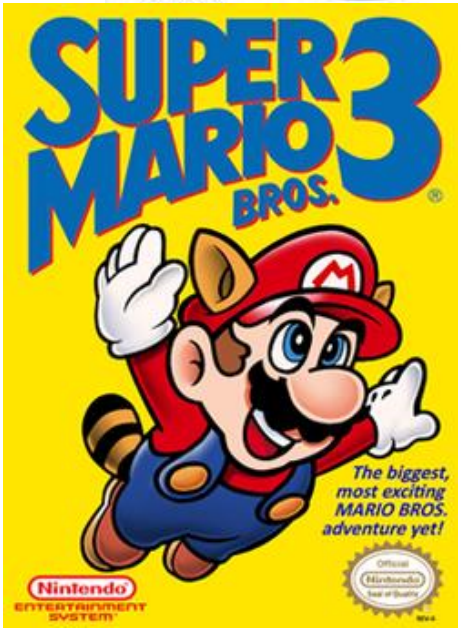


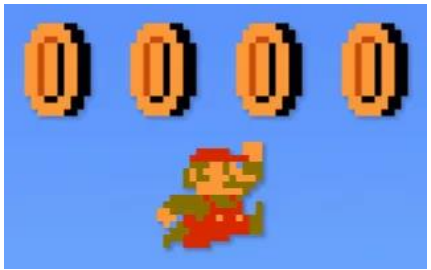
100% Speedrun is NP-hard: Mario

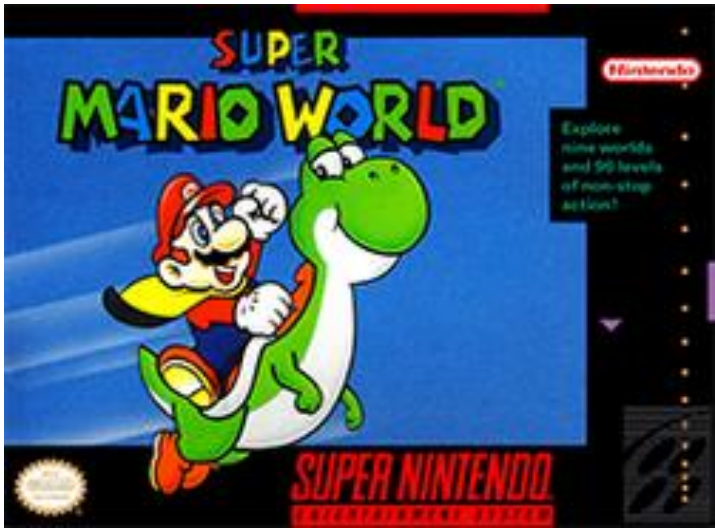


The Lost Levels

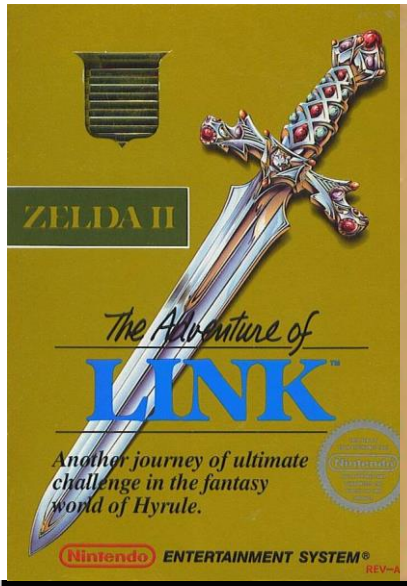


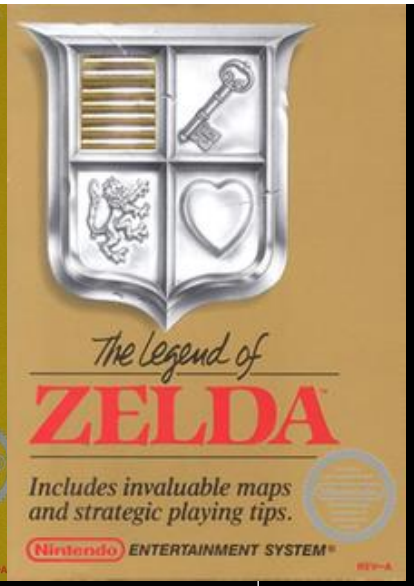






Speedrun is NP-hard: Zelda



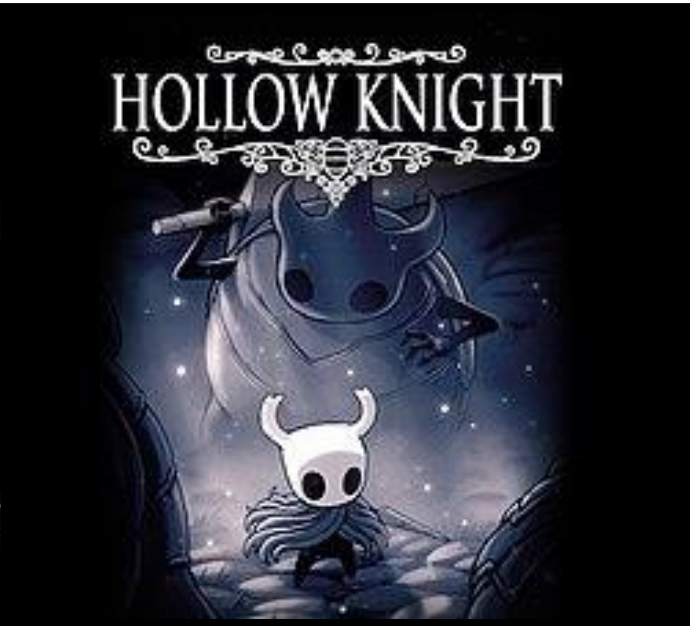


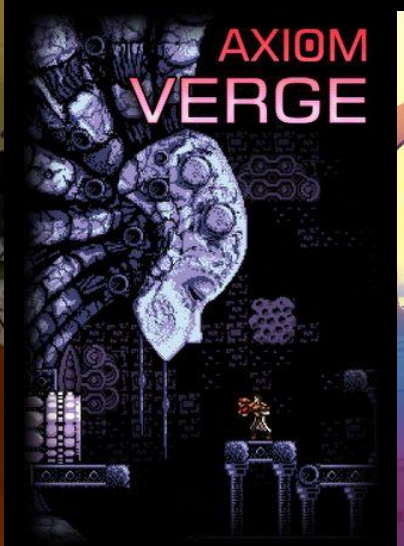
Small Key

Artwork of a Small Key from Ocarina of Time

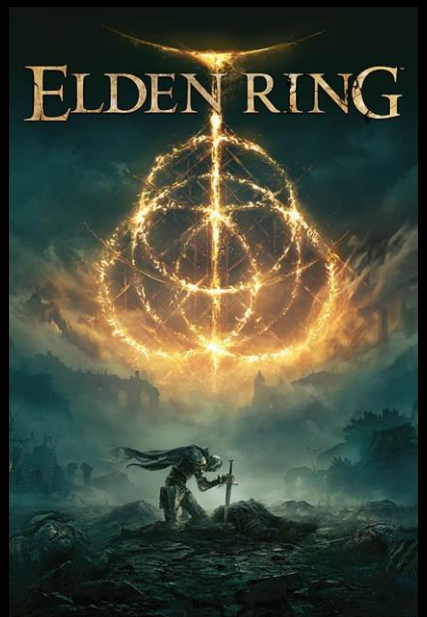
First appearance	*The Legend of Zelda* (1987)
Appearances	*The Legend of Zelda* *Adventure of Link* *A Link to the Past* *Link's Awakening* *Ocarina of Time* *Majora's Mask* *Oracle of Ages* *Oracle of Seasons* *The Wind Waker* *Four Swords* *The Minish Cap* *Four Swords Adventures* *Twilight Princess* *Phantom Hourglass* *Spirit Tracks* *Skyward Sword* *A Link Between Worlds* *Breath of the Wild*
Found	Dungeons (*Breath of the Wild*) Shrines
Use	Open small locks

Speedrun is NP-hard: Metroidvania





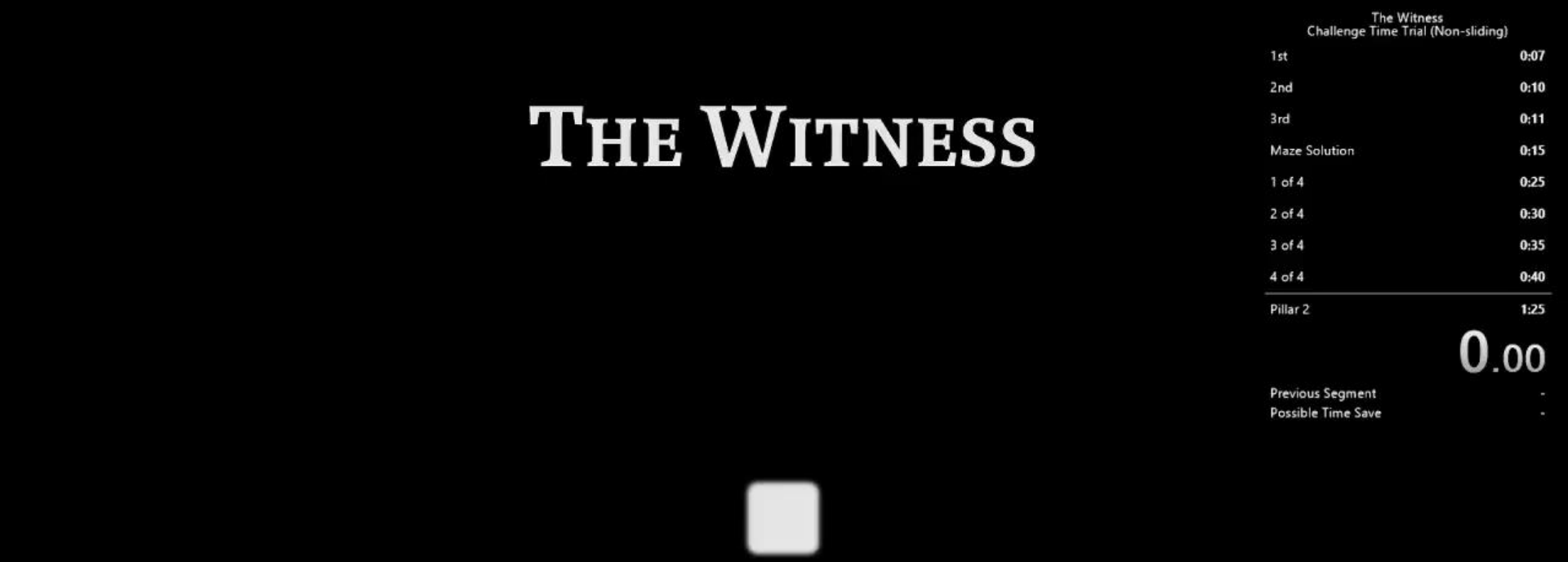
Speedrun is NP-hard: RPG



Playing is NP-hard: Katamari

The Witness

[Blow 2016]



https://youtu.be/zCU5XWyJvWg

RbdJellyfish, 2018

The Witness Clue Types

Start circle: Can begin path

End cap: Can end path

Square: ≤ 1 color per region

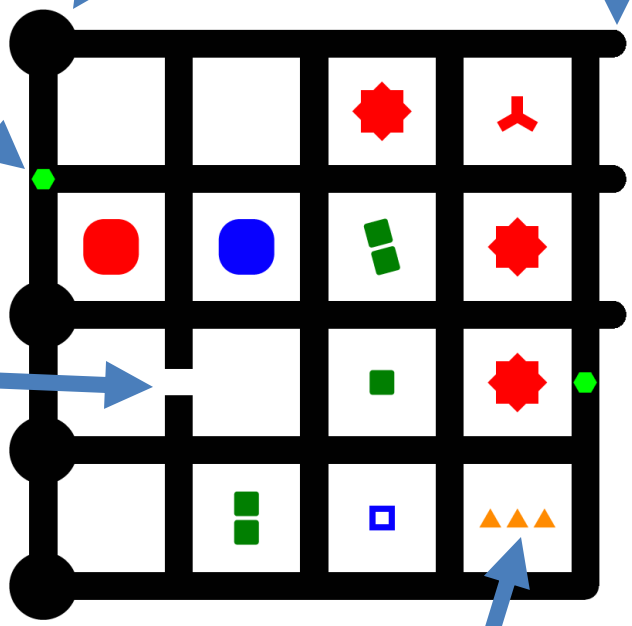
Star: 2 of color per region

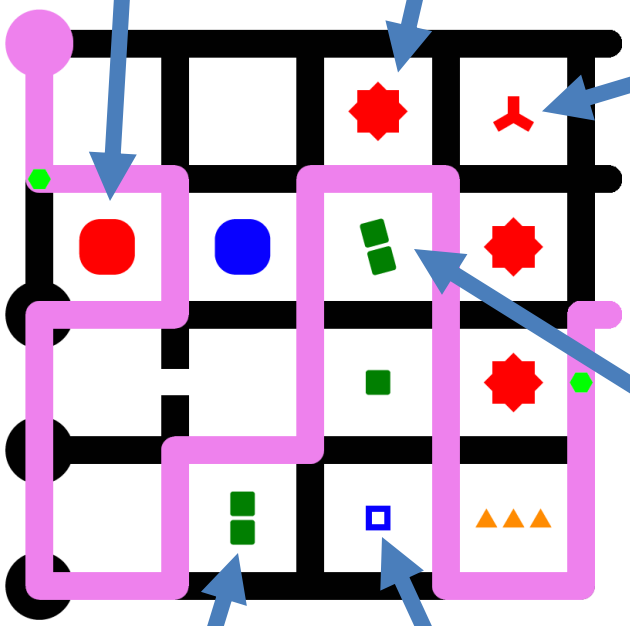
Hexagon: must be visited by path

Antibody: erase 1 clue in region

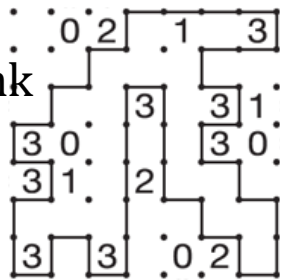
Broken edge: cannot be visited by path

Rotatable polyomino





Slitherlink



Triangle: # boundary edges in path

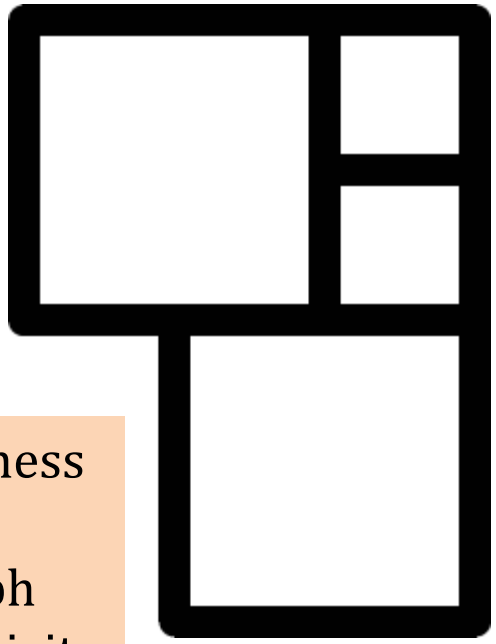
Polyomino: Must exact pack region

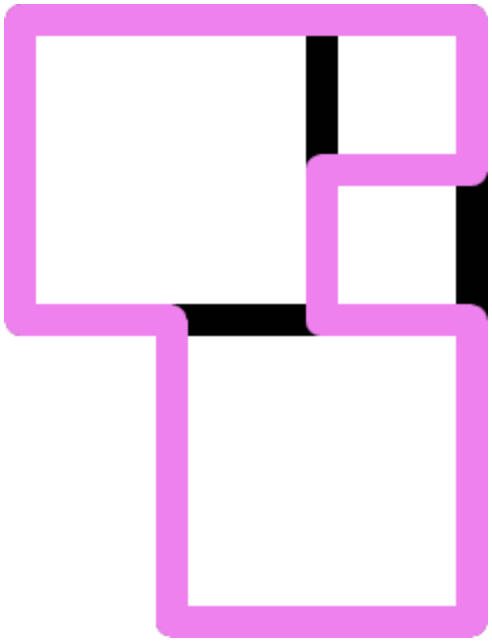
Antipolyomino: Cancels polyomino in packing

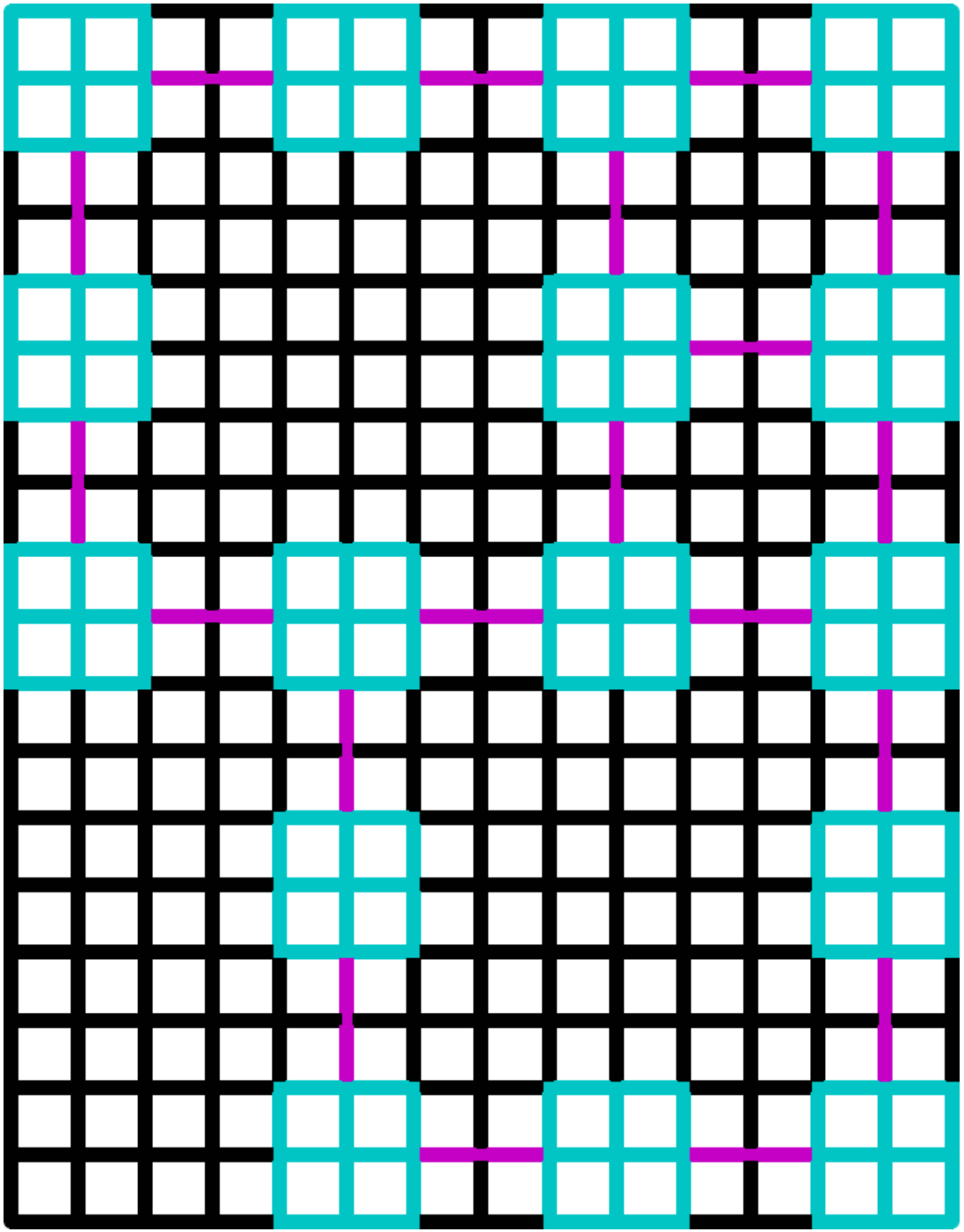
WHO WITNESSES THE WITNESS?

broken edge	hexagon	square	star	triangle	polyomino	antipolyomino	antibody	complexity
✓								$\in$ L
✓	✓ vertices							NP-complete
	✓ vertices							***OPEN***
	✓ edges							NP-complete
✓	✓ on boundary							$\in$ P
		✓ 1 color						always YES
		✓ 2 colors	(also Kostitsyna, Löffler, Sandag, Sonke, Wulms 2018)					NP-complete
			✓ 1 color					***OPEN***
			✓ n colors					NP-complete
✓				✓ any			(Yato 2000)	NP-complete
				✓ ▲				NP-complete
				✓ ▲▲				NP-complete
				✓ ▲▲▲				NP-complete
					✓ ■			$\in$ P
✓					✓ ■			$\in$ P
					✓ ■	✓ □		NP-complete
					✓ ■			NP-complete
					✓ ■			NP-complete
✓	✓	✓	✓	✓	✓	✓		$\in$ NP
✓	✓	✓	✓	✓			✓ n	$\in$ NP
					✓		✓ 2	Σ_2-complete
✓	✓	✓	✓	✓	✓		✓ 1	$\in$ NP
					✓	✓	✓ 1	Σ_2-complete
✓	✓	✓	✓	✓	✓	✓	✓ n	$\in \Sigma_2$

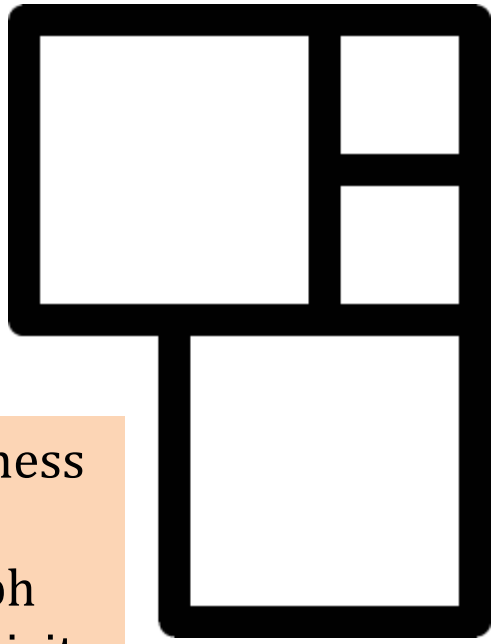
NP-hardness from grid-graph Hamiltonicity

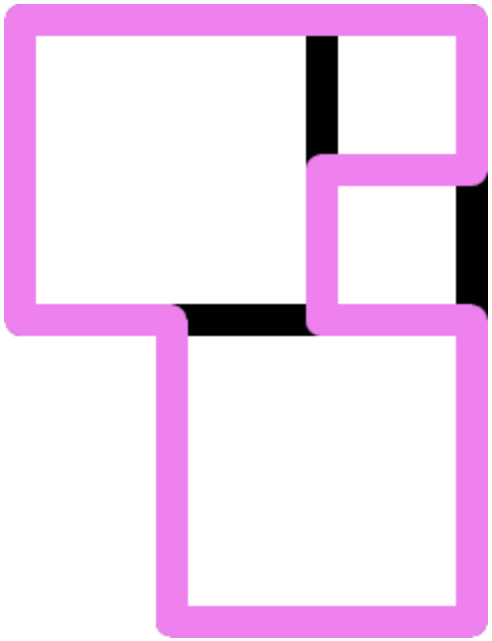


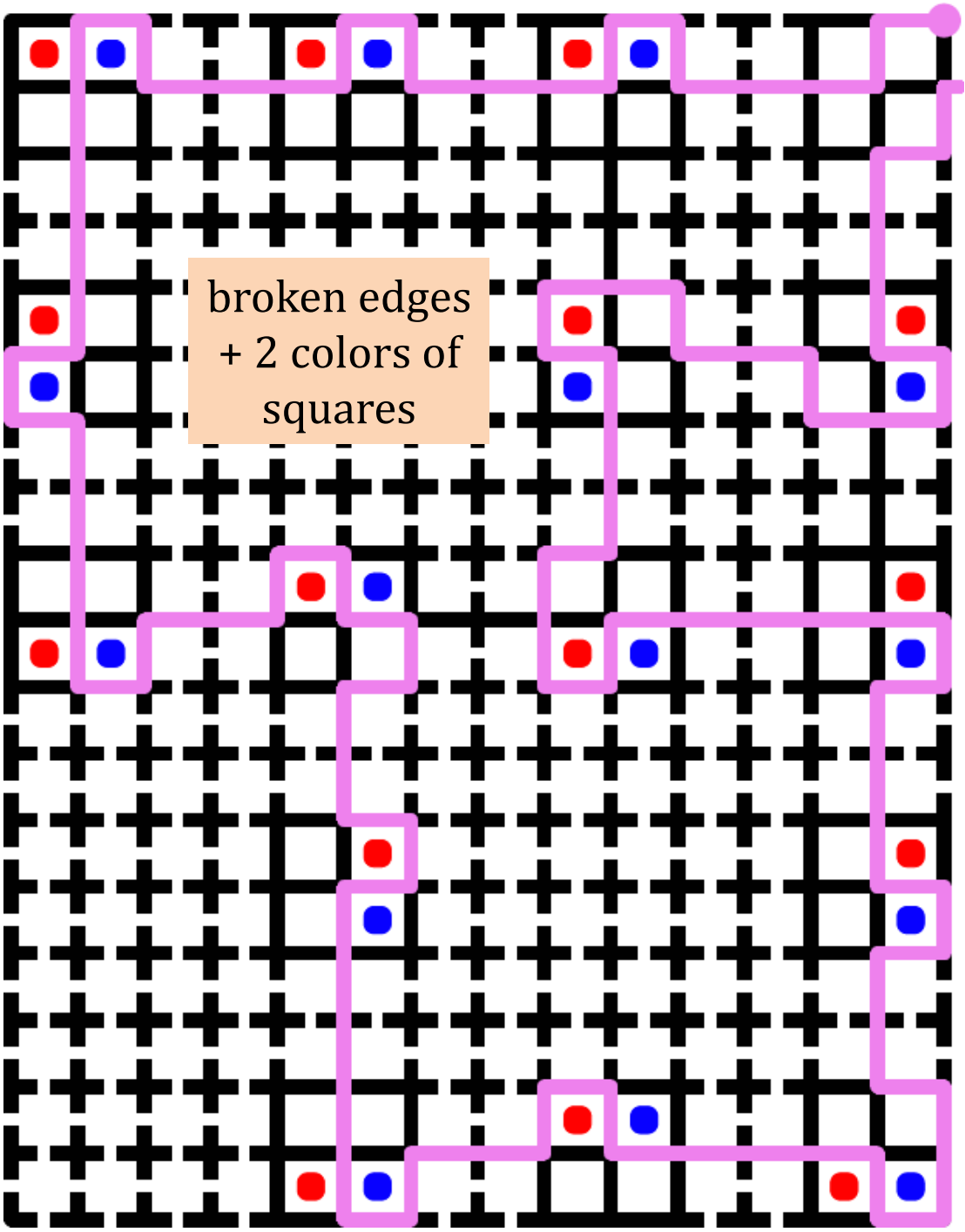




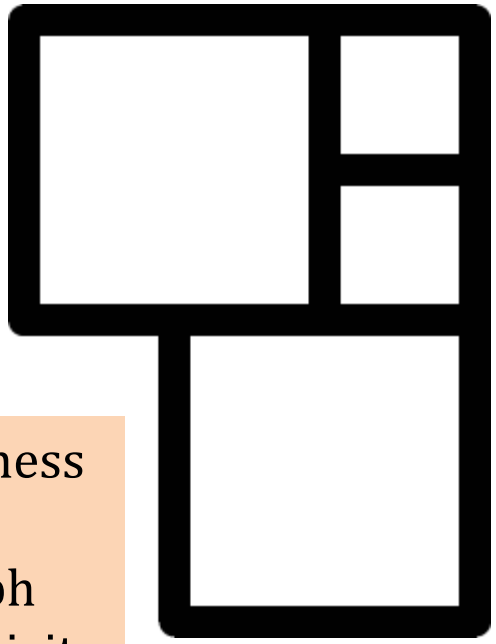
NP-hardness from grid-graph Hamiltonicity

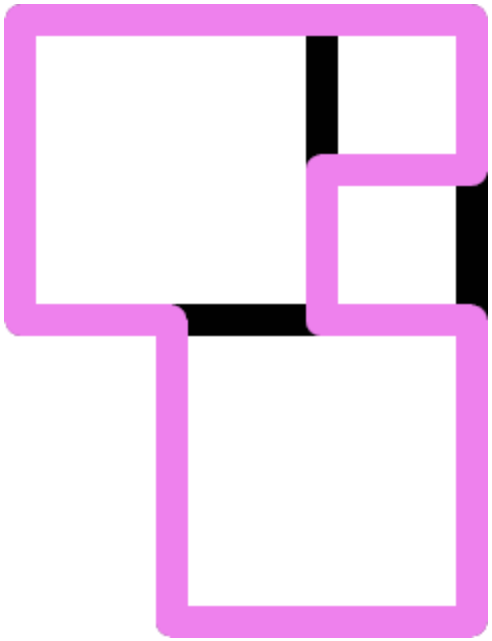


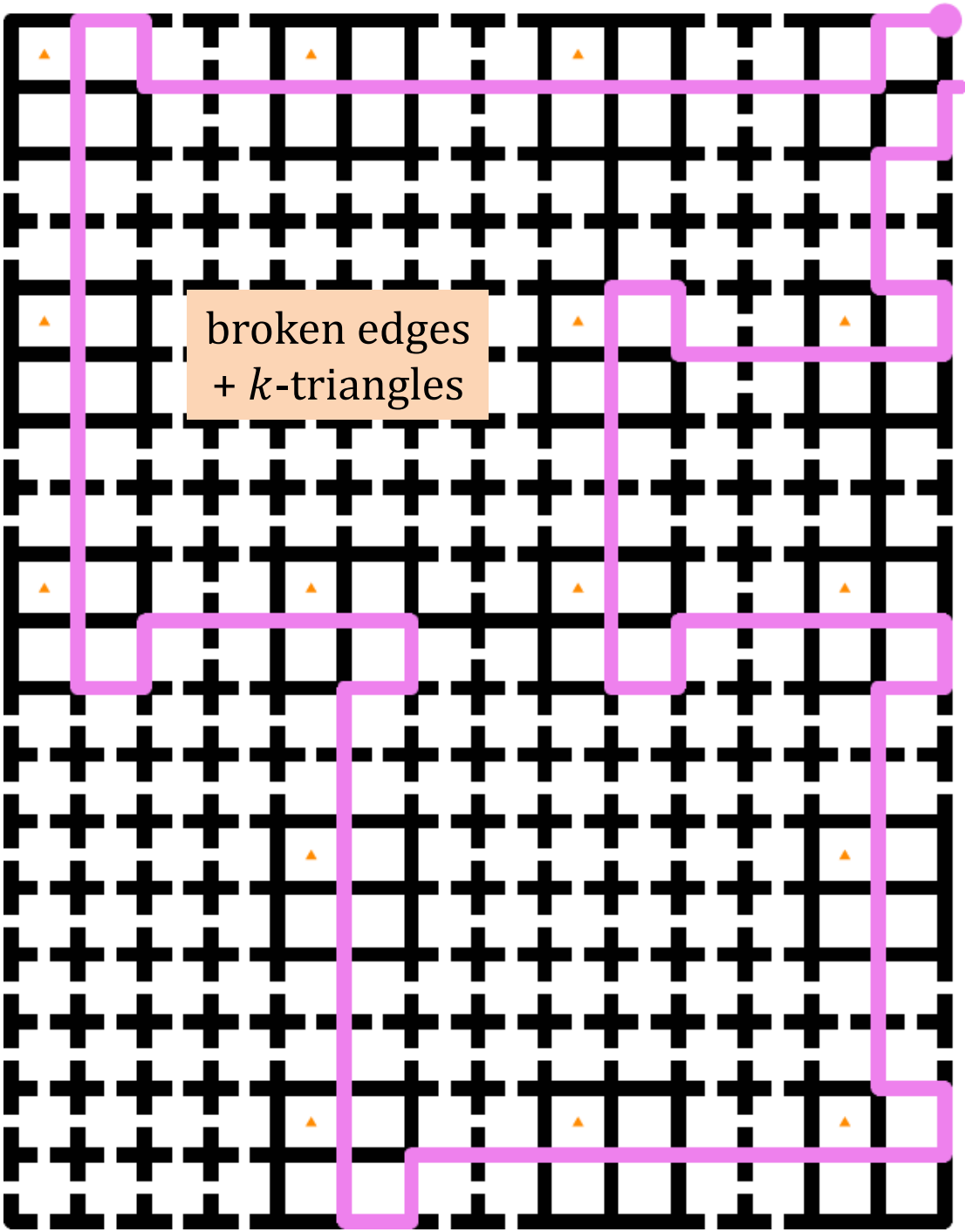




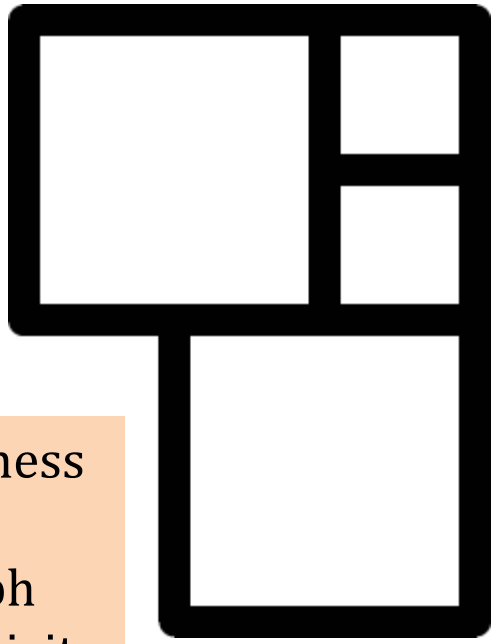
NP-hardness from grid-graph Hamiltonicity

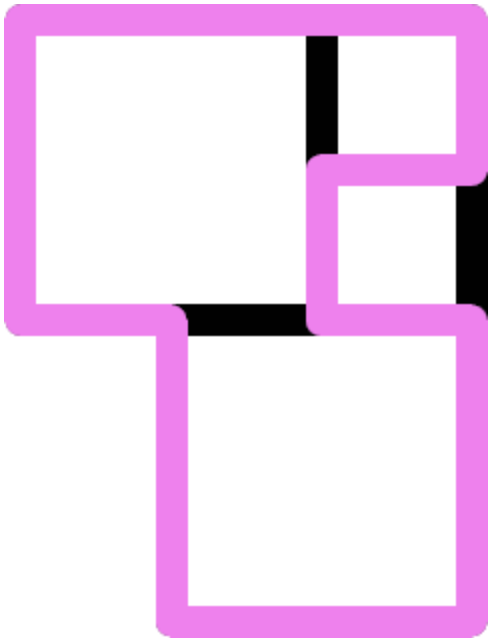


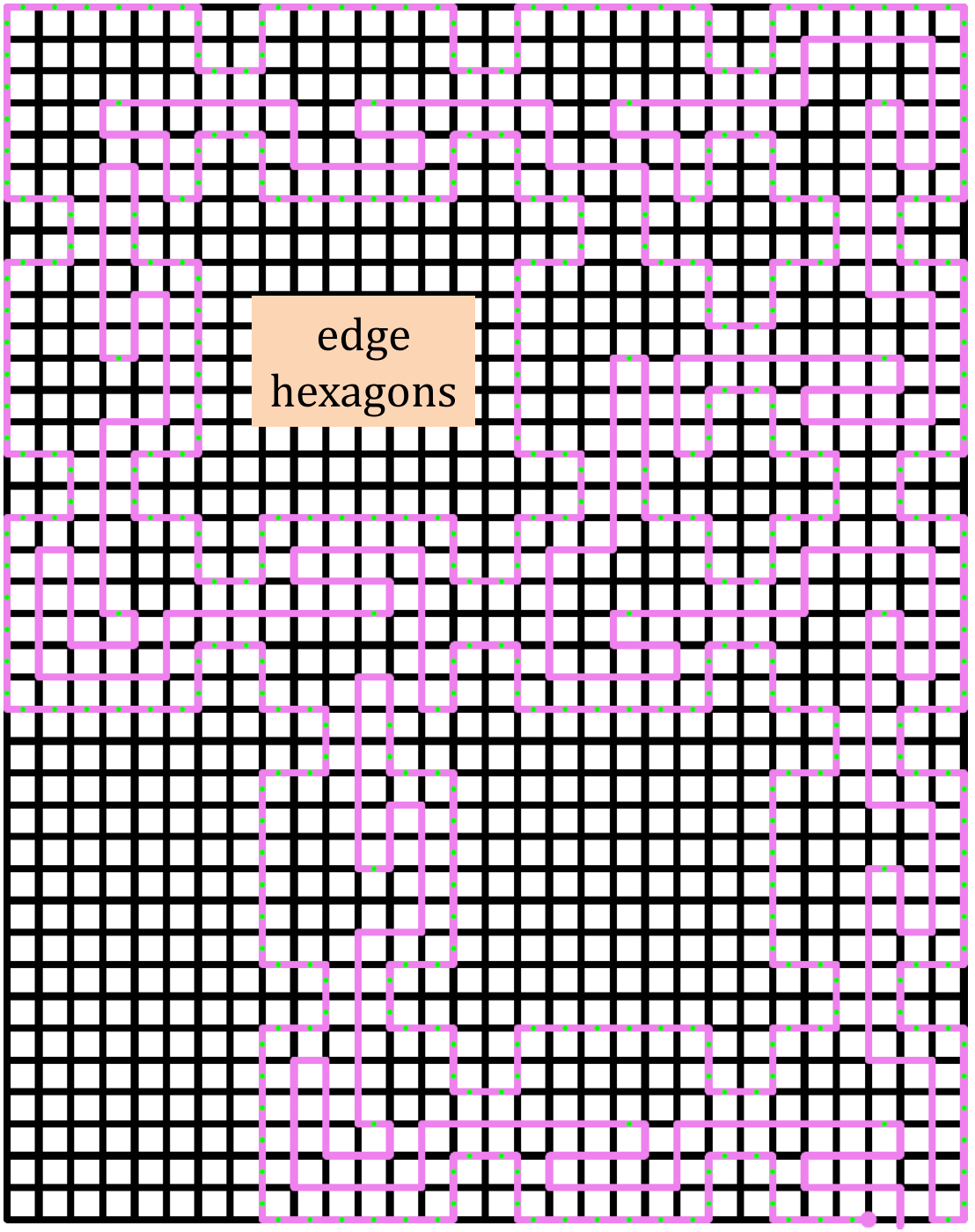




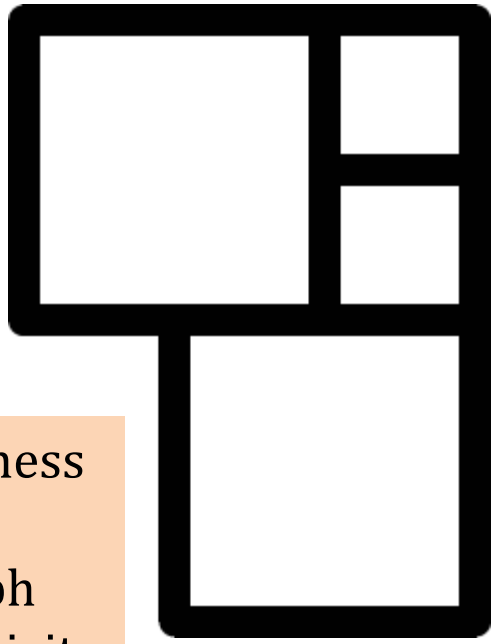
NP-hardness from grid-graph Hamiltonicity

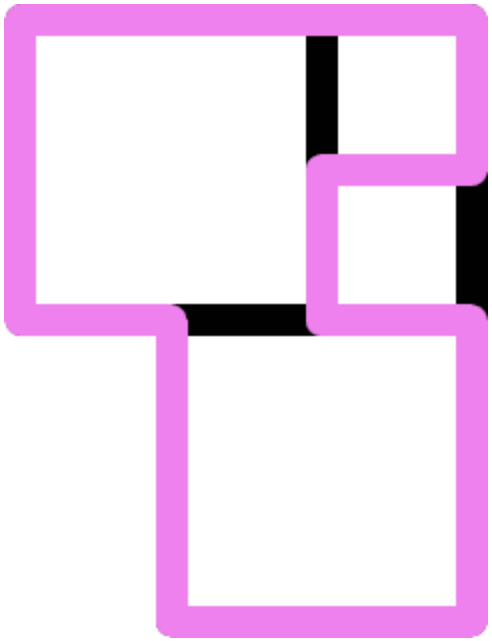


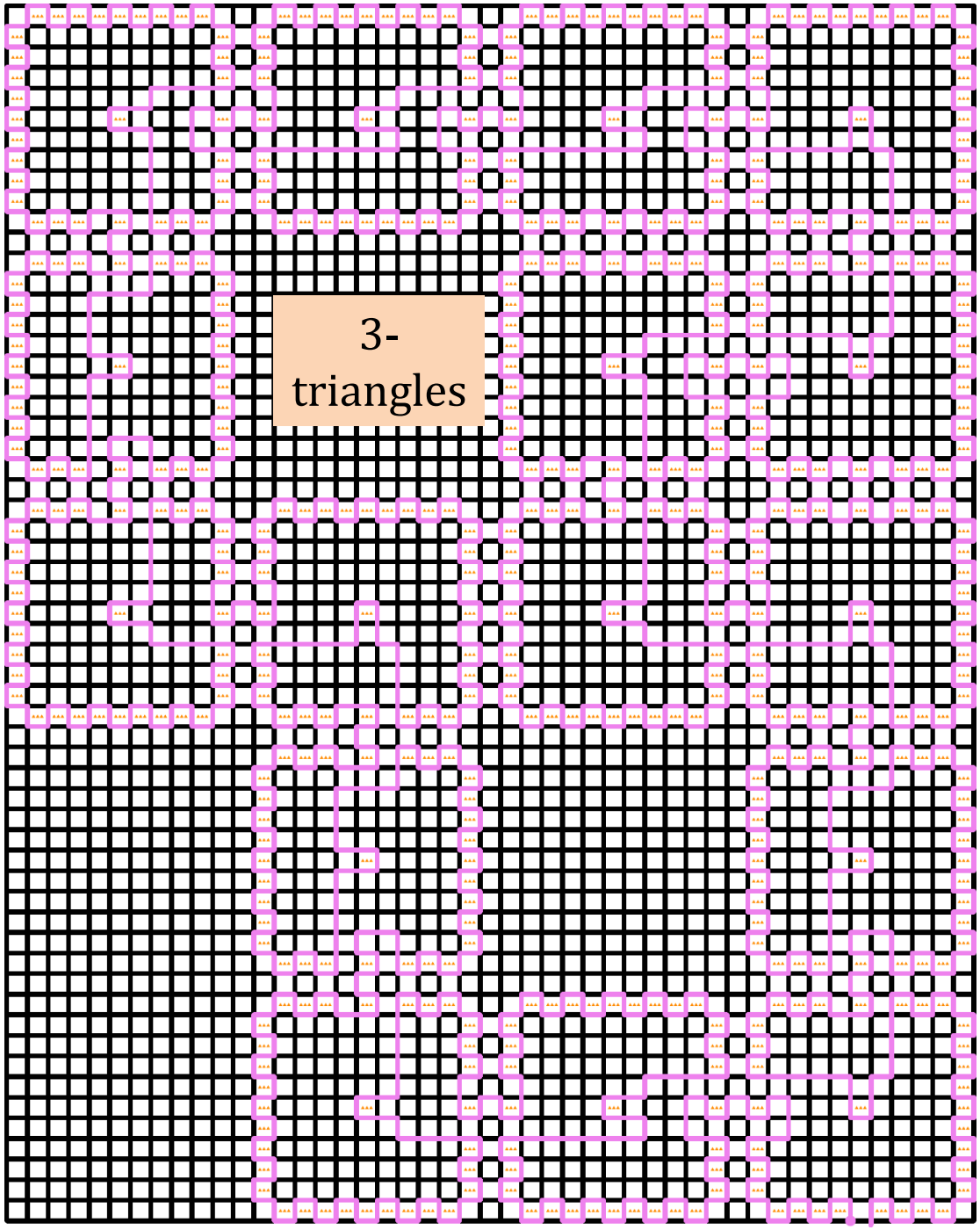


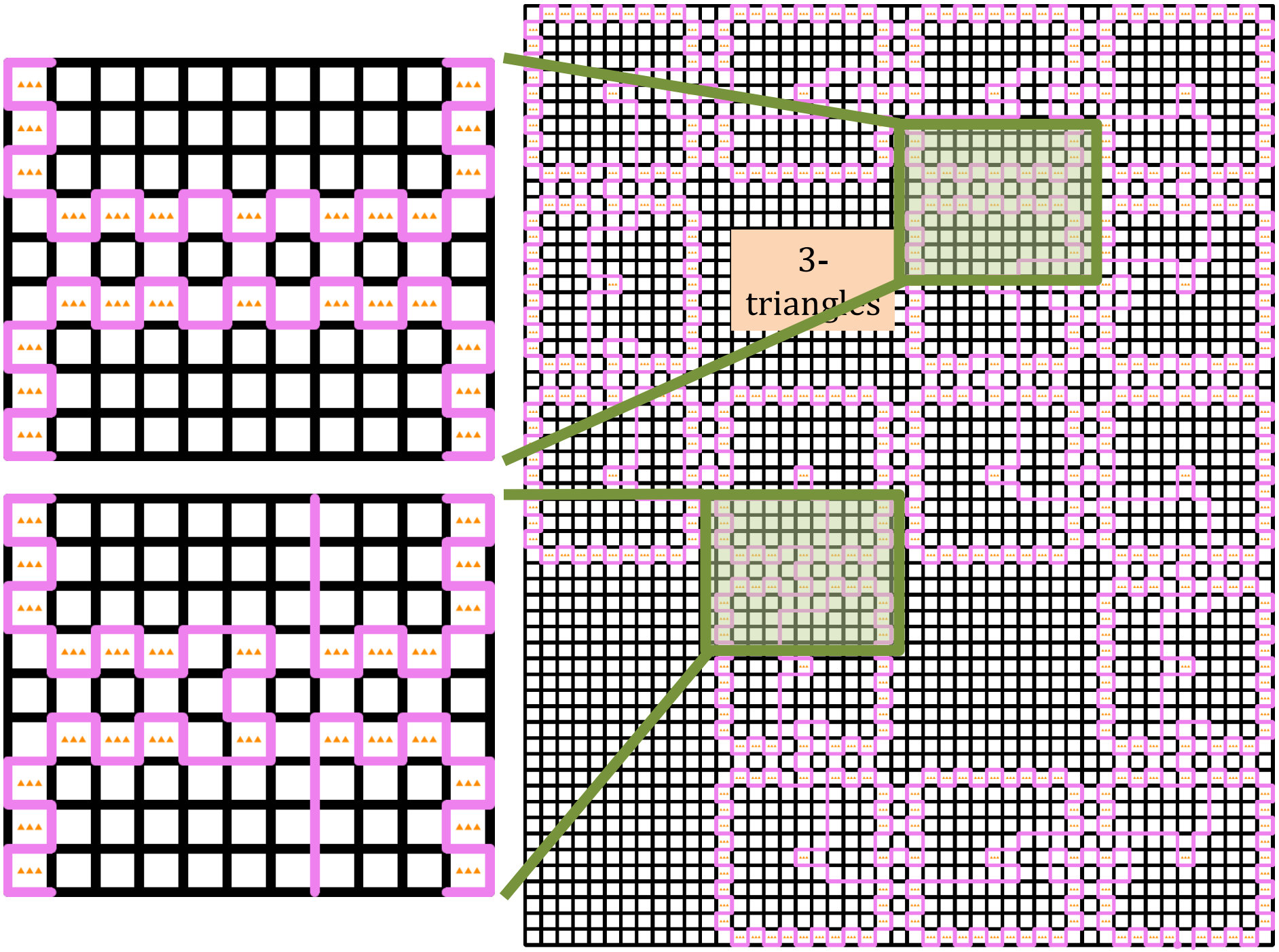


NP-hardness from grid-graph Hamiltonicity








3-
triangles