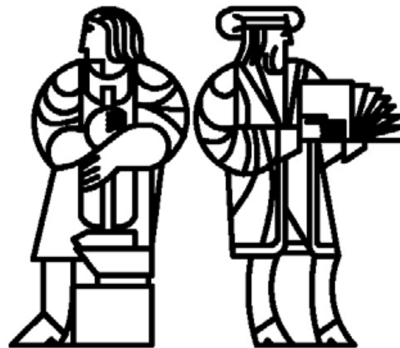
6.170 Lecture 8
Representation invariants and abstraction functions



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What is an ADT?

It's more than just a data structure

data structure + a set of conventions

Specification: only in terms of the abstraction

Never mentions the representation

Representation invariant: Object → boolean

Indicates whether a data structure is well-formed

Defines the set of valid values of the data structure

Abstraction function: Object → abstract value

What the data structure means (as an abstract value)

How the data structure is to be interpreted

How do you compute the inverse, abstract value→ Object ?

A Data Abstraction Is Defined by a Specification

A collection of procedural abstractions

- Not a collection of procedures

Together, these procedural abstractions provide

- A set of values
- ***All*** the ways of directly using that set of values
  - Creating
  - Manipulating
  - Observing

Creators and producers make new values

Mutators change the value (but don't affect = =)

Observers allow one to tell values apart

- The key to understanding

Implementation of an ADT is Provided by a Class

To implement a data abstraction

- Select the representation of instances, the *rep*
- Implement operations in terms of that rep

Choose a representation so that

- It is possible (preferably easy) to implement operations
- The most frequently used operations are efficient
  - But which will these be?
  - Abstraction allows changes to rep late in game

CharSet Abstraction

Overview: CharSets are finite sets of Characters

public CharSet ( )
 effects: creates a fresh, empty CharSet

public void insert (Character c);
 modifies: this
 effects: $this_{post} = this_{pre} \cup \{c\}$

public void delete (Character c);
 modifies: this
 effects: $this_{post} = this_{pre} - \{c\}$

public boolean member (Character c);
 returns: ($c \in this$)

public int size ( );
 returns: cardinality of this

A CharSet Implementation ?

```
class CharSet {
    private ArrayList<Character> elts
        = new ArrayList<Character> ();
    public void insert (Character c)   {
        elts.add (c);
    }
    public void delete (Character c)   {
        elts.remove (c);
    }
    public boolean member (Character c) {
        return elts.contains (c);
    }
    public int size ( ) {
        return elts.size ();
    }
}
```

```
CharSet s = new CharSet( );
Character a
    = new Character('a');
s.insert(a);
s.insert(a);
s.delete(a);
if (s.member(a))
    // print wrong;
else
    // print right;
```

Where Is the Error?

The answer to this question tells you what needs to be fixed

Perhaps delete is wrong

It should remove all occurrences

Perhaps insert is wrong

It should not insert a character that is already there

We have no way of knowing

Or do we?

Representation invariants tell us

A Representation Invariant

Captures information that must be shared across implementations of multiple operations

Write it this way:

```
class CharSet {
  private ArrayList<Character> elts;
   // Rep invariant: elts has no nulls and no duplicates
  …
```

Or, if you are the pedantic sort:

∀ indices i of elts . elts.elementAt(i) != null

∀ indices i, j of elts .

  i ≠ j ⇒ ! elts.elementAt(i).equals(elts.elementAt(j))

Now, where is the error?

```
public void insert (Character c) {
   elts.add (c);
}

public void delete (Character c) {
   elts.remove (c);
}
```

Rep invariant for Account

```
class Account {
    private List<Transaction> transactions;
    private int balance;
    …
}
```

// real-world constraints

balance ≥ 0

balance $= \Sigma_i$ transactions.get(i).amount

// implementation-related constraints

transactions != null

no nulls in transactions

Listing the elements of a CharSet

Consider adding the following method to CharSet

```
// returns: an ArrayList containing the members of this
public ArrayList<Character> getElts ( );
```

Consider this implementation:

```
public ArrayList<Character> getElts ( )  { return elts; }
```

Recall rep invariant: elts has no nulls and no duplicates

The implementation of getElts preserves the rep invariant

… sort of

Representation exposure

Consider the client code (outside implementation of the type)

```
CharSet s = new CharSet ( );
Character a = new Character('a');
s.insert (a);
s.getElts( ).add (a);
s.delete (a);
if (s.member (a)) …
```

This is almost always evil

If you do it, document why and how

And feel guilty about it

Avoiding Rep Exposure

Exploit immutability

```
Character choose ( ) {
    return elts.elementAt (0)
}
```

Character is immutable

Make a copy

```
ArrayList<Character> getElts ( ) {
    return new ArrayList<Character>(elts);
    // or: return (ArrayList<Character>) elts.clone ( );
}
```

Mutating a copy doesn’t affect the original

Checking rep Invariants

Should code check that rep invariant holds?

Yes, if inexpensive

Yes, as debugging code (even when expensive)

It’s quite hard to justify turning the checking off

Private methods need not check

Checking the Rep Invariant

```
public void delete (Character c) {
      checkRep ( );
      elts.remove (c)
      // Is this guaranteed to get called?
      // See handouts for a less error-prone way to check at exit
      checkRep ();
}
…
/** Verify that elts contains no duplicates **/
private void checkRep( ) {
      for (int i = 0; i < elts.size( ); i++) {
          assert elts.indexOf(elts.elementAt(i)) == i);
      }
}
```

An alternative implementation: repOK() returns a boolean, and callers of repOK must check its return value

Practice Defensive Programming

Assume that you will make mistakes

Write and incorporate code designed to catch them

- On entry:
  - Check rep invariant
  - Check preconditions (requires clause)
- On exit:
  - Check rep invariant
  - Check postconditions

Checking the rep invariant helps you discover errors

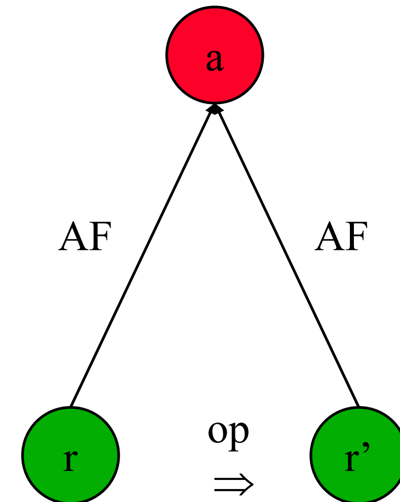
Reasoning about the rep invariant helps you avoid errors

- Or prove that they do not exist!
- We will discuss such reasoning, later in the semester

Benevolent Side Effects

Different implementation of member:

```
boolean member (Character c1) {
    int i = elts.indexOf (c1);
    if (i == -1) return false;
    // move-to-front optimization
    Character c2 = elts.elementAt (0);
    elts.setElementAt (c1, 0);
    elts.setElementAt (c2, i);
    return true;
}
```



Speeds up repeated membership tests

Mutates rep, but does not change *abstract* value

AF maps both reps to the same abstract value

Rep Invariant constrains structure, not meaning

New implementation of insert that preserves invariant

```
public void insert (Character c) {
    Character cc = new Character(encrypt(c));
    if (!elts.contains(cc))
        elts.addElement(cc);
}
public boolean member (Character c) {
    return elts.contains (c);
}
```

Program still does wrong thing

Where is the error?

```
CharSet s = new CharSet( );
Character a = new Character('a'));
s.insert (a);
if (s.member(a))
    // print right;
else
    // print wrong;
```

Abstraction functions

Abstraction function **relates the concrete representation to the abstract value it represents**

AF: Object → abstract value

AF(CharSet this) = { c | c is contained in this.elts }

- "set of Characters contained in this.elts"
- Typically not executable

Combined with rep invariant

- Allows us to examine operators independently
- "Correctness" is now a local issue

Once again we can place the blame

- Applying the abstraction function to the result of the call to insert yields AF(elts) U {encrypt('a')}
- What if we used this abstraction function?
  - { c | decrypt(c) is contained in this.elts }

The abstraction function is a function

Q: Why do we map concrete to abstract rather than vice versa?

It's not a function in the other direction.

E.g., lists [a,b] and [b,a] each represent the set {a, b}

It's not as useful in the other direction.

Can construct objects via the provided operators

Writing Abstraction Functions

Abstraction function needs to be defined properly on all representations that satisfy the rep invariant

Often a challenge

Whereas writing the rep invariant is usually easy

Problem lies in denoting range of abstraction function

Not a problem for mathematical entities like sets

For more complex abstractions, give them fields

AF defines the value of each “specification field”

Overview section of the specification is the key

Ideally, provides way of writing values of abstract type

Having a printed representation is valuable for debugging

Key Points

Rep invariant

Which concrete values represent abstract values

Abstraction function

Which abstract value each concrete value represents

Together, they modularize the implementation

Can examine operators one at a time

Neither one is part of the abstraction (the ADT)

In practice

Always write a representation invariant

Write an abstraction function when you need it

- Write an informal one for most non-trivial classes
- A formal one is harder to write and usually less useful