

6.170 Lecture 6

Procedure specifications



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Outline

Satisfying a specification; substitutability

Stronger and weaker specifications

- Comparing by hand
- Comparing via logical formulas
- Comparing via transition relations

Specification style; checking preconditions

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Satisfaction of a specification

Let P be an implementation and S a specification

P satisfies S iff

Every behavior of P is permitted by S
 “The behavior of P is a subset of S ”

The statement “ P is correct” is meaningless

Though often made!

If P does not satisfy S , either (or both!) could be “wrong”

“One person’s feature is another person’s bug.”

Usually better to change the program

Hate to publish amendments to library specifications

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General form of a procedure specification

qualifiers Returntype name (formals)

requires
modifies
throws
effects
returns

public static boolean isPrime (int i)

requires $i > 0$
modifies nothing
returns true iff i is a prime number

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A specification denotes a set of procedures

Some set of procedures satisfies a specification

Suppose a procedure takes an integer as an argument

Spec 1: “returns an integer \geq its argument”

Spec 2: “returns a non-negative integer \geq its argument”

Spec 3: “returns argument + 1”

Spec 4: “returns argument²”

Spec 5: “returns Integer.MAX_VALUE”

Consider these implementations

Code 1: `return arg * 2;`

Code 2: `return abs(arg);`

Code 3: `return arg + 5;`

Code 4: `return arg * arg;`

Code 5: `return Integer.MAX_VALUE;`

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Specification strength and substitutability

A stronger specification promises more


It constrains the implementation more
 The client can make more assumptions

Substitutability

A stronger specification can always be substituted for a weaker one

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Comparing specifications and procedures


We wish to compare procedures to specifications
 Determine whether the procedure satisfies the specification
 Indicates whether the implementer has succeeded

We wish to compare specifications to one another
 Determine which specification (if either) is stronger
 A procedure satisfying a stronger specification can be used anywhere that a weaker specification is required

Three ways to compare

1. By hand; examine each clause
2. Logical formulas representing the specification
3. Transition relations

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Comparing by hand (comparison technique 1)

We can weaken a specification by
 Making requires harder to satisfy (**strengthening requires**)
 Preconditions: *contravariant*, all other clauses: *covariant*
 Adding things to modifies clause (weakening modifies)
 Making effects easier to satisfy (weakening effects)
 Guaranteeing less about throws (weakening throws)
 Guaranteeing less about returns value (weakening returns)

The strongest (most constraining) spec has the following:
requires clause: true
modifies clause: nothing
effects clause: false
throws clause: nothing
returns clause: false
 (This particular spec is so strong as to be useless; in practice, some or all clauses must be relaxed.)

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Comparing logical formulas (comparison technique 2)

Specification S1 is stronger than S2
 When $\forall P, (P \text{ satisfies } S1) \Rightarrow (P \text{ satisfies } S2)$
 Treating each specification as a logical formula: $S1 \Rightarrow S2$

The specification promises that this property is true:
 $\text{requires} \Rightarrow (\text{not-modified} \wedge \text{effects})$

Convert each specification into a single logical formula
 Do it in two steps: 5 clauses \Rightarrow 3 clauses \Rightarrow 1 formula

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Desugaring step 1: eliminate throws and returns

6.170 style:
requires (unchanged)
modifies (unchanged)
throws
effects
returns } correspond to resulting "effects"


Example (from `java.util.Vector<T>`):

```
// requires: true
// modifies: this[index]
// throws: IndexOutOfBoundsException if index < 0 || index ≥ size()
// effects: this_post[index] = element
// returns: this_pre[index]
T set(int index, T element)
```

Desugaring (after step 1):

```
// requires: true
// modifies: this[index]
// effects: if index < 0 || index ≥ size() throws IndexOutOfBoundsException
//           else this_post[index] = element && returns this_pre[index]
T set(int index, T element)
```

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Desugaring step 2: eliminate requires and modifies


Single logical formula
 $\text{requires} \Rightarrow ((\text{not-modified}) \ \&\& \ \text{effects})$
 "not-modified" preserves every field not in modifies clause
 If precondition is false, formula is true
 Recall: $\forall x. x \Rightarrow \text{true}; \forall x. \text{false} \Rightarrow x; (x \Rightarrow y) \equiv (\neg x \vee y)$

Example:

```
// requires: true
// modifies: this[index]
// effects: E
T set(int index, T element)
```

Result:
 $\text{true} \Rightarrow ((\forall i \neq \text{index}. \text{this_pre}[i] = \text{this_post}[i]) \ \&\& \ E)$

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Transition relations (comparison technique 3)

Transition relation relates prestates to poststates
 Contains all possible <input,output> pairs


Transition relation maps procedure arguments to results

```
int increment(int i) {
    return i+1;
}

double mySqrt(double a) {
    if (Random.nextBoolean())
        return Math.sqrt(a);
    else
        return - Math.sqrt(a);
}
```

Specifications have transition relations, too
 Contains just as much information as other forms of specification

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Satisfaction via transition relations

A stronger specification has a smaller transition relation

Rule: P satisfies S iff P is a subset of S
(when both are viewed as transition relations)

Example

Specification


```
int sqrt (int x)
  requires x is a perfect square
  returns positive or negative square root
Transition relation: <0,0>, <1,1>, <1,-1>, <4,2>, <4,-2>, ...
```

Code

```
int sqrt (int x) {
  // ... always returns positive square root
}
Transition relation: <0,0>, <1,1>, <4,2>, ...
```

P satisfies S because P is a subset of S

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Transition relations in abbreviated form

Problem: “P satisfies S iff P is a subset of S” gives the **wrong answer** for transition relations written in abbreviated form

Specification


```
int anyOdd (int x)
  requires x = 0
  returns any odd integer
Transition relation: <0,1>, <0,3>, <0,5>, <0,7>, ...
```

Code

```
int anyOdd (int x) {
  return 3;
}
Transition relation: <0,3>, <1,3>, <2,3>, <3,3>, ...
```

The code satisfies the specification, but the rule says it does not. P is not a subset of S
because <1,3> is not in the specification’s transition relation

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Rule for abbreviated transition relations, option 1

The transition relation should make explicit everything an implementation may do

Observation: abbreviated transition relation for S does not indicate all possibilities

Expand the transition relation for S:

Specification

```
int anyOdd (int x)
  requires x = 0
  returns any odd integer
Transition relation: <0,1>, <0,3>, <0,5>, <0,7>, ...
<1,0>, <1,1>, <1,-1>, <1,2>, ..., <1,exception>, <1,infinite loop>, ...
<2,0>, <2,1>, <2,2>, ..., <2,exception>, <2,infinite loop>, ...
```

Code


```
int anyOdd (int x) {
  return 3;
}
Transition relation: <0,3>, <1,3>, <2,3>, <3,3>, ...
```

The rule now gives the right answer: P satisfies S iff P is a subset of S

Downside: writing the full transition relation is bulky and inconvenient

It’s more convenient to make the implicit notational assumption:
For elements not in the domain of S, any behavior is permitted.
(Recall that a relation maps a *domain* to a *range*.)

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Rule for abbreviated transition relations, option 2

P satisfies S iff $P \upharpoonright (\text{Domain of S})$ is a subset of S
where “ $P \upharpoonright D$ ” = “P restricted to the domain D”
i.e., remove from P all pairs whose first member is not in D
(recall that a relation maps a *domain* to a *range*)

Try our example again:

Specification

```
int anyOdd (int x)
  requires x = 0
  returns any odd integer
Transition relation: <0,1>, <0,3>, <0,5>, <0,7>, ...
```


Code

```
int anyOdd (int x) {
  return 3;
}
Transition relation: <0,3>, <1,3>, <2,3>, <3,3>, ...
```

Domain of S = { 0 }
 $P \upharpoonright (\text{domain of S}) = \{ \langle 0,3 \rangle \}$, which is a subset of S, so P satisfies S

The new rule gives the right answer even for abbreviated transition relations
We’ll use this version of the notation in 6.170

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Abbreviated transition relations, summary

The abbreviated version of the transition relation can be misleading


The true transition relation contains all the pairs

When doing comparisons

Use the expanded transition relation, or
Restrict the domain when comparing

Either approach makes the “smaller is stronger rule” work

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Review: strength of a specification

A stronger specification is satisfied by fewer procedures

A stronger specification has

- weaker preconditions (note contravariance)
- stronger postcondition
- fewer modifications

Advantage of this view: can be checked by hand


A stronger specification has a (logically) stronger formula

Advantage of this view: mechanizable in tools

A stronger specification has a smaller transition relation

Advantage of this view: captures intuition of “stronger = smaller” (fewer choices)

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Specification style

Typically have only one of effects and returns
A procedure has a side effect or is called for its value
Exception: return old value, as for `HashMap.put`

The point of a specification is to be helpful
Formalism helps, overformalism doesn't

A specification should be
coherent (not too many cases)
informative (bad example: `HashMap.get`)
strong enough (to do something useful, to make guarantees)
weak enough (to permit (efficient) implementation)

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Checking preconditions

Checking preconditions

- makes an implementation more robust
- provides better feedback to the client
- avoids silent errors

A quality implementation checks preconditions whenever it is *inexpensive and convenient* to do so

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