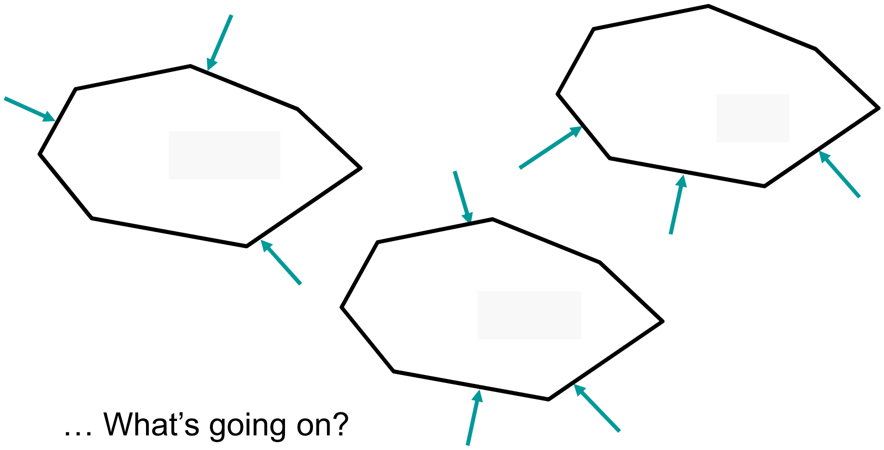
Manipulation: Mechanisms, Grasping and Inverse Kinematics

RSS Lectures 14 & 15
Monday / Wednesday, 31 March / 2 April 2014
Prof. Seth Teller

MIT HKN is hosting another study break TODAY
with cupcakes from Georgetown Cupcakes!
3pm until we run out at the Chu Lounge (38-201).

Force-Direction Closure

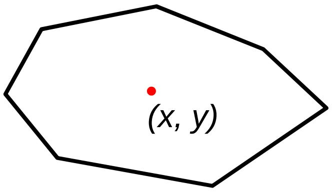
- Under what conditions will a set of point contact forces resist arbitrary planar *translation*?



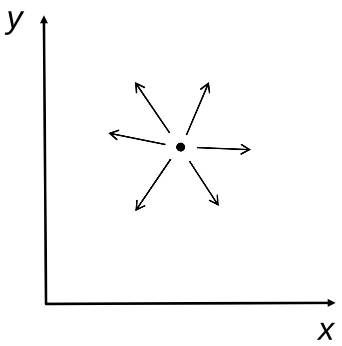
… What's going on?

How many contacts are needed?

- Analyze situation in c-space with DOF argument
  - First: how many c-space DOFs for object origin?



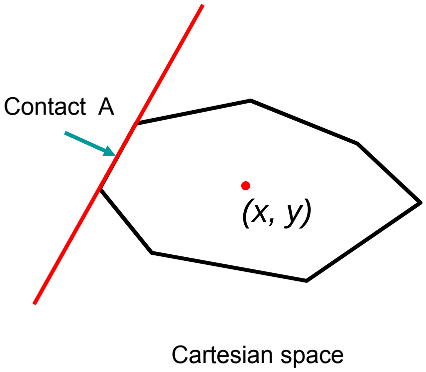
Cartesian space



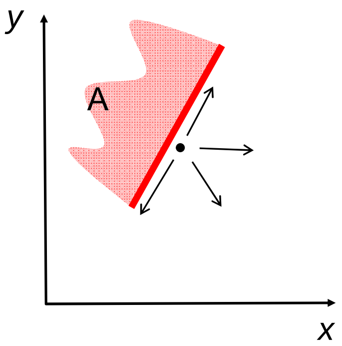
Configuration space

How many contacts are needed?

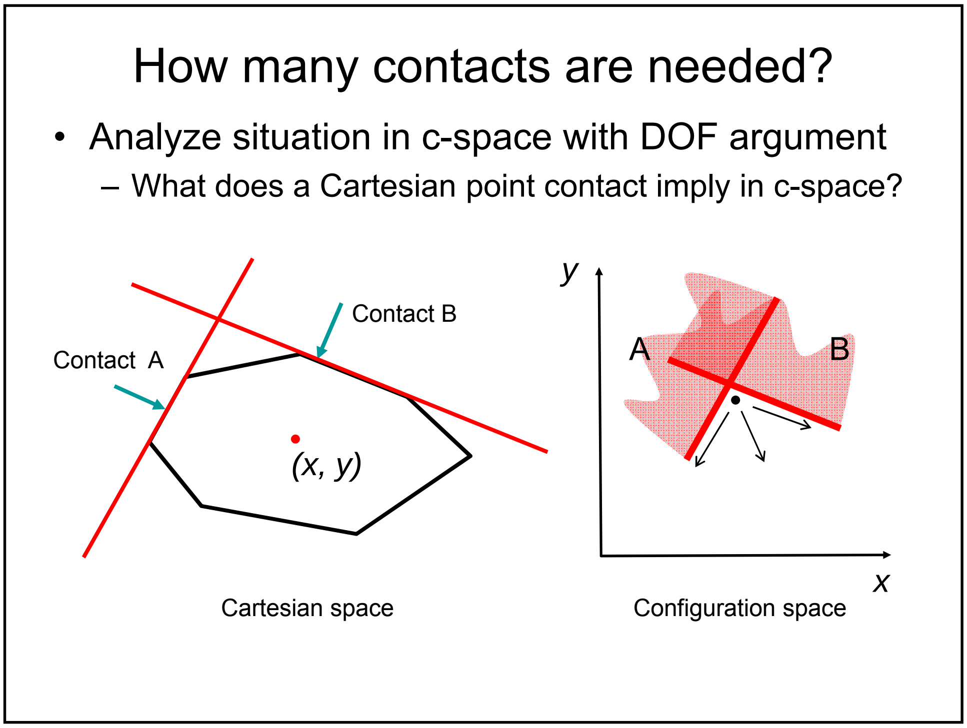
- Analyze situation in c-space with DOF argument
  - What does a Cartesian point contact imply in c-space?



Cartesian space



Configuration space



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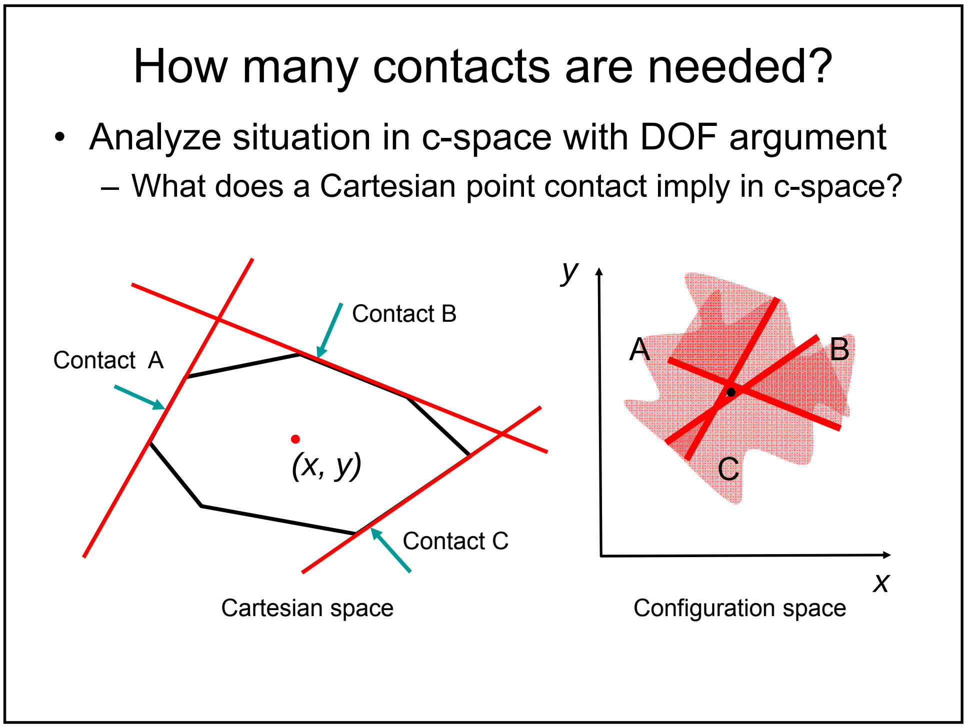
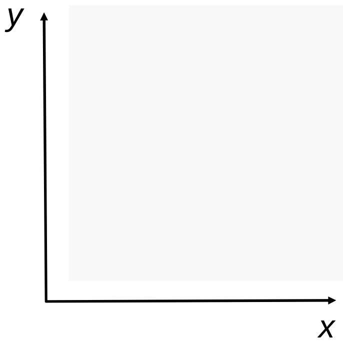


How many contacts are needed?

- Analyze situation in c-space with DOF argument
  - What does a Cartesian point contact imply in c-space?

DOF Counting for Translation

- Conclude that contacts are needed in general
  - Are there situations in which more are required?



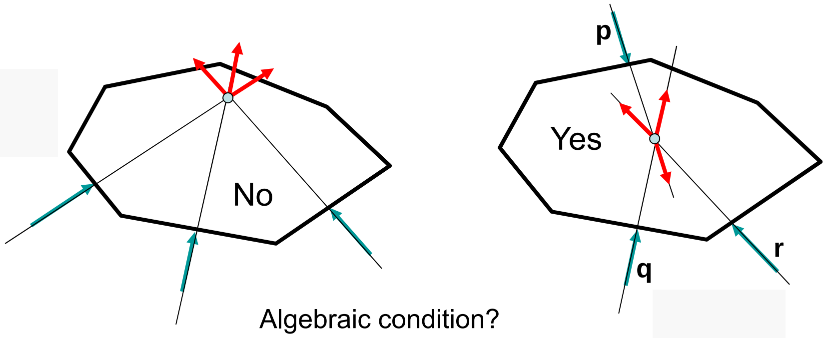
Cartesian space

Configuration space

  - Example of

Conditions for Force-Direction Closure

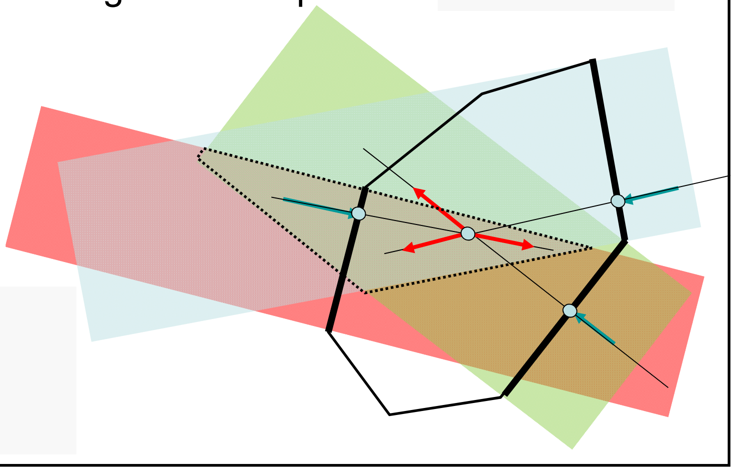
- Force vectors must
- Some positive combination of forces



Algebraic condition?
For force vectors **p**, **q**, **r**,
there must exist $\alpha, \beta, \gamma > 0$
s.t.

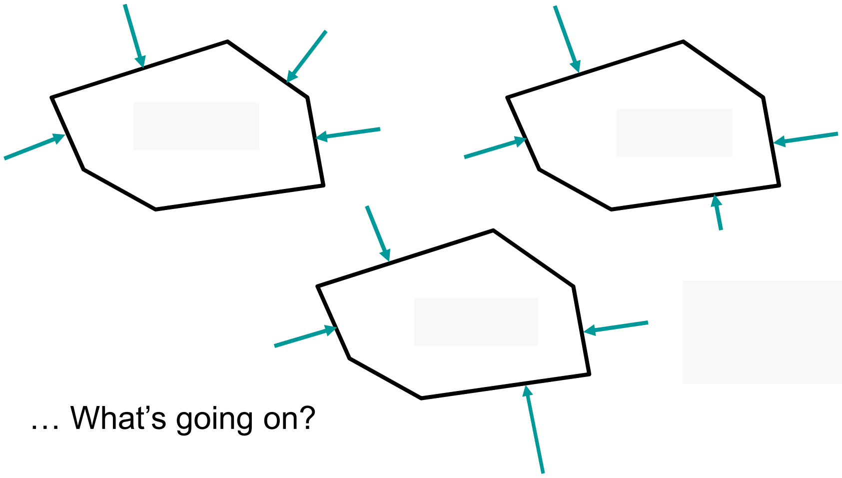
Synthesizing a Force-Direction Grasp

1. Choose admitting a
2. Project onto per-edge
3. Scale force magnitudes to produce



Torque Closure

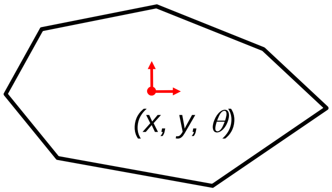
- Under what conditions will a set of point contact forces resist arbitrary planar *rotations*?



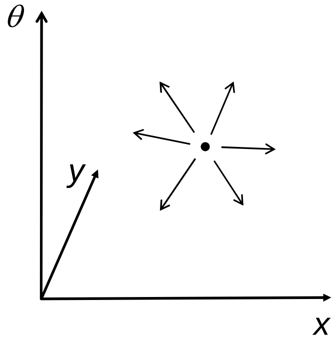
… What's going on?

How many contacts to “pin” rotation?

- Use analogous DOF argument in c-space
  - First: how many c-space DOFs for object pose?



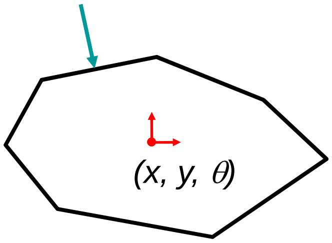
Cartesian space



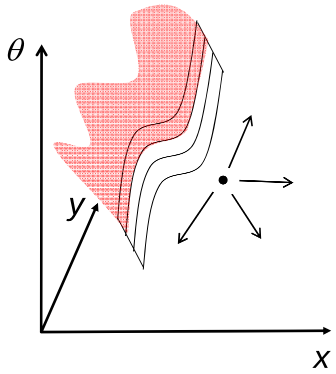
Configuration space

How many contacts to pin rotation?

- Introduce point contact in Cartesian space
  - Implies c-space constraint with 2D manifold boundary



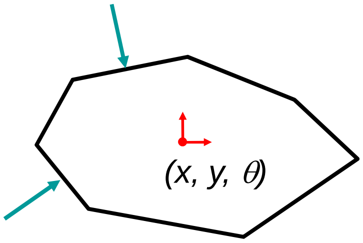
Cartesian space



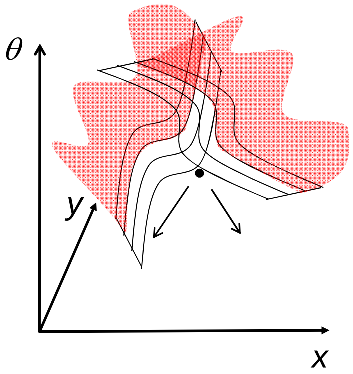
Configuration space

How many contacts to pin rotation?

- Introduce point contact in Cartesian space
  - Implies c-space constraint with 2D *manifold* boundary



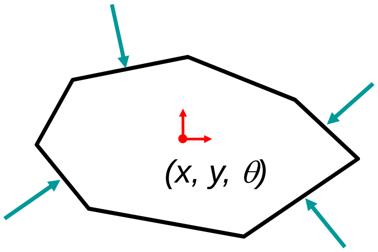
Cartesian space



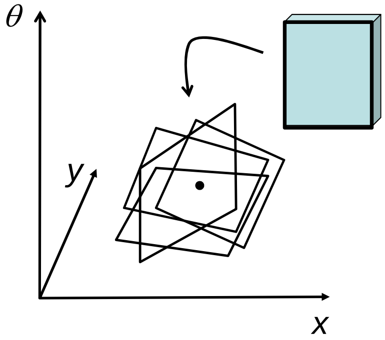
Configuration space

How many contacts to pin rotation?

- Locally, each constraint has a planar boundary
  - … So, how many *halfspaces* needed to pin point?



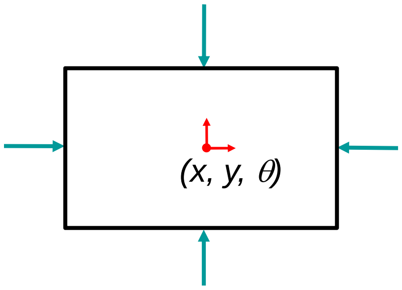
Cartesian space



Configuration space

Are There Degeneracies?

- Polygon with sides not in general position…
- Might we need more? What about ?



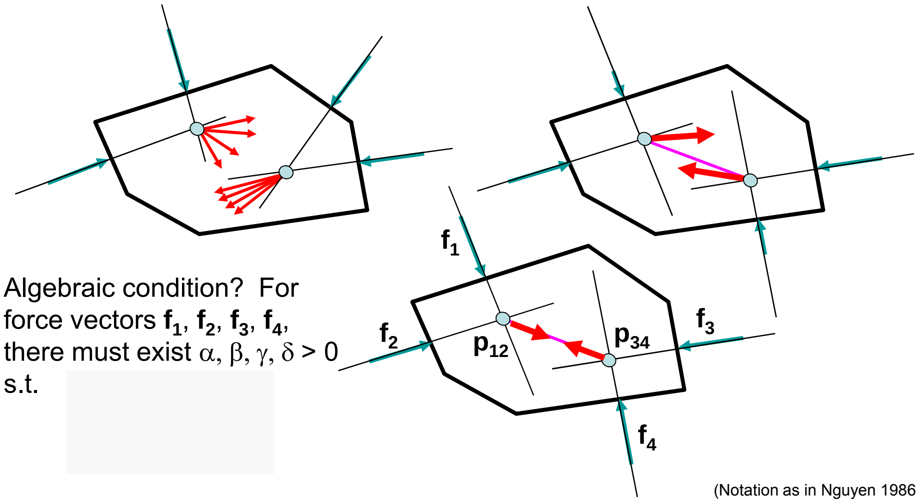
Cartesian space

Cartesian space

- For polyhedra in 3D: need
  - *Frictionless* contacts cannot pin

Conditions for Torque Closure

- Each normal cone must contain the other's apex
- Pairwise effective forces must cancel each other



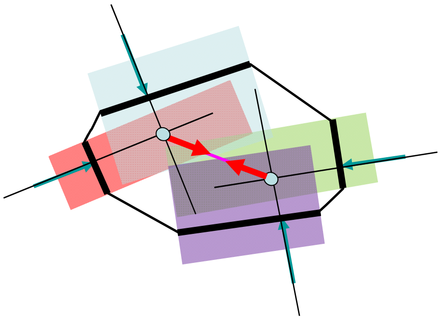
Algebraic condition? For force vectors $\mathbf{f}_1$, $\mathbf{f}_2$, $\mathbf{f}_3$, $\mathbf{f}_4$, there must exist $\alpha, \beta, \gamma, \delta > 0$ s.t.

(Notation as in Nguyen 1986)

Synthesizing a Torque-Closure Grasp

1. Choose two edge pairs* admitting force centers
2. Choose centers inducing mutual normal cones
3. Project centers to respective edge contact points
4. Scale forces to produce alignment, cancellation

*Edge pairs need not be contiguous

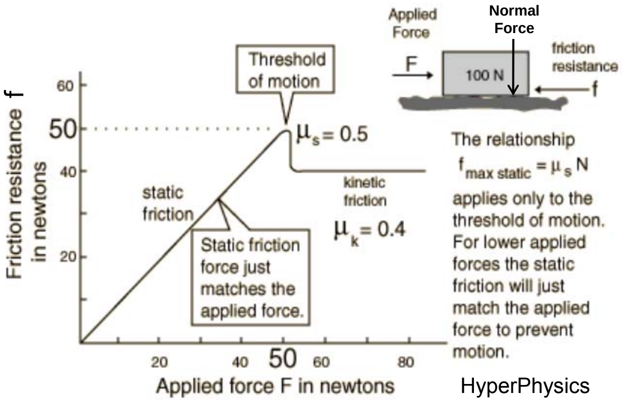


Does rotation closure imply translation closure?

Kinetic and Static Friction ("Stiction")

$F_f <= \mu_s * F_n$ (at rest): coefficient of *static* friction μ_s

$F_f <= \mu_k * F_n$ (moving): coefficient of *kinetic* friction μ_k



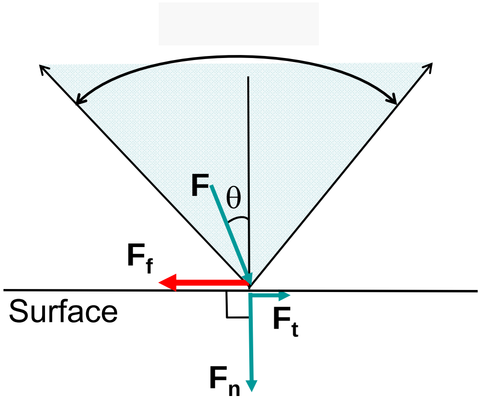
The relationship $f_{max\ static} = \mu_s N$ applies only to the threshold of motion. For lower applied forces the static friction will just match the applied force to prevent motion.

HyperPhysics

(Stiction makes things difficult both for humans and robots. Why?)

Point Contact with Friction

- Consider a point contact exerting force **F** at an angle θ to the surface normal. What happens?



For contact at rest,

$|\mathbf{F_t}| < |\mathbf{F_f}| = \mu\, |\mathbf{F_n}|$

At critical angle θ_{crit},

$|\mathbf{F_t}| =$

Substituting gives

$|\mathbf{F}| \sin \theta_{crit} =$

Which yields

$\mu =$

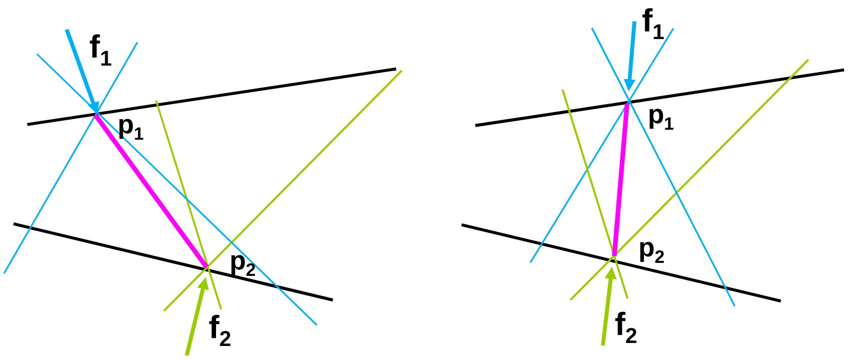
So that

$\theta_{crit} =$

- Produces of directions, s.t. point will not when **F** is applied

Grasp Analysis With Friction

Consider forces $\mathbf{f_1}$, $\mathbf{f_2}$ at frictional contacts $\mathbf{p_1}$, $\mathbf{p_2}$



When can $\mathbf{f_1}$, $\mathbf{f_2}$ oppose one another without sliding?

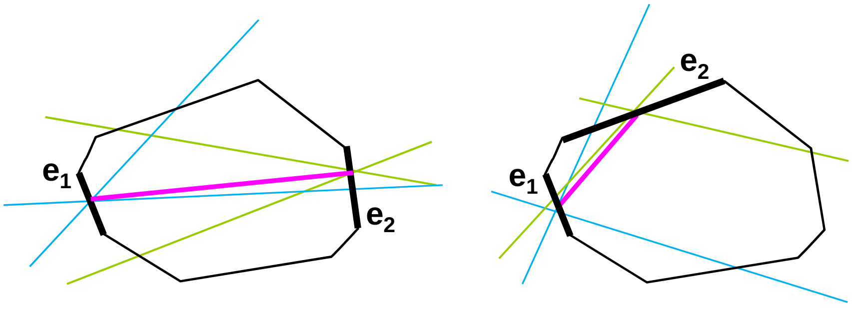
Each force must

Point $\mathbf{p_1}$ (resp. $\mathbf{p_2}$) must

Grasp Synthesis With Friction

Choose a *compatible* pair of edges $\mathbf{e_1}$, $\mathbf{e_2}$

Intuition? Using what data? How to choose?

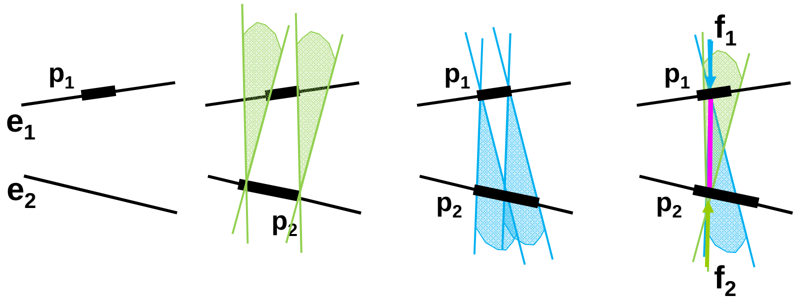


Grasp Synthesis With Friction

Choose target region for contact point $\mathbf{p_1}$

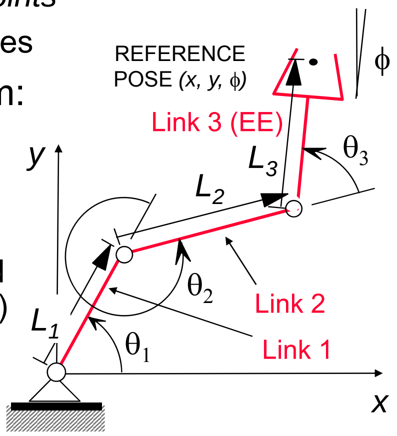
Determine feasible target region for contact $\mathbf{p_2}$

Orient and scale $\mathbf{f_1}$, $\mathbf{f_2}$ so as to cancel along $\overline{\mathbf{p_1 p_2}}$



Forward and Inverse Kinematics

- So far, have cast computations in Cartesian space
- But manipulators controlled in *configuration space:*
  - Rigid *links* constrained by *joints*
  - For now, focus on joint values
- Example 3-link mechanism:
  - Joint coordinates θ_1, θ_2, θ_3
  - Link lengths L_1 , L_2 , L_3
- End effector coordinates
  - "Reference pose" described by x, y, and ϕ (w.r.t. vertical)
- How can we relate EE to configuration variables?

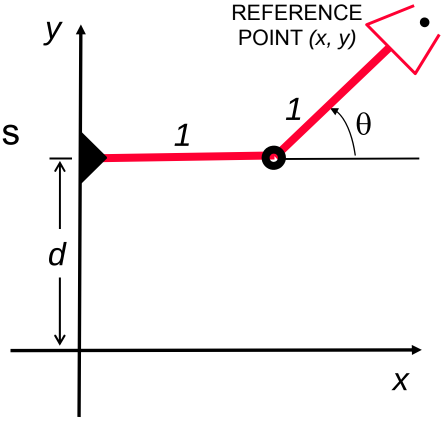


Forward Kinematics

- Given mechanism description and joint values, express end effector pose in Cartesian coordinates
  - Example: two-link arm with one sliding, one rotating joint
- Configuration variables:
  - Joint coordinates d, θ
  - Link lengths (both 1)
- End effector coordinates
  - "Reference point" (x, y)
- Challenge: express as

$x = x\,(d, \theta) =$

$y = y\,(d, \theta) =$



Inverse Kinematics

- Given end effector pose in Cartesian coordinates, identify the joint values that *yield* the desired pose
- Challenge: solve for joint values in terms of pose

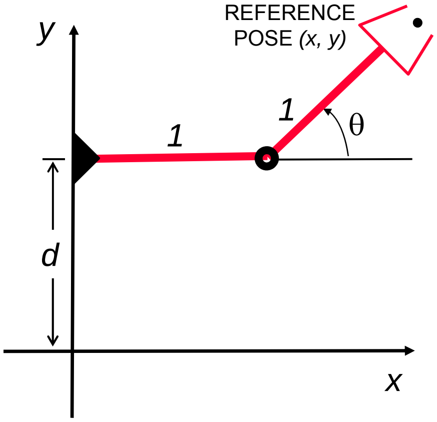
$\theta = \theta(x, y)$

$=$

$d = d(x, y)$

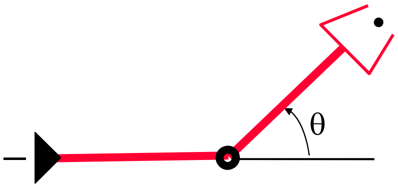
$=$

Hints:



Why is IK difficult?

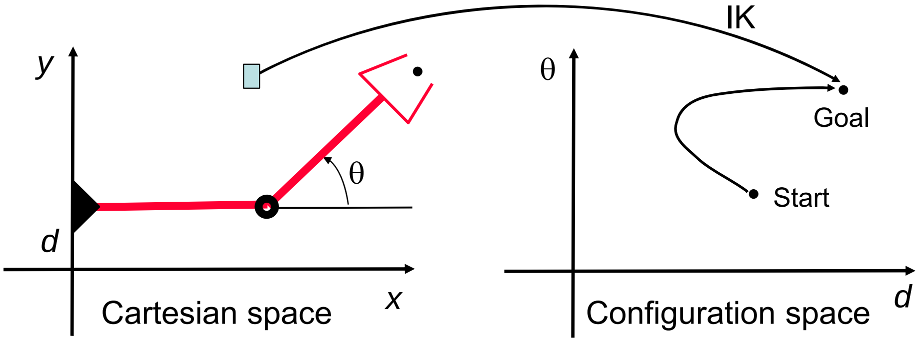
- Nonlinear
  - Revolute joints → inverse trigonometry



- Multi-valued
  - Often multiple solutions for a single Cartesian pose
- Discontinuities and singularities
  - Can lose one or more DOFs in some configurations
- Possibly over-constrained (no exact solution)
  - Use of approximation and iterative algorithms
- Dynamics
  - In reality, want to apply forces and torques (while respecting physical constraints), not just move arm!

Putting it All Together: Grasping

- Input workspace, obstacles, and manipuland:
  - Determine a feasible grasp (set of contact points)
  - Use IK to solve for target end-effector pose in c-space
  - Plan a collision-free reach to the computed pose
  - Control end-effector along desired trajectory



What have we swept under the rug?

- Sensing
  - Shape, pose of target object, accessibility of surfaces
  - Classification of material type from sensor data
  - Freespace through which grasping action will occur
- Material properties
  - Estimation of object’s mass, moments, friction coefficients
  - Internal, articulated, passive vs. active degrees of freedom
- Uncertainty & compliance
  - Tolerate noise inherent in sensing and actuation
  - Ensure that slight sensing, actuation errors won’t cause damage
  - Handle soft fingers making contact over a finite (not zero) area
- Dynamics
  - All of the above factors may be changing in real time