

# Navigation: Mapping

RSS Lecture 13

Wednesday 19 March 2014

Prof. Teller

Text: Siegwart and Nourbakhsh Ch. 5, 6

Dudek and Jenkin Ch. 8

## Navigation Overview

- Where am I?
  - Localization (Lecture 12)
  - Assumes perfect map, imperfect sensing
- How can I get there from here?
  - Planning (Lectures 8-10)
  - Assumes perfect map, sensing, and actuation
- What did I observe during my excursion?
  - Mapping (Today)
  - Assumes perfect localization, noisy sensing
- Can I build a map *and* localize in it, on-line?
  - Yes; using SLAM
  - Assumes no prior knowledge of the world

## Lecture Overview

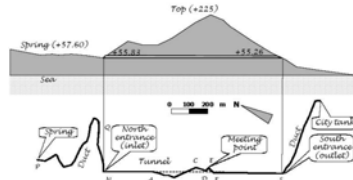
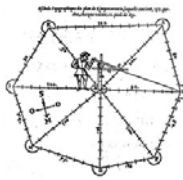
- What are maps?
- Map representations
- Fusing observations
- Uncertainty: noise and outliers
- Feature and free-space complexity

## What are maps?

- Collection of elements or features at some scale of interest, and a representation of the geometric and/or topological relationships among them
- Also *semantic information (metadata)*
  - Segmentation, place/object naming, function, etc.
- We will focus on *geometry* and *topology*
  - But *semantics* are also critical in real-world applications!

# History

- Early surveying, mapping methods:
  - Egyptians (c. 1400 B.C.): Nile floods, taxation
    - Plumb bobs, sighting instruments, area measurement
  - Greeks (c. 550 B.C.): Trade, warfare, engineering
    - Coastal, nautical maps for marine navigation
    - Dug Eupalinos tunnel from both ends, 1036m long!
  - Europeans (16<sup>th</sup> century onward): foundational computational methods
    - Gauss, method of least squares (1809)



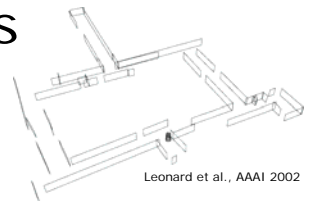
Demetris Koutsoyiannis Triangulation of Hanover, 1820-1850

# Why maps? From where?

- Essential for a wide variety of human, robotic activities (localization, planning)
- Maps are highly labor-intensive to create:
  - Exploration (global coverage)
  - Measurement (local coverage)
  - Validity (correctness, error bounds)
  - Currency (freshness)
  - As-planned vs. as-built building models
  - On top of all that: metadata/semantics ...
- Map creation is an ideal robotics task!
  - Achieving a robust, sustained, large-area, fully autonomous mapping capability has been an “open” (i.e., unsolved) problem for decades

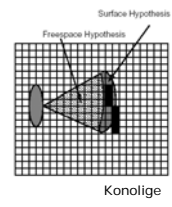
# Some robot map types

- Continuous / "vector" format
  - Points, linear or curved segments, surface patches

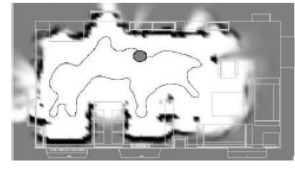


Leonard et al., AAAI 2002

- Discrete / "raster" format
  - Occupancy grids

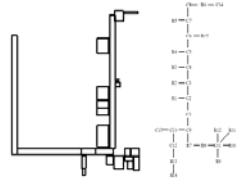


Konolige



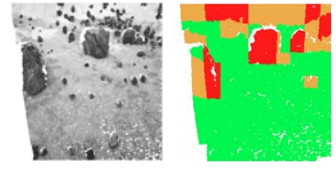
Chatila, SSS 2004

- Metrical / Topological



Metrical / Topological

- Global / Local



Chatila, SSS 2004

- Hybrid

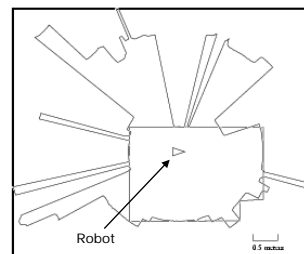
Local, Metrical, Qualitative

# Commonly used range sensors

Polaroid sonar ring  
12 range returns,  
one per 30  
degrees, at ~4 Hz



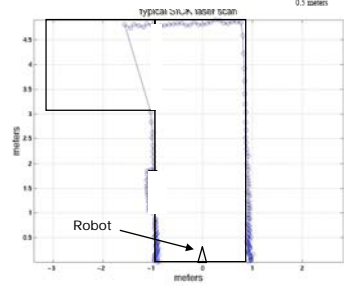
(+ servoed rotation)



SICK laser scanner  
180 range returns,  
one per degree,  
at 5-75 Hz



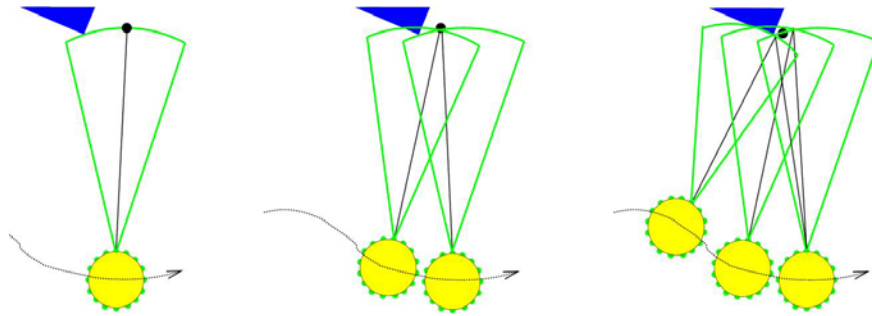
→



Other possibilities: Stereo/monocular vision; Robot body (e.g. bump/stall sensing)

## Fusing multiple returns

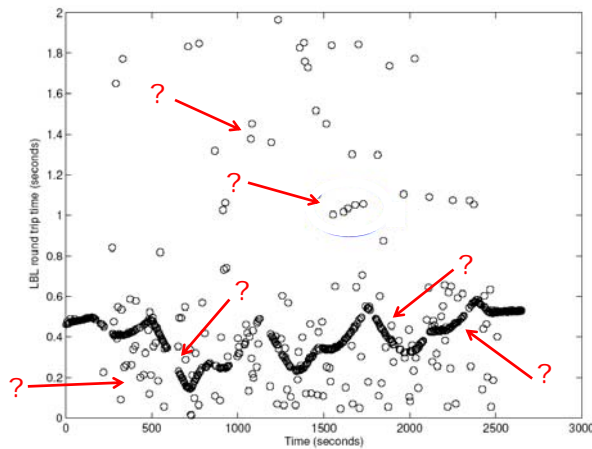
- Crucial assumption: pose estimation (e.g., odometry, dead reckoning) is accurate over short times and distances



- Can then localize features using conventional triangulation (sonar beam width complicates things)

## Digression: sensing challenges

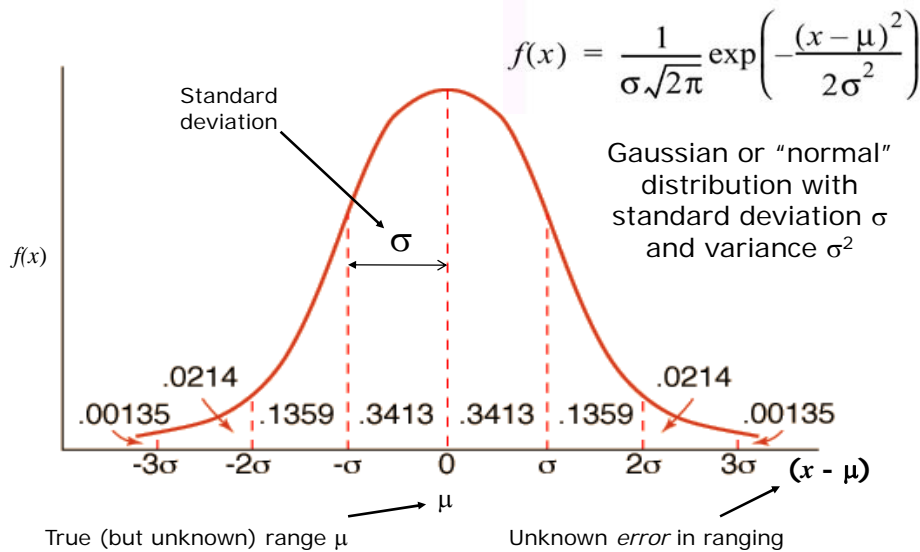
- Time series of round-trip-time to one acoustic beacon for an underwater autonomous vehicle



(Olson, Leonard, Teller, Robust Range-Only Beacon Localization, IEEE AUV, June 2004)

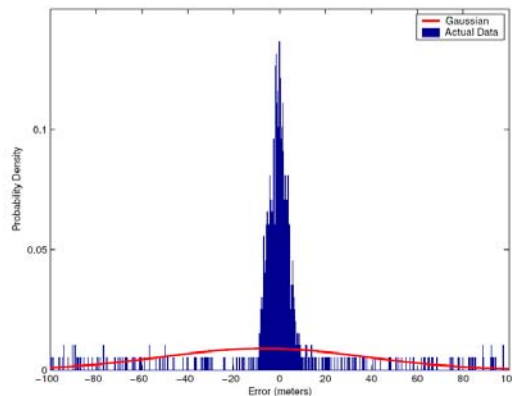
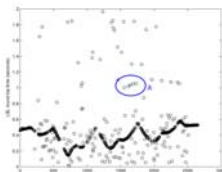
# Gaussian noise model

- Measurement returns a corrupted value



# Outliers

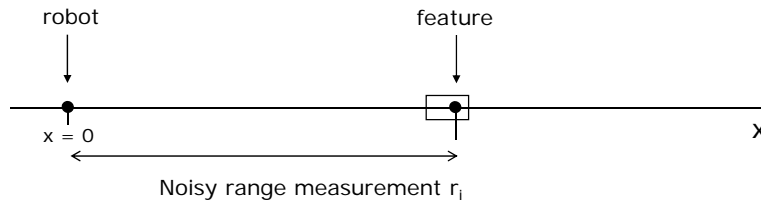
- Many measurements are *outliers*; their frequency is not well-modeled by a Gaussian distribution



... what to do?

## Filtering

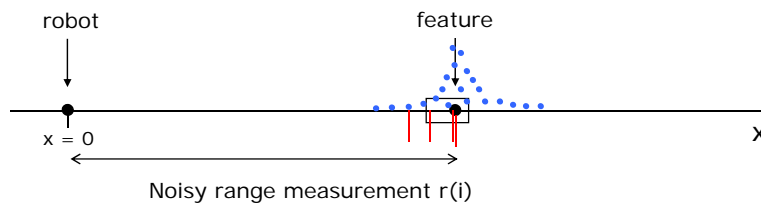
- Consider one-dimensional localization:
  - Robot measures range  $r(i)$  at  $i^{\text{th}}$  time step
  - Ranges *corrupted* by Gaussian noise, outliers



- *Filter* measurements; combine over time
  - Incorporate each measurement as it arrives
  - *Recursive (on-line)* filtering (contrast *batch*)

## Filtering with no outliers

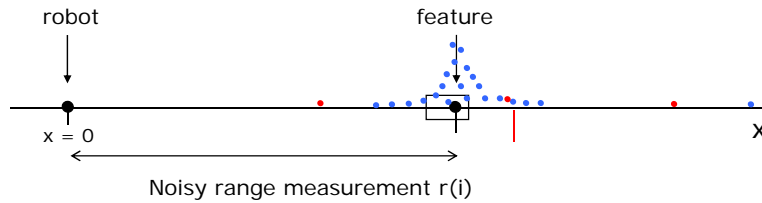
- Suppose neither robot nor feature moves
  - What should our filtering strategy be?
  - Call  $x(t)$  our *estimate* of  $x$  after  $t$  time steps



- Compute the mean (arithmetic average)
  - $x(i) = (r(1) + r(2) + \dots + r(i)) / i$  (batch)
  - $x(i) = [x(i-1) * (i-1) / i] + [r(i) / i]$  (on-line or "recursive")
  - ... if no outliers, no change over time, filter is optimal
- Computational complexity of each update?

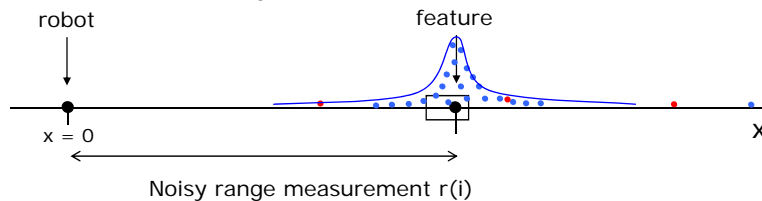
## Handling outliers

- Suppose a fraction of  $r(i)$  are wildly wrong
  - Classify  $r(i)$  as *inliers* ● or *outliers* ●
  - How to do this?



## Modeling measurement noise

- Estimate sample *variance* as well as mean



- Reject unlikely samples (e.g.,  $p < 1\%$ )
  - Filter only inliers, by averaging as before
- ... But where does variance come from?
  - Determine it *a priori* (e.g. from bench tests)
  - Or, estimate it *on-line*, in addition to mean
    - Chicken-and-egg problem (could be unlucky)
    - If “outliers” become frequent, what can you do?

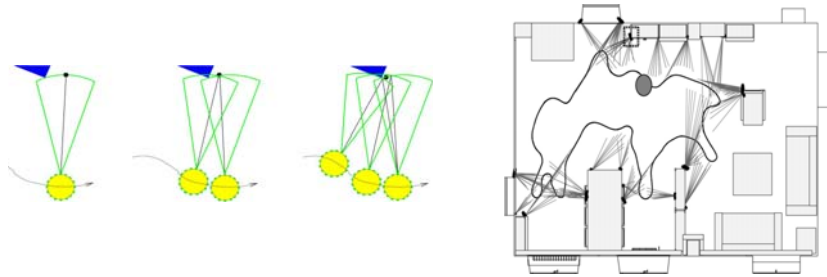


## Estimating variance

- Define  $\sigma^2(i)$  as variance after  $i$  steps
- Batch computation:
  - As before,  $x(i)$  is the mean after  $i$  steps
  - Then variance  $\sigma^2(i)$  is  $[\sum(r(i)-x(i))^2] / i$
- Recursive (on-line) computation:
  - Estimate  $x(i)$  recursively as before
  - Define  $\sigma^2(1) = 0$ ; then for  $i > 1$ :

$$\sigma^2(i) = \frac{(i-1)}{i} * \sigma^2(i-1) + \frac{1}{i} * (r(i) - x(i))^2$$

## Local vs. global data fusion



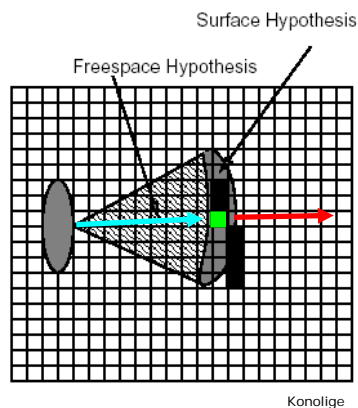
- Crucial assumption: that robot can solve strong localization (global pose estimation) throughout
- This is a very difficult problem without a map! (It's difficult even *with* a map or a partial map.)
- SLAM: Simultaneous Localization and Mapping
- For now, we assume localization; o/wise, need SLAM

## Representation considerations

- We want our robot to be able to plan and execute high-level motions among obstacles
- What do we want from our map?
  - Consistent global, or locally metrical, coordinate system
  - Identification and localization of substantial *features*, e.g., obstacles that may hinder or damage the robot
  - All of this should be well-defined and computationally accessible (data model, data structure, API)
  - Scalability (reasonable search, access times as exploration continues, and map gets really large)
- ... Is that all we need/want from a map?

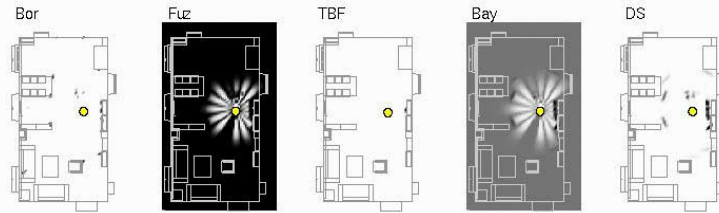
## Alternative 1: Discretize

- *Occupancy grid of cells*
  - Regular subdivision of region
  - Models free & occupied space
- Cells accumulate *evidence* of presence of obstacle surface
- Grid is updated on-line with recent measurements
- Range return from obstacle implies three grid intervals:
  - From robot to obstacle (FS)
  - At (quantized) obstacle depth
  - Beyond obstacle (from robot's point of view)



## Many occupancy grid methods

- Example: sonar data, varying update rules
  - White: freespace; black: obstacle; grey: unknown



Wijk 2001

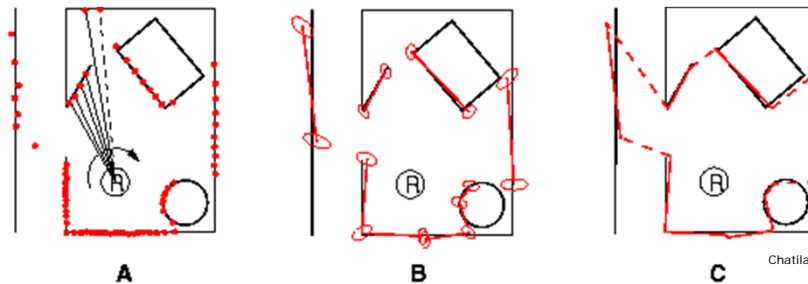
- Bor: Histogramic (Borenstein 1991); accumulates hits
- Fuz: Fuzzy (Zadeh 1973; Ribo and Pinz 1999); with weights
- TBF: Triangulation-Based Fusion (Wijk 2000); local triangulation
- Bay: Bayesian (Elfes 1988); probabilistic occupancy/emptiness
- DS: Dempster-Shafer (Shafer 1976; Pagac 1996); with "ignorance"

## Pitfalls of occupancy grids

- Quantization error
  - Cells too large: not faithful to environment or robot task
  - Cells too small: too numerous (expensive) to process efficiently
  - Task-dependent: grid size can be simultaneously too small *and* too large!
- Blurring
  - Caused by pose estimation error, sensor uncertainty, grid quantization

## Alternative 2: Line Features

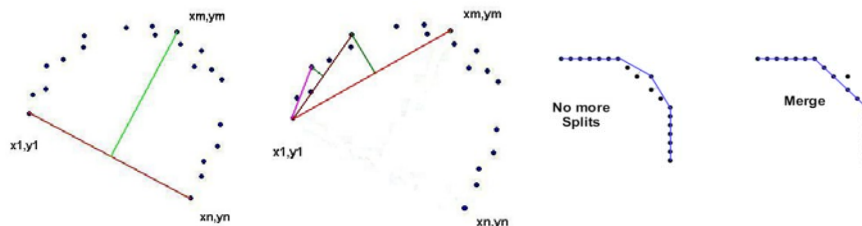
- Piecewise linear approximation of sequence of point features (i.e., ranges)



- How are individual ranges, point features grouped into useable line segments?
- How to counteract noise inherent in data?

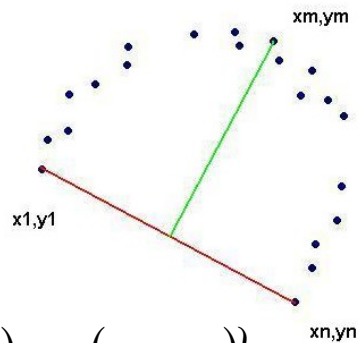
## Split, Merge, Fit algorithm

- Used for *ordered sets* of laser or sonar returns
- Takes two thresholds: split distance, merge angle
- Split phase:
  - Recursively split until (max) distance criterion is met
- Merge phase:
  - Merge adjacent segments until (min) angle criterion is met
- Fit phase (perhaps with explicit outlier handling):
  - Fit line segments to resulting (noisy) point sequences



## Split phase

- Given points  $P = \{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$
- Find  $(x_m, y_m)$ : point with maximum distance to line  $L = \{(x_1, y_1), (x_n, y_n)\}$

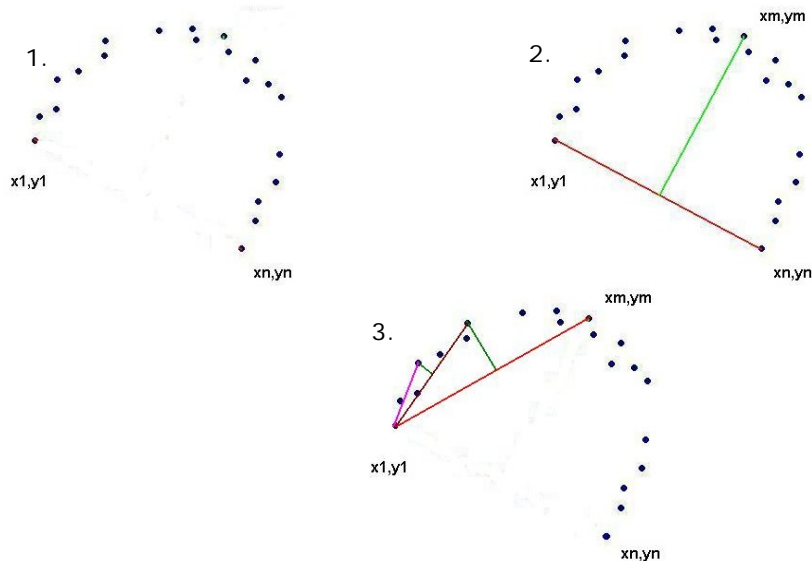


- Split into two subsets:

$$P' = \{(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)\}$$

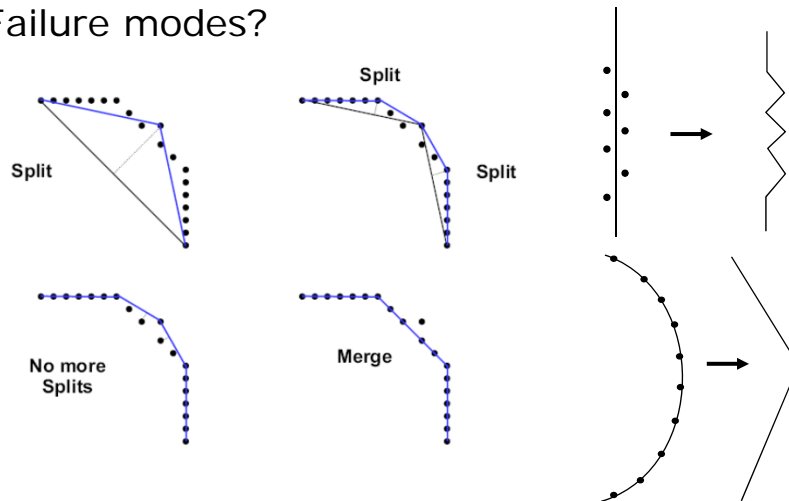
$$P'' = \{(x_m, y_m), (x_{m+1}, y_{m+1}), \dots, (x_n, y_n)\}$$

## Splitting is recursive



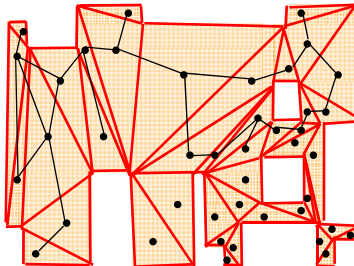
## Segment merging phase

- Merge adjacent segments if nearly collinear
- Failure modes?



## Storing extracted features

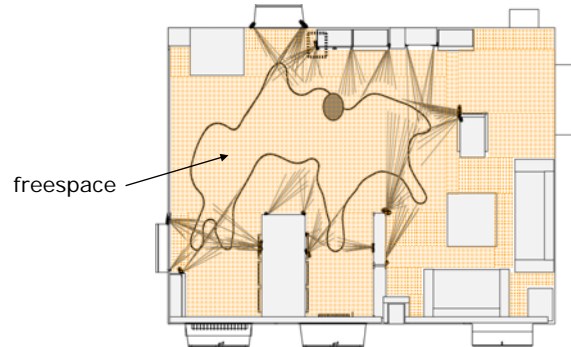
- Store as linear list
  - Advantage: very simple. Drawbacks: ?
- Or, store in *proximity data structure*
  - E.g., constrained Delaunay triangulation



- CDT has many nice properties:
  - Linear size; logarithmic search; temporal coherence; maximum minimum angle; dual to Voronoi diagram; etc.

## Alternative 3: Free-space Map

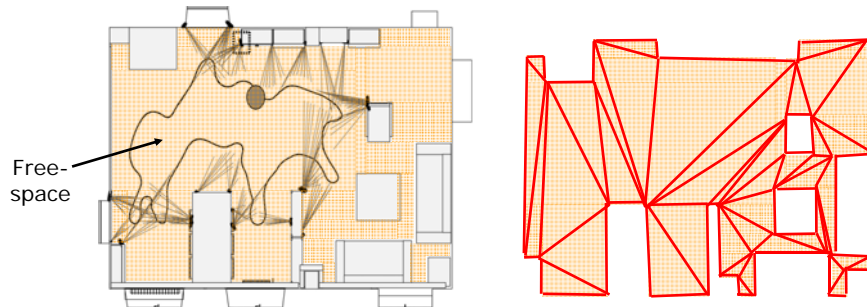
- Robot spends most time well away from obstacles



- Call this area "free-space," *i.e.*, the region through which the robot can expect to be free to move
- The *complement* of the union of all *obstacles*

## Free-space complexity

- It's empty, but that doesn't mean its representation is compact! What's the *descriptive complexity* of FS?



- Free-space is *more complex* than obstacle union  $n$ 
  - 2D simple polygon (no holes):
  - 2D segments:
  - 3D polyhedron:

## Mapping summary

- Maps are critical to many tasks
- Assumed localization for now
- Saw several map representations, data fusion algorithms
- Considered scaling requirements

## Coming Up In RSS

- Today in lab:
  - Final Lab briefings
  - Teamwork on CDO's (due **Friday 1pm**)
- Friday
  - No forum
- Next week
  - Spring Break! Get some R&R.