Manipulation: Mechanisms, Grasping and Inverse Kinematics

RSS Lectures 14 & 15
Monday & Wednesday, 1 & 3 April 2013
Prof. Seth Teller

Overview

• Mobility and Manipulation
  – Manipulation Strategies
• Mechanism Analysis
  – Instantaneous Center
  – Reuleaux’s Method
• Multi-Finger Manipulation
  – Grasp Analysis
  – Grasp Synthesis
  – Forward Kinematics
  – Inverse Kinematics
  – Grasp Planning
• Lab 7 Preview
Mobility / Manipulation Duality

- **Mobility:**
  - Earth is fixed
  - Legs apply forces to earth
  - Reaction forces move body

- **Manipulation:**
  - Body is fixed to earth
  - Arms apply forces to manipuland
  - Forces move manipuland

- **Goal of Field: Mobile Manipulation**
  - Use of *coordinated whole-body motion* to effect desired manipulation of manipuland, environment
  - Examples: Lifting a sandbag, throwing a baseball, shoveling snow, replacing a ceiling smoke detector

Manipulation by Pushing

- **Stable push:**
  - Motions that keep object in *line* contact w/ manipulator

  - Motion planning, but with additional constraints
Fixturing

- Use of designed pegs, surfaces, prior knowledge of manipuland geometry to achieve desired pose

![Fixturing Diagram](https://via.placeholder.com/150)

- Goldberg’s “part squeezer” ([Try it!](#))

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Soft-finger Manipulation

- Can exploit visual/tactile sensing & feedback

![Soft-finger Manipulation](https://via.placeholder.com/150)
Mobile, Two-handed Manipulation

- Challenges: mass distribution; uncertainty

End Effectors

- The component that usually comes into intentional contact with the manipuland
- Often attached interchangeably to robot arm
  - … like a human hand picking up a specialized tool
- Many designs (here ordered roughly by time)
Manipulation Challenges

CMU robot “Herb”
(Home Exploring Robot Butler, also after Herb Simon)

How can the robot perceive the object’s type and pose?
How can the robot reach for the object?
How can the robot grasp the object?
How can the robot move the object where desired?
… Today we’ll focus on grasping.

Mechanism Analysis

- Given some set of constraints, how can the motion of an object be characterized?
  - Rotating links
  - Sliding links
  - Point contacts

Figures from Mason, MoRM
Rotation Center (RC)

- Consider finite planar displacement of rigid object
  - Some point in the plane is left fixed by displacement
  - This point is called the “rotation center” (RC)
- What if the displacement is a pure translation?
  - Where is the RC?

Instantaneous Center (IC):

- Consider a differential displacement (i.e. velocity)
  - Displacement still has a fixed point; where is it?
- What if the displacement is a pure translation?
  - Where can the IC lie?
Use of IC for Mechanism Analysis

• Example four-bar linkage:
  • Base link
  • Two sliding+rotating links A, B
  • Coupler link connecting AB

• Example four-bar linkage:
  • Base link
  • Four rotating links A, B, C, D
  • Coupler link connecting AB

• Constraints on A, B dictate coupler motion
• IC completely determined; characterizes linkage

IC for Mechanism Analysis (cont.)

• Consider this mechanism:
  – IC is

• Another possibility:
  – “False instantaneous center”
Unilateral constraints

• Point contact with boundary of manipuland
• Manipuland cannot violate constraint (but it can separate from it: thus “unilateral”)

• How does this point contact constrain the possible motions of the manipuland?

Reuleaux’s method (1876)

• Each unilateral constraint partitions space of ICs into regions left, right and on line of contact normal

For any IC in this region, only rotations are possible!

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• Why is the “line of contact normal” key to analysis?
  – Along it, differential rotation of either sign is possible (for now, we are assuming frictionless point contacts)
Reuleaux’s method (cont.)

1. Construct line of contact for each contact
2. Label plane regions as $\oplus$ or $\ominus$ w.r.t. this constraint

3. Each remaining region with consistent labels is a locus of possible instantaneous centers
   → Can the IC locus become empty? If so, how?

Multi-Finger Manipulation

- Frictionless contacts
- Force-direction closure
- Torque closure
- Contacts with friction
Frictionless Point Contacts

- Force must be normal to object boundary (why?)
- Force must point into object’s interior (why?)

Force-Direction Closure

- Under what conditions will a set of point contact forces resist arbitrary planar translation?

… What’s going on?
How many contacts are needed?

• Analyze situation in c-space with DOF argument
  – First: how many c-space DOFs for object origin?

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DOF Counting for Translation

- Conclude that contacts are needed in general
  - Are there situations in which more are required?

- Example of degeneracy / degenerate conditions

Conditions for Force-Direction Closure

- Force vectors must
- Some positive combination of forces

Algebraic condition? For force vectors $p, q, r$, there must exist $\alpha, \beta, \gamma > 0$ s.t.
Synthesizing a Force-Direction Grasp

1. Choose contact edges admitting a force center
2. Project force center onto per-edge
3. Scale force magnitudes to produce zero net force

Torque Closure

- Under what conditions will a set of point contact forces resist arbitrary planar rotations?

... What's going on?
How many contacts to “pin” rotation?

• Use analogous DOF argument in c-space
  – First: how many c-space DOFs for object pose?

How many contacts to pin rotation?

• Introduce point contact in Cartesian space
  – Implies c-space constraint with 2D manifold boundary
How many contacts to pin rotation?

• Introduce point contact in Cartesian space
  – Implies c-space constraint with 2D manifold boundary

How many contacts to pin rotation?

• Locally, each constraint has a planar boundary
  – ... So, how many halfspaces needed to pin point?
Are There Degeneracies?

- Polygon with sides not in general position…
- Might we need more? What about circles?
- For polyhedra in 3D: need 7 contacts (6 DOF + 1)
  - Frictionless contacts cannot pin

Conditions for Torque Closure

- Each normal cone must contain the other’s apex
- Pairwise effective forces must cancel each other

Algebraic condition? For force vectors $f_1, f_2, f_3, f_4$, there must exist $\alpha, \beta, \gamma, \delta > 0$ s.t.

(Notation as in Nguyen 1996)
Synthesizing a Torque-Closure Grasp

1. Choose two edge pairs* admitting force centers
2. Choose centers inducing mutual normal cones
3. Project centers to respective edge contact points
4. Scale forces to produce alignment, cancellation

*Edge pairs need not be contiguous

Does rotation closure imply translation closure?

Kinetic and Static Friction ("Stiction")

\[ F_f \leq \mu_s \cdot F_n \] (at rest): coefficient of static friction \( \mu_s \)

\[ F_f \leq \mu_k \cdot F_n \] (moving): coefficient of kinetic friction \( \mu_k \)

(Stiction makes things difficult both for humans and robots. Why?)
Point Contact with Friction

• Consider a point contact exerting force $F$ at an angle $\theta$ to the surface normal. What happens?

For contact at rest, $|F_t| < |F_f| = \mu |F_n|$ At critical angle $\theta_{\text{crit}}$, $|F_f| = \mu |F_n|$ Substituting gives $|F| \sin \theta_{\text{crit}} = \mu |F| \cos \theta_{\text{crit}}$ Which yields $\mu = \frac{\sin \theta_{\text{crit}}}{\cos \theta_{\text{crit}}}$ So that $\theta_{\text{crit}} = \tan^{-1} \mu$

• Produces a friction cone of directions, s.t. point will not slide along surface when $F$ is applied

Grasp Analysis With Friction

Consider forces $f_1$, $f_2$ at frictional contacts $p_1$, $p_2$

When can $f_1$, $f_2$ oppose one another without sliding? Each force must Point $p_1$ (resp. $p_2$) must
Grasp Synthesis With Friction

Choose a compatible pair of edges $e_1, e_2$

Intuition? Using what data? How to choose?

Grasp Synthesis With Friction

Choose target region for contact point $p_1$

Determine feasible target region for contact $p_2$

Orient and scale $f_1, f_2$ so as to cancel along $\overrightarrow{p_1p_2}$
Forward and Inverse Kinematics

- So far, have cast computations in Cartesian space
- But manipulators controlled in configuration space:
  - Rigid links constrained by joints
  - For now, focus on joint values
- Example 3-link mechanism:
  - Joint coordinates $\theta_1, \theta_2, \theta_3$
  - Link lengths $L_1, L_2, L_3$
- End effector coordinates
  - “Reference pose” described by $x, y,$ and $\phi$ (w.r.t. vertical)
- How can we relate EE to configuration variables?

Forward Kinematics

- Given mechanism description and joint values, express end effector pose in Cartesian coordinates
  - Example: two-link arm with one sliding, one rotating joint
- Configuration variables:
  - Joint coordinates $d, \theta$
  - Link lengths (both 1)
- End effector coordinates
  - “Reference point” $(x, y)$
- Challenge: express as $x = x(d, \theta) =$
  $y = y(d, \theta) =$
Inverse Kinematics

- Given end effector pose in Cartesian coordinates, identify the joint values that yield the desired pose.

- Challenge: solve for joint values in terms of pose
  \[ \theta = \theta (x, y) \]
  \[ d = d (x, y) \]

Hints:
\[ x = 1 + \cos \theta \]
\[ y = d + \sin \theta \]
\[ \cos^2 \theta = (x-1)^2 \]
\[ \sin^2 \theta = (y-d)^2 \]
\[ 1 = (x-1)^2 + (y-d)^2 \]
\[ (y-d)^2 = (x-1)^2 - 1 \]

Why is IK difficult?

- Nonlinear
  - Revolute joints → inverse trigonometry

- Multi-valued
  - Often multiple solutions for a single Cartesian pose

- Discontinuities and singularities
  - Can lose one or more DOFs in some configurations

- Possibly over-constrained (no exact solution)
  - Use of approximation and iterative algorithms

- Dynamics
  - In reality, want to apply forces and torques (while respecting physical constraints), not just move arm!
Putting it All Together: Grasping

- Input workspace, obstacles, and manipuland:
  - Determine a feasible grasp (set of contact points)
  - Use IK to solve for target end-effector pose in c-space
  - Plan a collision-free reach to the computed pose
  - Control end-effector along desired trajectory

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What have we swept under the rug?

- Sensing
  - Shape, pose of target object, accessibility of surfaces
  - Classification of material type from sensor data
  - Freespace through which grasping action will occur

- Prior knowledge
  - Estimation of mass, moments, friction coefficients from material
  - Internal, articulated, passive vs. active degrees of freedom

- Uncertainty & compliance
  - Tolerate noise inherent in sensing and actuation
  - Ensure that slight sensing, actuation errors won’t cause damage
  - Handle soft fingers making contact over a finite area (not a point)

- Dynamics
  - All of the above factors may be changing in real time
Confidence vs. Arrogance

Confident:
Having strong belief, firm trust, or sure expectation. [OED]

From Latin *com-* (intensive prefix) + *fidere* “to trust”

Arrogant:
Making or implying strong or unwarrantable claims to dignity, authority or knowledge. [OED]

From Latin *arrogare* “to claim for oneself, assume”