

# Manipulation: Mechanisms, Grasping and Inverse Kinematics

RSS Lectures 14 & 15  
Monday & Wednesday, 1 & 3 April 2013  
Prof. Seth Teller

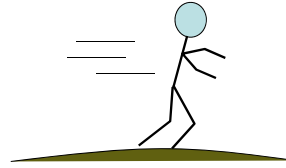
## Overview

- Mobility and Manipulation
  - Manipulation Strategies
- Mechanism Analysis
  - Instantaneous Center
  - Reuleaux's Method
- Multi-Finger Manipulation
  - Grasp Analysis
  - Grasp Synthesis
  - Forward Kinematics
  - Inverse Kinematics
  - Grasp Planning
- Lab 7 Preview

## Mobility / Manipulation Duality

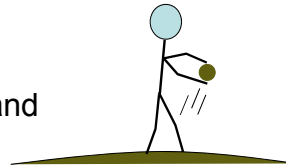
- Mobility:

- Earth is fixed
- Legs apply forces to earth
- Reaction forces move body



- Manipulation:

- Body is fixed to earth
- Arms apply forces to manipuland
- Forces move manipuland



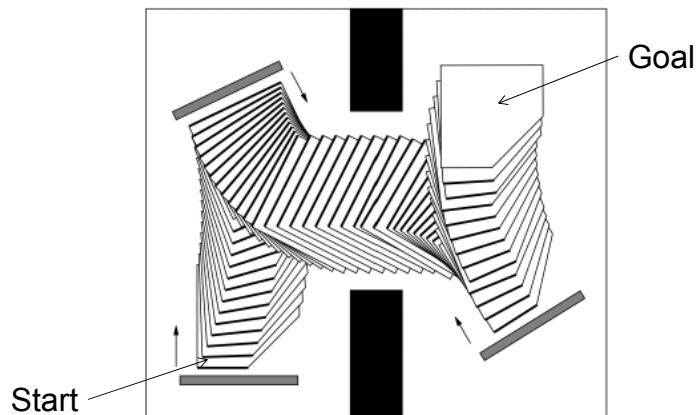
- Goal of Field: Mobile Manipulation

- Use of *coordinated whole-body motion* to effect desired manipulation of manipuland, environment
- Examples: Lifting a sandbag, throwing a baseball, shoveling snow, replacing a ceiling smoke detector

## Manipulation by Pushing

- Stable push:

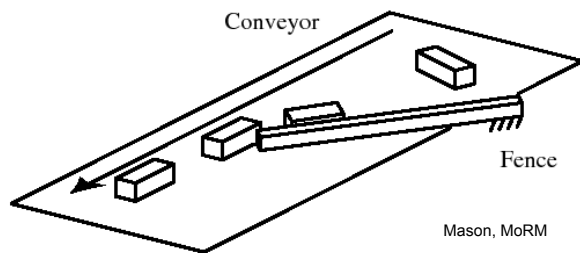
- Motions that keep object in *line* contact w/ manipulator



- Motion planning, but with additional constraints

## Fixturing

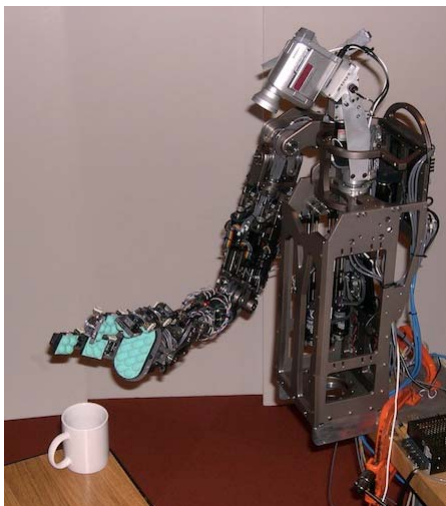
- Use of designed pegs, surfaces, prior knowledge of manipuland geometry to achieve desired pose



- Goldberg's "part squeezer" ([Try it!](#))

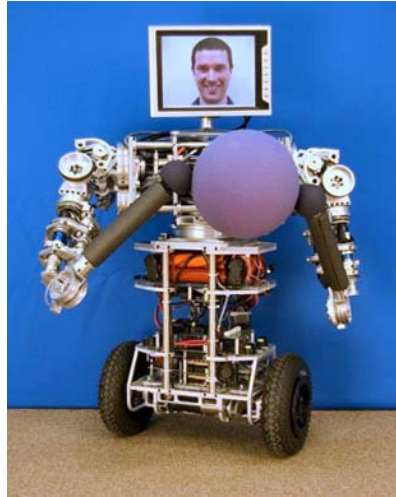
## Soft-finger Manipulation

- Can exploit visual/tactile sensing & feedback



## Mobile, Two-handed Manipulation

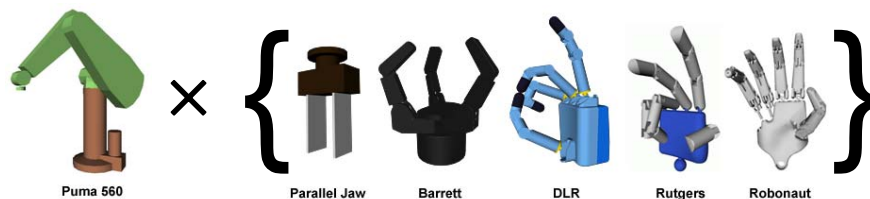
- Challenges: mass distribution; uncertainty



uBot / UMass Amherst

## End Effectors

- The component that usually comes into intentional contact with the manipuland
- Often attached interchangeably to robot arm
  - ... like a human hand picking up a specialized tool
- Many designs (here ordered roughly by time)



# Manipulation Challenges

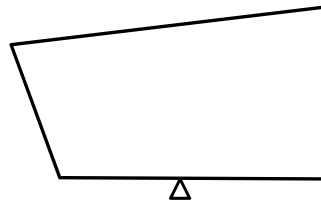
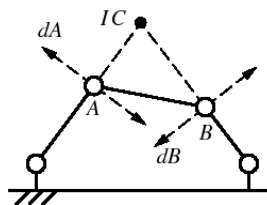
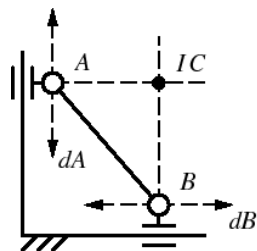


CMU robot “[Herb](#)”  
(Home Exploring  
Robot Butler, also  
after Herb Simon)

- How can the robot *perceive* the object’s type and pose?
- How can the robot *reach* for the object?
- How can the robot *grasp* the object?
- How can the robot *move* the object where desired?
- ... Today we’ll focus on grasping.

# Mechanism Analysis

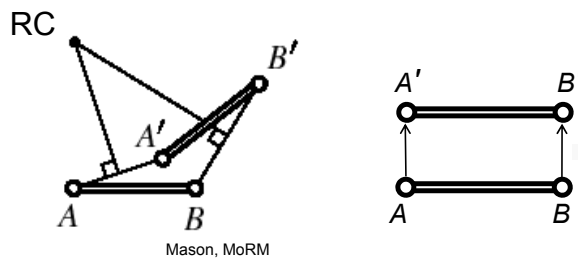
- Given some set of constraints, how can the motion of an object be characterized?
  - Rotating links
  - Sliding links
  - Point contacts



Figures from Mason, MoRM

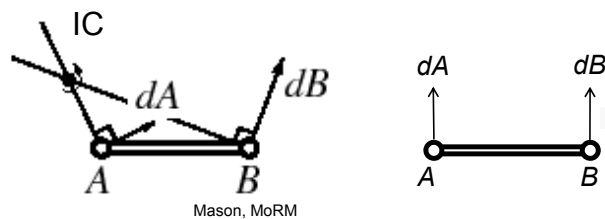
## Rotation Center (RC)

- Consider *finite* planar displacement of rigid object
  - Some point in the plane is left fixed by displacement
  - This point is called the “rotation center” (RC)
- What if the displacement is a pure translation?
  - Where is the RC?



## Instantaneous Center (IC):

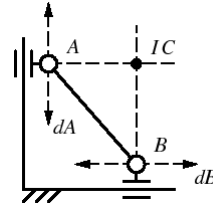
- Consider a *differential* displacement (i.e. velocity)
  - Displacement still has a fixed point; where is it?
- What if the displacement is a pure translation?
  - Where can the IC lie?



## Use of IC for Mechanism Analysis

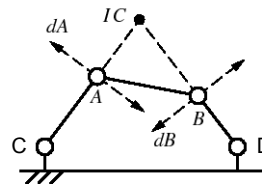
- Example four-bar linkage:

- Base link
- Two sliding+rotating links A, B
- Coupler link connecting AB



- Example four-bar linkage:

- Base link
- Four rotating links A, B, C, D
- Coupler link connecting AB



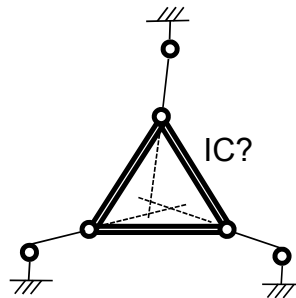
- Constraints on A, B dictate coupler motion
- IC completely determined; characterizes linkage

Figures from Mason, MoRM

## IC for Mechanism Analysis (cont.)

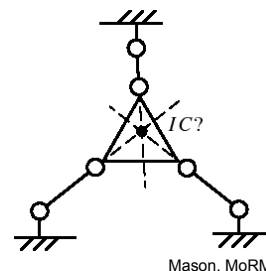
- Consider this mechanism:

- IC is



- Another possibility:

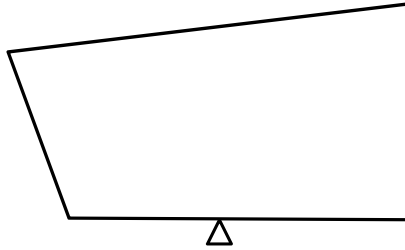
- “False instantaneous center”



Mason, MoRM

## Unilateral constraints

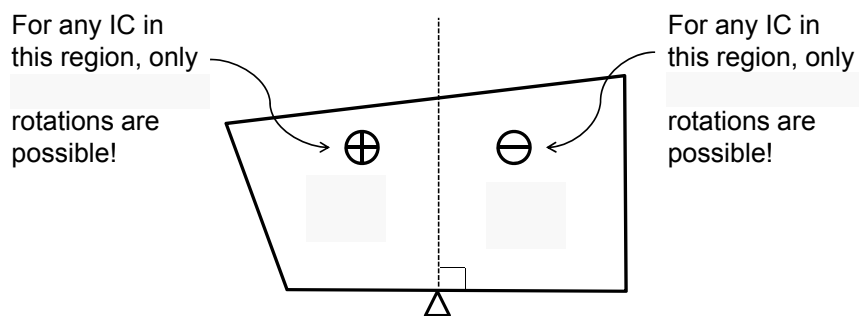
- Point contact with boundary of manipuland
- Manipuland cannot *violate* constraint (but it can *separate* from it: thus “unilateral”)



- How does this point contact constrain the possible motions of the manipuland?

## Reuleaux's method (1876)

- Each unilateral constraint partitions space of ICs into regions left, right and on *line of contact normal*

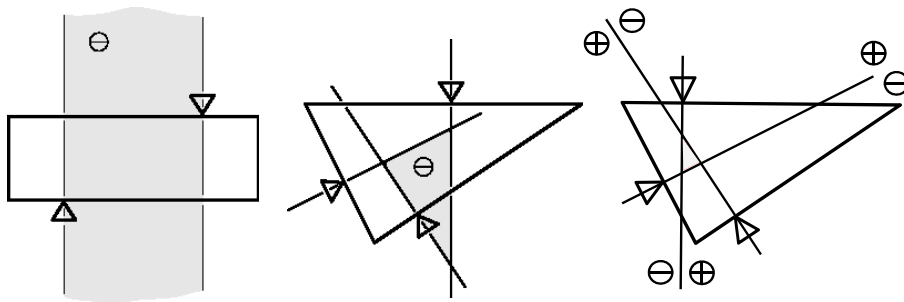


- Why is the “line of contact normal” key to analysis?
  - Along it, differential rotation of either sign is possible (for now, we are assuming frictionless point contacts)



## Reuleaux's method (cont.)

1. Construct line of  $\ominus$  for each contact
2. Label plane regions as  $\ominus$  w.r.t. this constraint



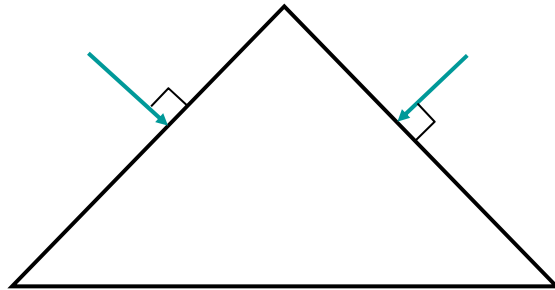
3. Each remaining region with  $\ominus$  is a locus of possible instantaneous centers  
→ Can the IC locus become *empty*? If so, how?

## Multi-Finger Manipulation

- Frictionless contacts
- Force-direction closure
- Torque closure
- Contacts with friction

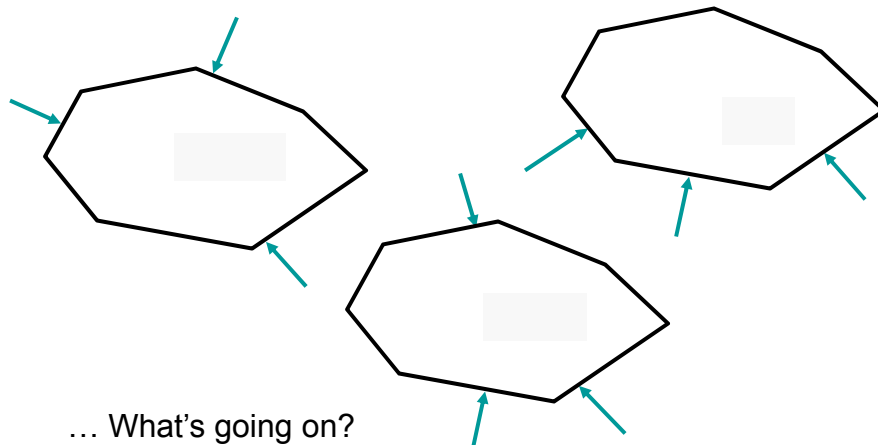
## Frictionless Point Contacts

- Force must be normal to object boundary (why?)
- Force must point into object's interior (why?)



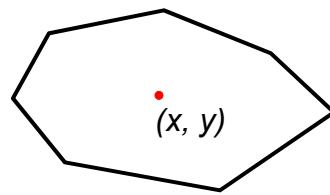
## Force-Direction Closure

- Under what conditions will a set of point contact forces resist arbitrary planar *translation*?

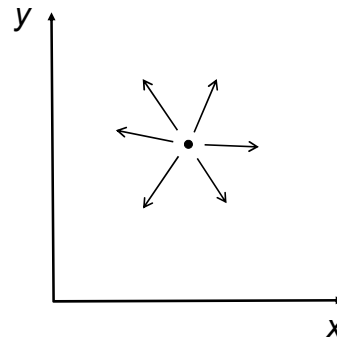


## How many contacts are needed?

- Analyze situation in c-space with DOF argument
  - First: how many c-space DOFs for object origin?



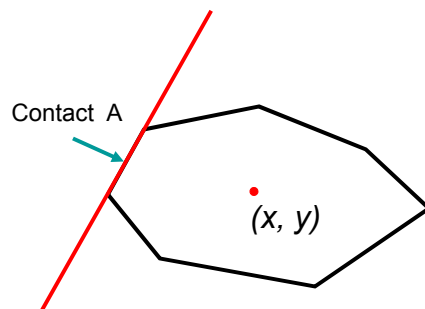
Cartesian space



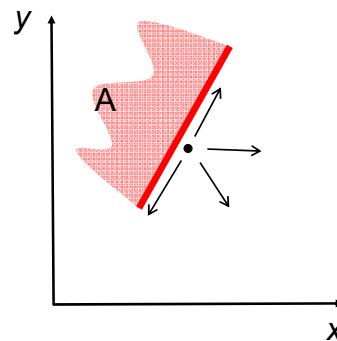
Configuration space

## How many contacts are needed?

- Analyze situation in c-space with DOF argument
  - What does a Cartesian point contact imply in c-space?



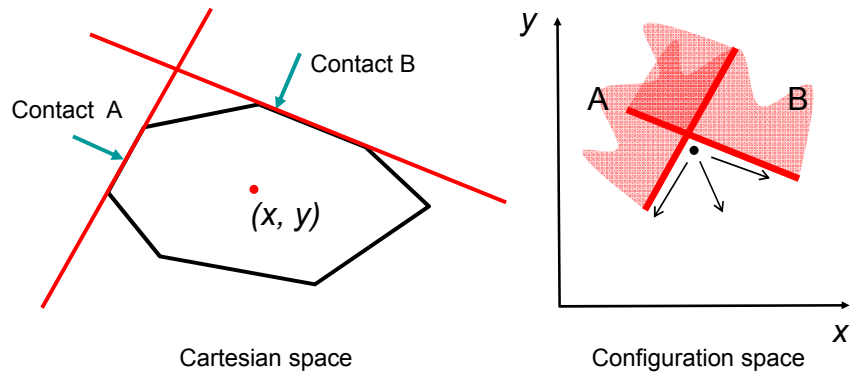
Cartesian space



Configuration space

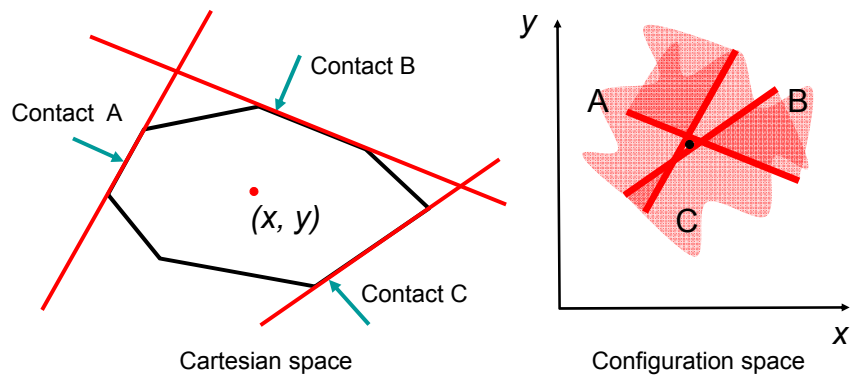
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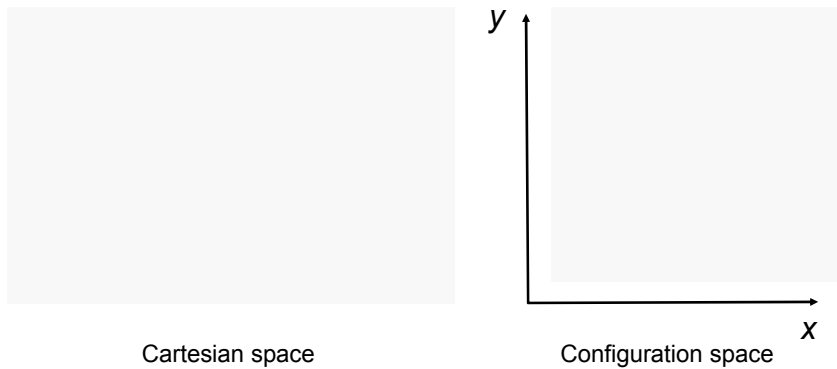
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- Analyze situation in c-space with DOF argument
  - What does a Cartesian point contact imply in c-space?



## DOF Counting for Translation

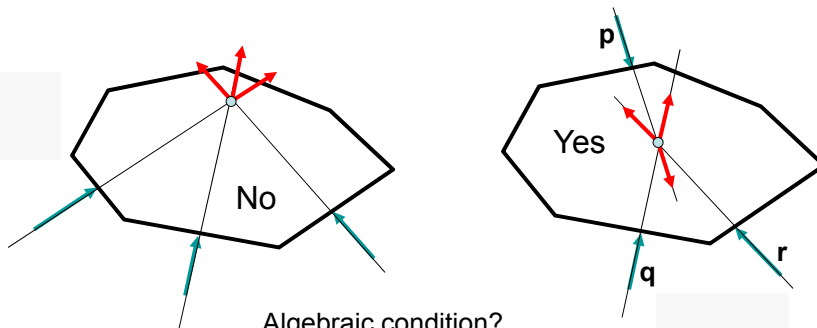
- Conclude that  $\dots$  contacts are needed in general
  - Are there situations in which more are required?



- Example of  $\dots$

## Conditions for Force-Direction Closure

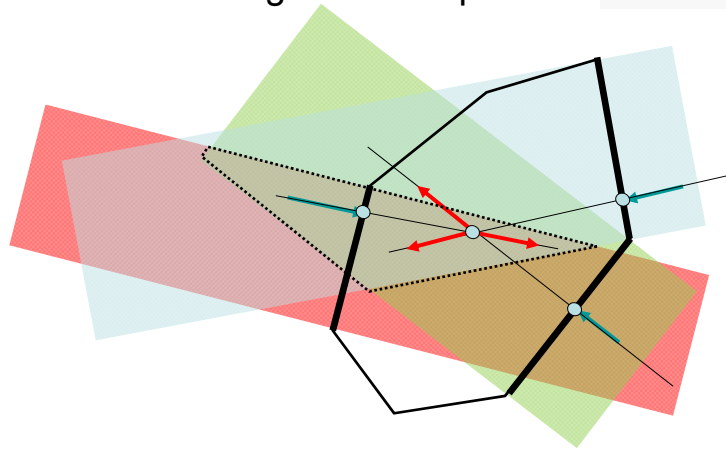
- Force vectors must  $\dots$
- Some positive combination of forces  $\dots$



Algebraic condition?  
 For force vectors  $\mathbf{p}$ ,  $\mathbf{q}$ ,  $\mathbf{r}$ ,  
 there must exist  $\alpha, \beta, \gamma > 0$   
 s.t.  $\dots$

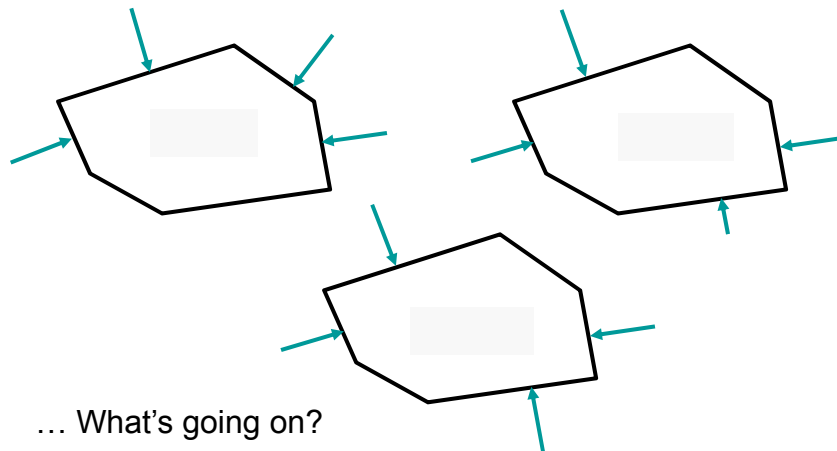
## Synthesizing a Force-Direction Grasp

1. Choose  admitting a
2. Project  onto per-edge
3. Scale force magnitudes to produce



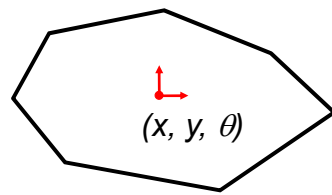
## Torque Closure

- Under what conditions will a set of point contact forces resist arbitrary planar *rotations*?

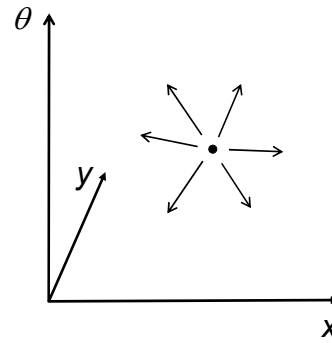


## How many contacts to “pin” rotation?

- Use analogous DOF argument in c-space
  - First: how many c-space DOFs for object pose?



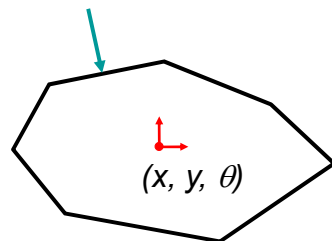
Cartesian space



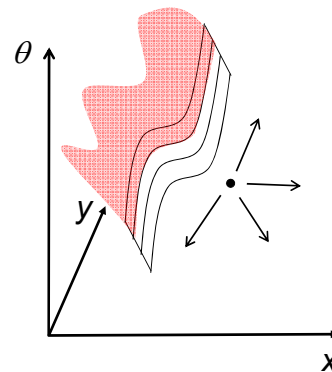
Configuration space

## How many contacts to pin rotation?

- Introduce point contact in Cartesian space
  - Implies c-space constraint with 2D manifold boundary



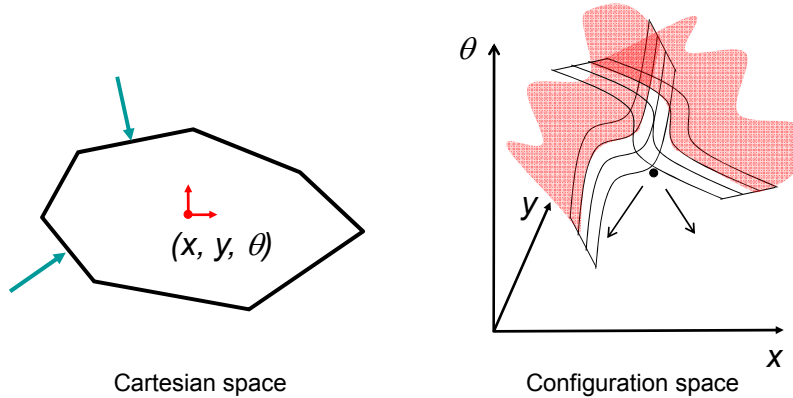
Cartesian space



Configuration space

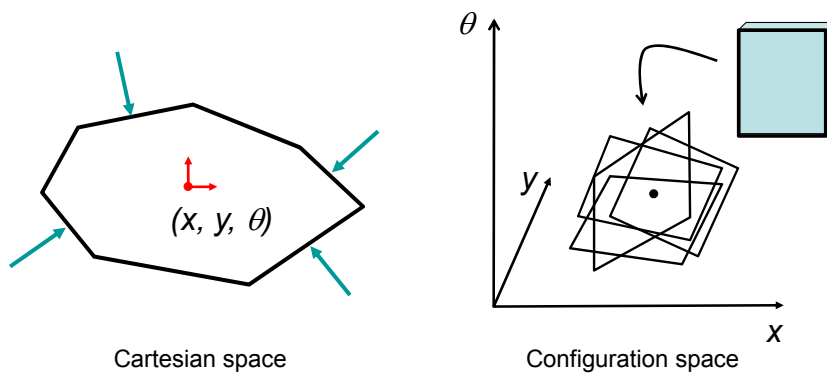
## How many contacts to pin rotation?

- Introduce point contact in Cartesian space
  - Implies c-space constraint with 2D *manifold* boundary



## How many contacts to pin rotation?

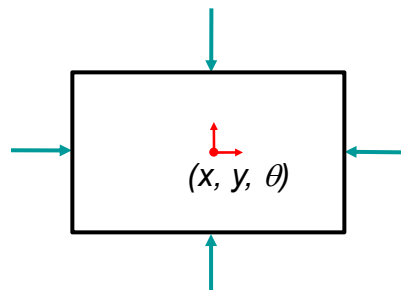
- Locally, each constraint has a planar boundary
  - ... So, how many *halfspaces* needed to pin point?





## Are There Degeneracies?

- Polygon with sides not in general position...
- Might we need more? What about ?



Cartesian space

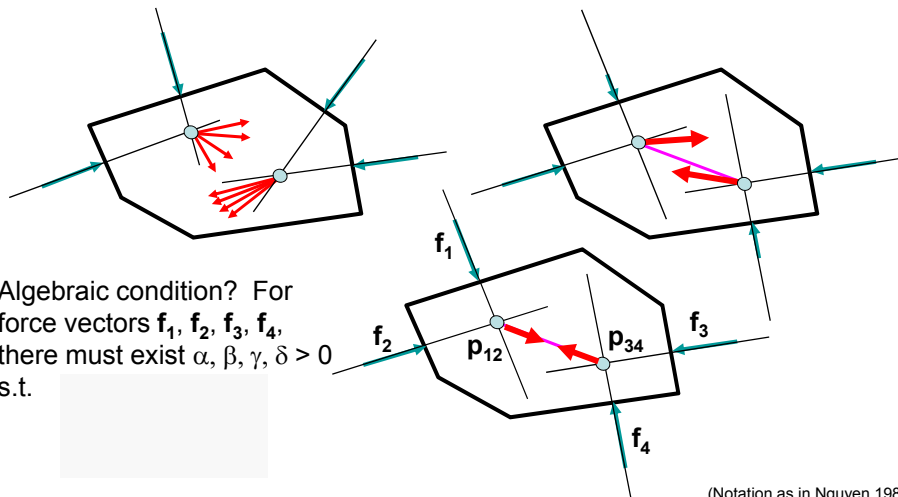


Cartesian space

- For polyhedra in 3D: need
  - *Frictionless* contacts cannot pin

## Conditions for Torque Closure

- Each normal cone must contain the other's apex
- Pairwise effective forces must cancel each other

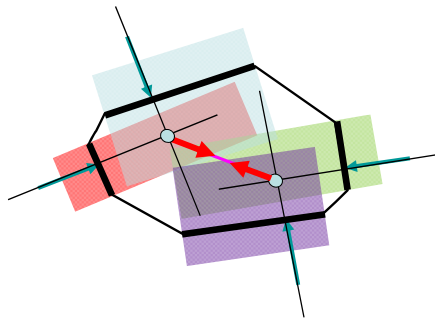


(Notation as in Nguyen 1986)

## Synthesizing a Torque-Closure Grasp

1. Choose two edge pairs\* admitting force centers
2. Choose centers inducing mutual normal cones
3. Project centers to respective edge contact points
4. Scale forces to produce alignment, cancellation

\*Edge pairs need not be contiguous

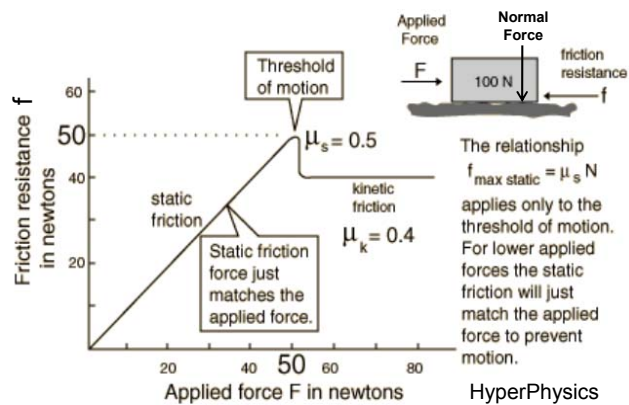


Does rotation closure imply translation closure?

## Kinetic and Static Friction (“Stiction”)

$F_f \leq \mu_s * F_n$  (at rest): coefficient of *static* friction  $\mu_s$

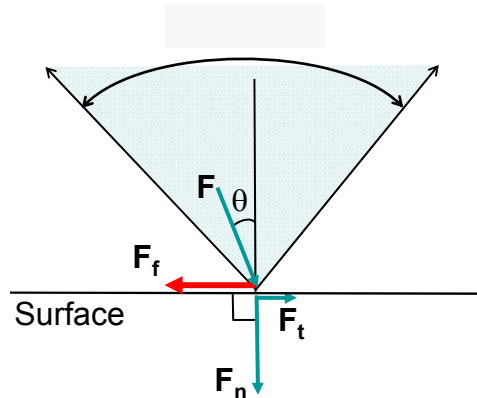
$F_f \leq \mu_k * F_n$  (moving): coefficient of *kinetic* friction  $\mu_k$



(Stiction makes things difficult both for humans and robots. Why?)

## Point Contact with Friction

- Consider a point contact exerting force  $\mathbf{F}$  at an angle  $\theta$  to the surface normal. What happens?



For contact at rest,

$$|\mathbf{F}_t| < |\mathbf{F}_f| = \mu |\mathbf{F}_n|$$

At critical angle  $\theta_{\text{crit}}$ ,

$$|\mathbf{F}_t| =$$

Substituting gives

$$|\mathbf{F}| \sin \theta_{\text{crit}} =$$

Which yields

$$\mu =$$

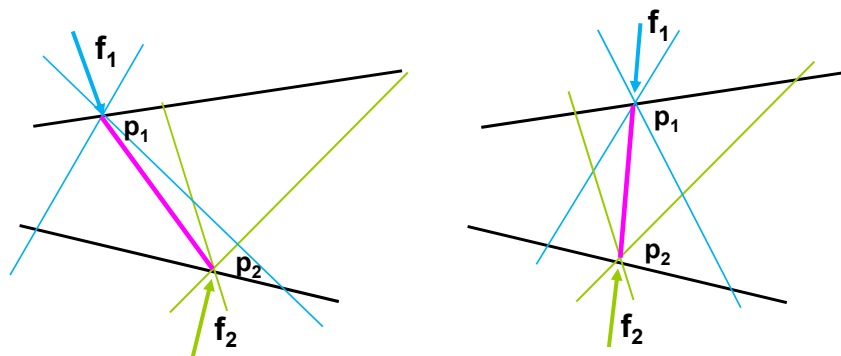
So that

$$\theta_{\text{crit}} =$$

- Produces \_\_\_\_\_ of directions, s.t. point will not \_\_\_\_\_ when  $\mathbf{F}$  is applied

## Grasp Analysis With Friction

Consider forces  $\mathbf{f}_1, \mathbf{f}_2$  at frictional contacts  $\mathbf{p}_1, \mathbf{p}_2$



When can  $\mathbf{f}_1, \mathbf{f}_2$  oppose one another without sliding?

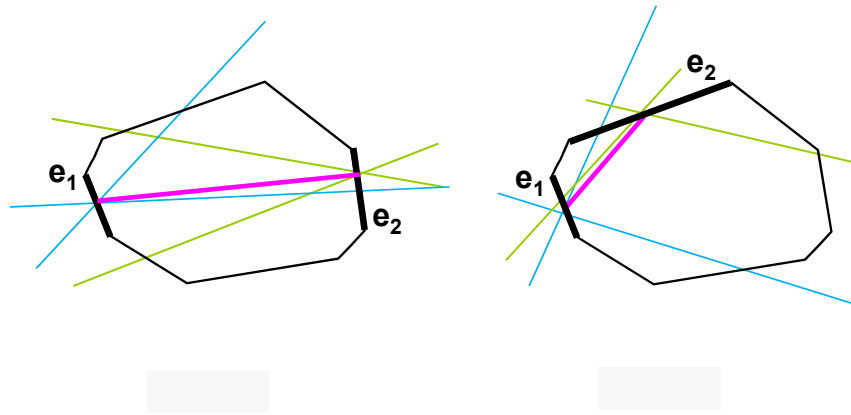
Each force must \_\_\_\_\_

Point  $\mathbf{p}_1$  (resp.  $\mathbf{p}_2$ ) must \_\_\_\_\_

## Grasp Synthesis With Friction

Choose a *compatible* pair of edges  $e_1, e_2$

Intuition? Using what data? How to choose?

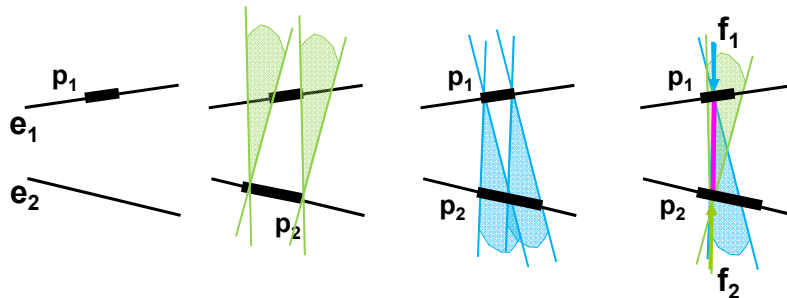


## Grasp Synthesis With Friction

Choose target region for contact point  $p_1$

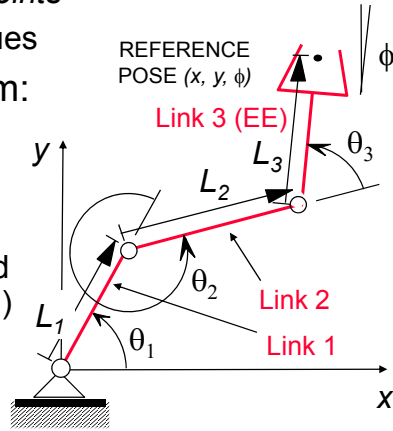
Determine feasible target region for contact  $p_2$

Orient and scale  $f_1, f_2$  so as to cancel along  $\overline{p_1 p_2}$



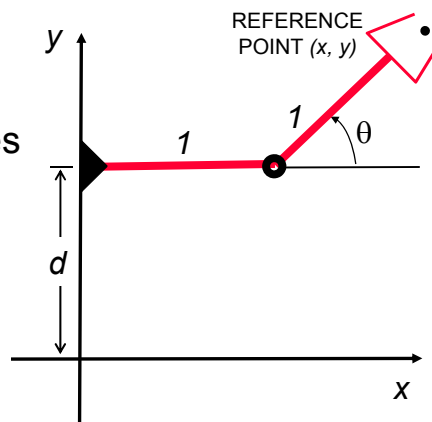
## Forward and Inverse Kinematics

- So far, have cast computations in Cartesian space
- But manipulators controlled in *configuration space*:
  - Rigid *links* constrained by *joints*
  - For now, focus on joint values
- Example 3-link mechanism:
  - Joint coordinates  $\theta_1, \theta_2, \theta_3$
  - Link lengths  $L_1, L_2, L_3$
- End effector coordinates
  - “Reference pose” described by  $x, y,$  and  $\phi$  (w.r.t. vertical)
- How can we relate EE to configuration variables?



## Forward Kinematics

- Given mechanism description and joint values, express end effector pose in Cartesian coordinates
  - Example: two-link arm with one sliding, one rotating joint
- Configuration variables:
  - Joint coordinates  $d, \theta$
  - Link lengths (both 1)
- End effector coordinates
  - “Reference point”  $(x, y)$
- Challenge: express as
  - $x = x(d, \theta) =$
  - $y = y(d, \theta) =$



## Inverse Kinematics

- Given end effector pose in Cartesian coordinates, identify the joint values that *yield* the desired pose
- Challenge: solve for joint values in terms of pose

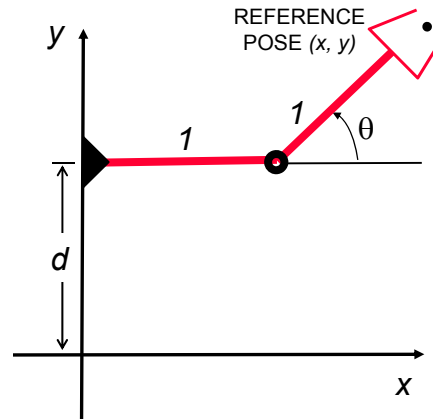
$$\theta = \theta(x, y)$$

=

$$d = d(x, y)$$

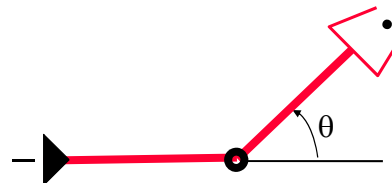
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Hints:



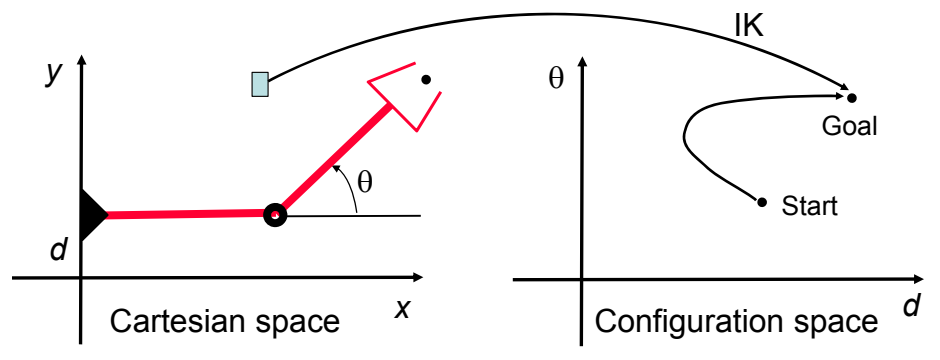
## Why is IK difficult?

- Nonlinear
  - Revolute joints  $\rightarrow$  inverse trigonometry
- Multi-valued
  - Often multiple solutions for a single Cartesian pose
- Discontinuities and singularities
  - Can lose one or more DOFs in some configurations
- Possibly over-constrained (no exact solution)
  - Use of approximation and iterative algorithms
- Dynamics
  - In reality, want to apply forces and torques (while respecting physical constraints), not just move arm!



## Putting it All Together: Grasping

- Input workspace, obstacles, and manipuland:
  - Determine a feasible grasp (set of contact points)
  - Use IK to solve for target end-effector pose in c-space
  - Plan a collision-free reach to the computed pose
  - Control end-effector along desired trajectory



## What have we swept under the rug?

- Sensing
  - Shape, pose of target object, accessibility of surfaces
  - Classification of material type from sensor data
  - Freespace through which grasping action will occur
- Prior knowledge
  - Estimation of mass, moments, friction coefficients from material
  - Internal, articulated, passive vs. active degrees of freedom
- Uncertainty & compliance
  - Tolerate noise inherent in sensing and actuation
  - Ensure that slight sensing, actuation errors won't cause damage
  - Handle soft fingers making contact over a finite area (not a point)
- Dynamics
  - All of the above factors may be changing in real time

## Confidence vs. Arrogance

### **Confident:**

Having strong belief, firm **trust**, or sure expectation. [OED]

From Latin *com-* (intensive prefix) + *fidere* “to trust”

### **Arrogant:**

Making or implying strong or **unwarrantable** claims to dignity, authority or knowledge. [OED]

From Latin *arrogare* “to claim for oneself, assume”