Rapidly-exploring **R**andom **T**rees **(RRTs)** for Efficient Motion Planning

RSS Lecture #13

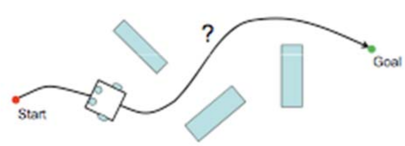
Wednesday, 20 March 2013

Prof. Seth Teller

(Thanks to Sertac Karaman for animations)

Recap of Previous Lectures:

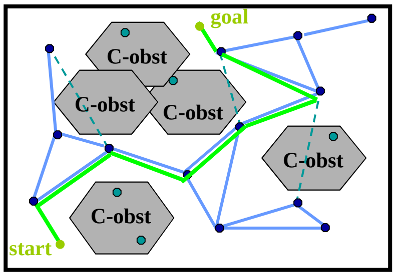
- Recall the motion planning problem:



- We discussed:
  - Cell decomposition
  - Guided search using A*
  - Potential fields
  - Configuration space
  - Probabilistic Road Maps

Recap: PRMs [Kavraki et al. 1996]

C-space



Roadmap Construction (Pre-processing)

1. Randomly generate robot configurations (nodes)
   - Discard invalid nodes (how?)
2. Connect pairs of nodes to form **roadmap edges**
   - Use simple, deterministic *local planner*
   - Discard invalid edges (how?)

Plan Generation (Query processing)

1. Link *start* and *goal* poses into roadmap
2. Find path from *start* to *goal* within roadmap
3. Generate motion plan for each edge used

Primitives Required:

1. Method for sampling C-Space points
2. Method for “validating” C-space points and edges

Today’s Focus

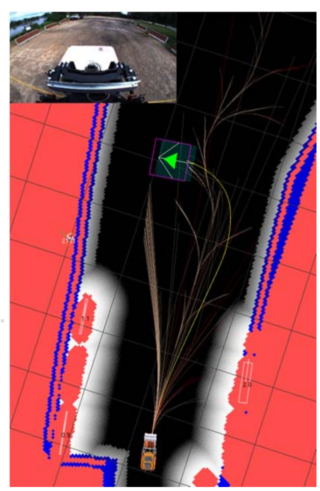
- Retain assumptions:
  - Perfect map
  - Perfect localization
- Incorporate additional elements:
  - Unstable **dynamics**
    - Cars, helicopters, humanoids, …
    - Agile maneuvering aircraft
  - High-dimensional configuration space
  - Real-time and online
    - Trajectory design & execution

Today's Focus

- Retain assumptions:
  - Perfect map
  - Perfect localization
- Incorporate additional elements:
  - Unstable **dynamics**
    - Cars, helicopters, humanoids, …
    - Agile maneuvering aircraft
  - High-dimensional configuration space
  - Real-time and online
    - Trajectory design & execution

Today's Focus

- Retain assumptions:
  - Perfect map
  - Perfect localization
- Incorporate additional elements:
  - Unstable **dynamics**
    - Cars, helicopters, humanoids, …
    - Agile maneuvering aircraft
  - High-dimensional configuration space
  - Real-time and online
    - Trajectory generation & execution



Motion Planning Revisited

- **Given:**
  - Robot's dynamics
  - A map of the environment
    (perfect information, but discovered online)
  - Robot's pose in the map
  - A goal pose in the map
- **Find a sequence of**
  - Actuation commands
    (such as steer, gas/brake, transmission)
  - In real time (requires efficient algorithms)

… that drive system to the goal pose

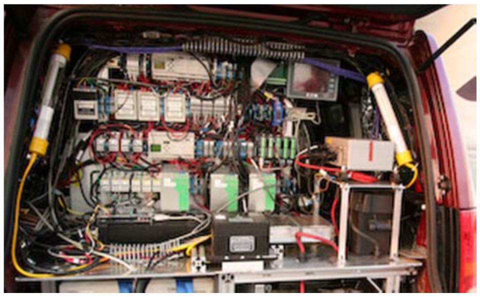
- Problem is essential in almost all robotics applications irrespective of size, type of actuation, sensor suite, task domain, etc.

Motion Planning Revisited

- **Challenges in (most) practical applications:**
  - **Safety:** do not collide with anything;
    ensure that system is stable; etc.
  - **Computational effectiveness:** problem is (provably) computationally very challenging
  - **Optimize:** fuel, efficiency etc.
  - **Social challenges (in human-occupied environments):** motion should seem natural; robot should be accepted by humans

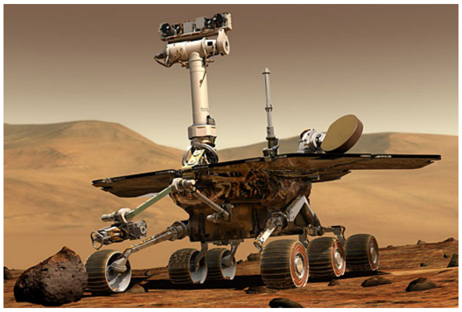
Motion Planning Revisited

- **Challenges in (most) practical applications:**
  - **Safety:** do not collide with anything;
    ensure that system is stable; etc.
  - **Computational effectiveness:** problem is (provably) computationally very challenging
  - **Optimize:** fuel, efficiency etc.
  - **Social challenges (in human-occupied environments):** motion should seem natural; robot should be accepted by humans



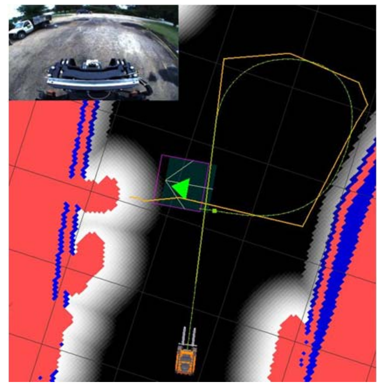
Motion Planning Revisited

- **Challenges in (most) practical applications:**
  - **Safety:** do not collide with anything;
    ensure that system is stable; etc.
  - **Computational effectiveness:** problem is (provably) computationally very challenging
  - **Optimize:** fuel, efficiency etc.
  - **Social challenges (in human-occupied environments):** motion should seem natural; robot should be accepted by humans



Motion Planning Revisited

- **Challenges in (most) practical applications:**
  - **Safety:** do not collide with anything; ensure that system is stable; etc.
  - **Computational effectiveness:** problem is (provably) computa-tionally very challenging
  - **Optimize:** fuel, efficiency etc.



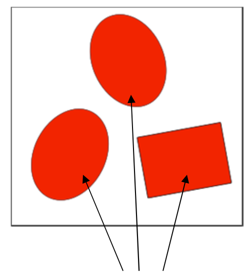
  - **Social challenges (in human-occupied environments):** motion should seem natural; robot should be accepted by humans

Different Approaches

- Algebraic Planners
- Cell Decomposition
- Potential Fields
- Sampling-Based Methods

Motion Planning Approaches

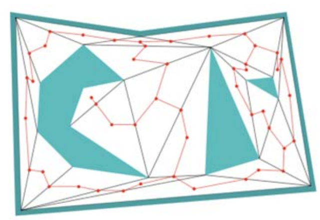
- **Algebraic Planners**
  - Explicit (algebraic) representation of obstacles
  - Use algebraic expressions (of visibility computations, projections etc.) to find the path
  - Complete (finds a solution if one exists, otherwise reports failure)
  - Computationally very intensive – impractical
- Cell Decomposition
- Potential Fields.
- Sampling-Based Methods



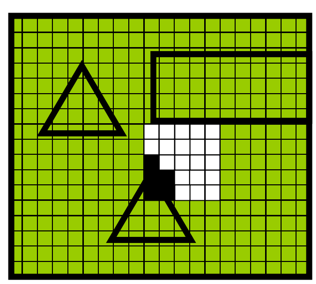
1. Represent with polynomial inequalities
2. Transform inequalities to c-space
3. Solve inequalities in c-space to check feasibility and find a plan

Motion Planning Approaches

- Algebraic Planners
- **Cell Decomposition**
  - Analytic methods don't scale well with dimension (too many cells in high *d*)
  - Gridding methods are only "resolution complete" (i.e., will find a solution only if the grid resolution is fine enough, and if enough grid cells are inspected)
- Potential Fields.
- Sampling-Based Methods



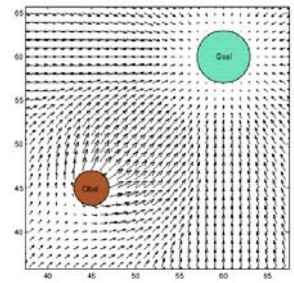
Analytic subdivision

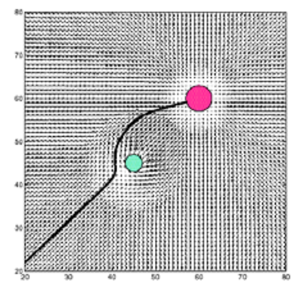


Gridded subdivision

Motion Planning Approaches

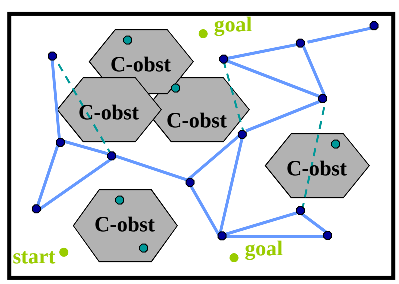
- Algebraic Planners
- Cell Decomposition
- **Potential Fields**
  - No completeness guarantee (can get stuck in local minima)
  - Of intermediate efficiency; don't handle dynamic environments well
- Sampling-Based Methods



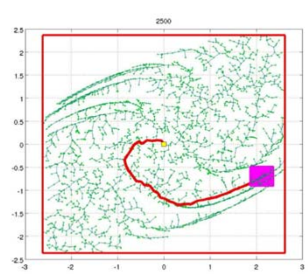


Motion Planning Approaches

- Algebraic Planners
- Cell Decomposition
- Potential Fields

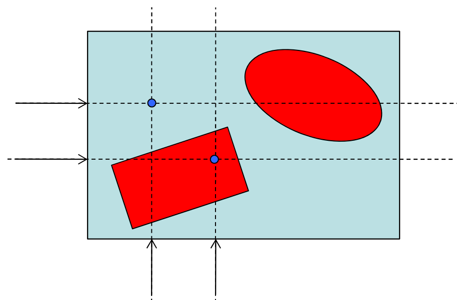


- **Sampling-Based Methods**
  - (Randomly) construct a set of feasible (that is, collision-free) trajectories
  - "Probabilistically complete" (if run long enough, very likely to find a solution)
  - Quite efficient; methods scale well with increasing dimension, # of obstacles



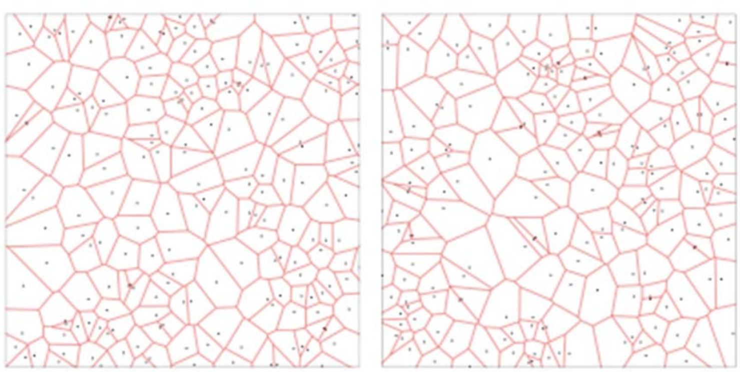
Sampling Strategies

- How can we draw random samples from within c-space?
- Normalize all c-space dimensions to lie inside [0..1]
- Then, simple idea:
  1. Generate a random point in *d*-dimensional space
     - Independently generate *d* random numbers between 0 and 1
     - Aggregate all *d* numbers into a single point in c-space
  2. Check whether sample point (i.e., robot pose) lies within any obstacle



Example Sample Sets

Uniform sampling:
From a given axis, sample each coordinate with equal likelihood

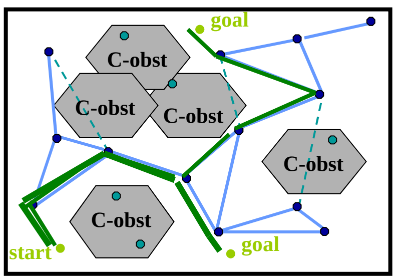


(200 random samples) (200 random samples)

Observe:
Significant local variation, but sample sets are globally consistent
(Later, we'll see that this yields consistent performance across runs)

Sampling-based Motion Planning

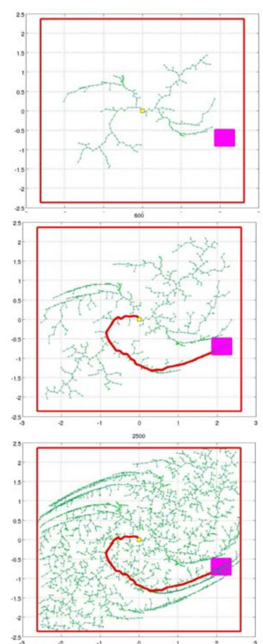
- **Basic idea:**
  - Randomly sample *n* points from c-space
  - Connect them to each other (if no collision with obstacles)
  - Recall the two primitive procedures:
    - Check if a point is in the obstacle-free space
    - Check if a trajectory lies in the obstacle-free space



This is the **P**robabilistic **R**oad **M**ap (**PRM**) algorithm

PRM is a **multiple-query** algorithm (can reuse the roadmap for many queries)

Incremental Sampling-based Motion Planning

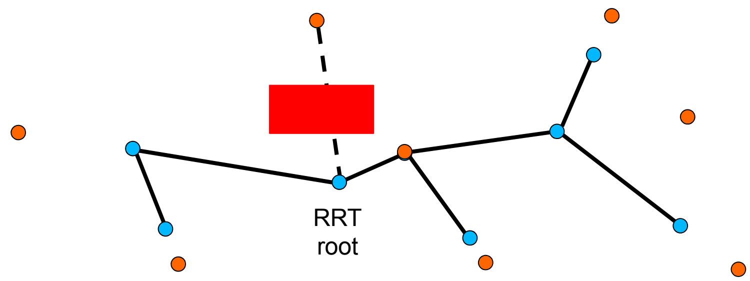


- Sometimes building a roadmap *a priori* might be inefficient (or even impractical)
  - Assumes that all regions of c-space will be utilized during actual motions
- Building a roadmap requires global knowledge
  - But in real settings, obstacles are not known *a priori*; rather, they are discovered *online*
- **We desire an *incremental* method:**
  - Generate motion plans for a single start, goal pose
  - Expending more CPU yields better motion plans
- **The Rapidly-exploring Random Tree (RRT) algorithm meets these requirements**

RRT Data Structure, Algorithm

T = (nodes V, edges E): tree structure

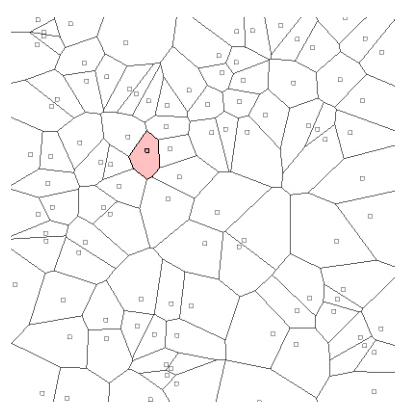
- **Initialized as single root vertex (the robot's current pose)**



1. $x \leftarrow \texttt{Sample}()$ // Sample a node *x* from c-space
2. $v \leftarrow \texttt{Nearest}(T, x)$ // Find nearest node *v* in tree
3. $v' \leftarrow \texttt{Extend}(v, x)$ // Extend nearest node toward sample
4. If $(\texttt{ObstacleFree}(v, v'))$ then // If extension is collision-free
5. $\quad V \leftarrow V \cup \{v'\}; E \leftarrow E \cup \{(v, v')\}$ // Add new node and edge to tree

Digression: Voronoi Diagrams

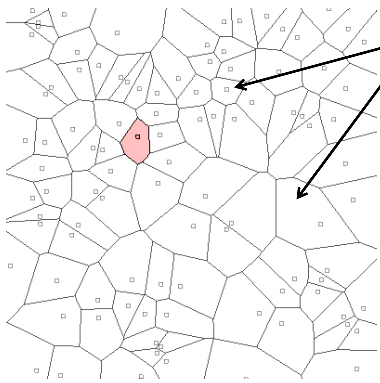
Given *n sites* in *d* dimensions, the *Voronoi diagram* of the sites is a partition of $\boldsymbol{R}^d$ into regions, one region per site, such that all points in the interior of each region lie closer to that region's site than to any other site



(AKA Dirichlet tesselations, Wigner-Seitz regions, Thiessen polygons, Brillouin zones, …)

Rapidly-exploring Random Trees:
Clearly random! Why rapidly-exploring?

- RRTs tend to grow toward *unexplored* portions of the state-space
  - Unexplored regions are (in some sense) more likely to be sampled
  - This is called a ***Voronoi bias***



For an RRT at a given iteration, some nodes are associated with large Voronoi regions of c-space, some with smaller Voronoi regions

The **unexplored areas** of c-space tend to coincide with the larger Voronoi regions

(Uniform) samples will tend to fall into relatively larger Voronoi regions

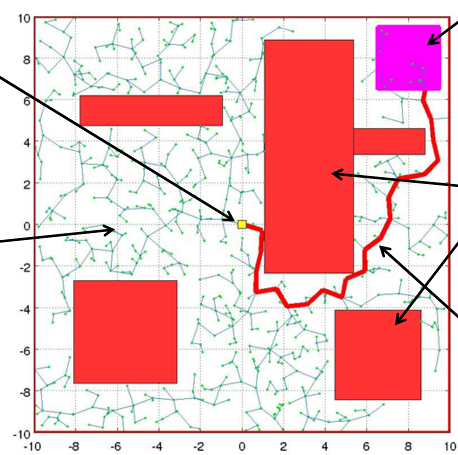
Thus unexplored regions will tend to shrink!

Main advantage of RRT: Samples "**grow**" tree toward unexplored regions of c-space!

Rapidly-exploring Random Trees in simulation

Goal pose region

Initial pose



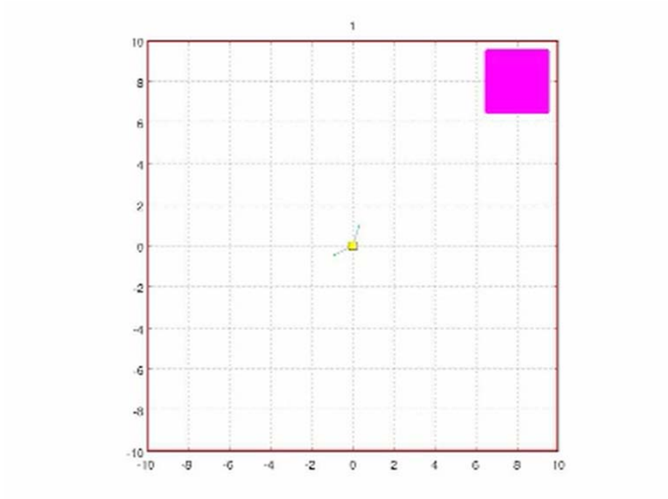
Obstacles

The tree

Best path in the tree (identified through search)

Rapidly-exploring Random Trees in simulation

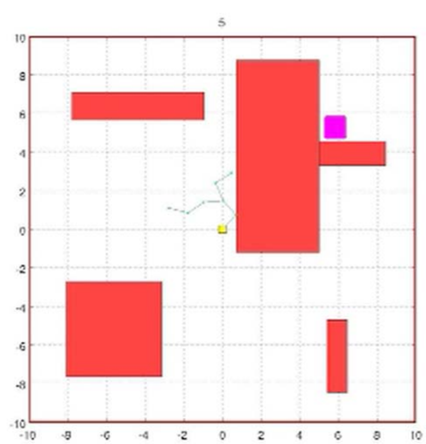
Movie shows the RRT exploring empty c-space



Goal pose region

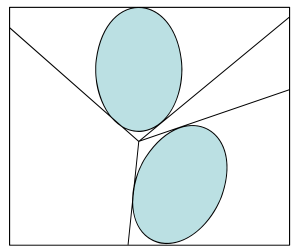
Rapidly-exploring Random Trees in simulation

Let's look at initial exploration phase more carefully:



Performance of Sampling-based Methods

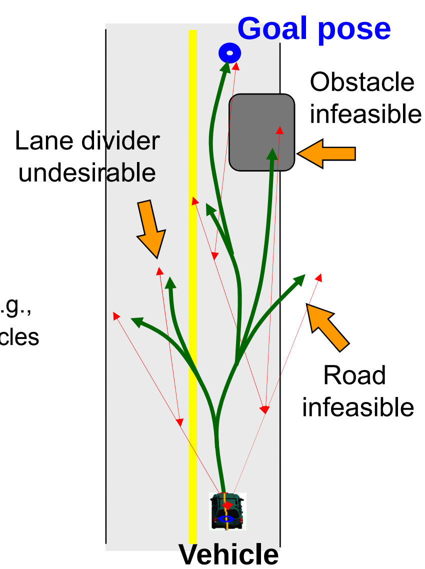
- Why do the PRM and RRT methods work so well?
- **Probabilistic Completeness:**
  - The probability that the RRT will find a path approaches 1 as the number of samples increases — **if** a feasible path exists.
  - The approach rate is exponential — **if** the environment has good "**visibility**" properties
- **ϵ-goodness:**
  - A **point** is **ϵ-good** if it "sees" at least an ϵ fraction of the obstacle-free space
  - An **environment** is **ϵ-good** if all freespace points in it are **ϵ-good**



Good performance of PRMs and RRTs has been tied to the fact that, in practice, most applications feature environments with good visibility guarantees ***(Latombe et al., IJRR '06)***.

Example: Unmanned Driving

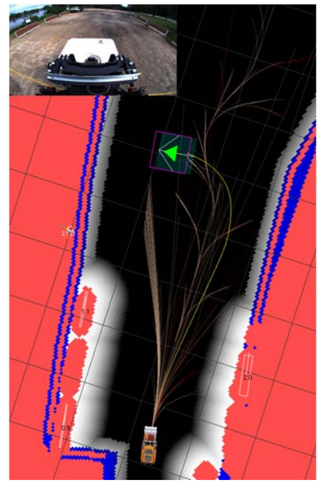
- **Tree of trajectories** is grown by sampling configurations randomly
- **Rapidly explores** several configurations that the robot can reach.
  - Many test trajectories generated (tens of thousands per second)
- **Safety** of any trajectory is guaranteed
  - As of instantaneous world state at the time of trajectory generation
- **Choose best** one that reaches the goal, e.g.,
  - Maximizes minimum distance to obstacles
  - Minimizes total path length
- **Supports dynamic replanning**; if current trajectory becomes infeasible:
  - Choose another one that is feasible
  - If none remain, then E-stop



Real-world Implementation

A few details:

- CPU limitations and sampling method
- Dynamical feasibility constraints
- Grid map with local obstacle awareness
- Stop nodes for safety

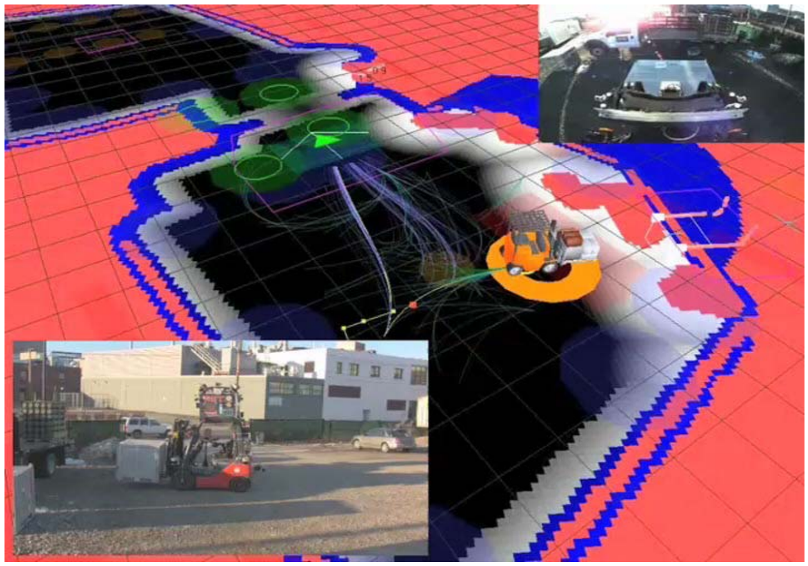


Legend for images, videos you'll see next:

- Instantaneous vehicle pose
- Goal pose
- Obstacle
- High-cost regions
- Reaching, low cost
- Reaching, high cost
- Non-reaching

RRT at work: Urban Challenge

RRT at work: Autonomous Forklift



Summary

- The Rapidly-exploring Random Tree (RRT) algorithm
- Discussed challenges for motion planning methods in real-world applications
- Intuition behind good performance of sampling-based methods
- Two applications:
  - Urban Challenge vehicle, Agile Robotics forklift