6.141:
Robotics systems and science
Lecture 9: Configuration Space and Motion Planning

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Reading: Chapter 3, and Craig: Robotics

http://courses.csail.mit.edu/6.141/

Challenge: Build a Shelter on Mars

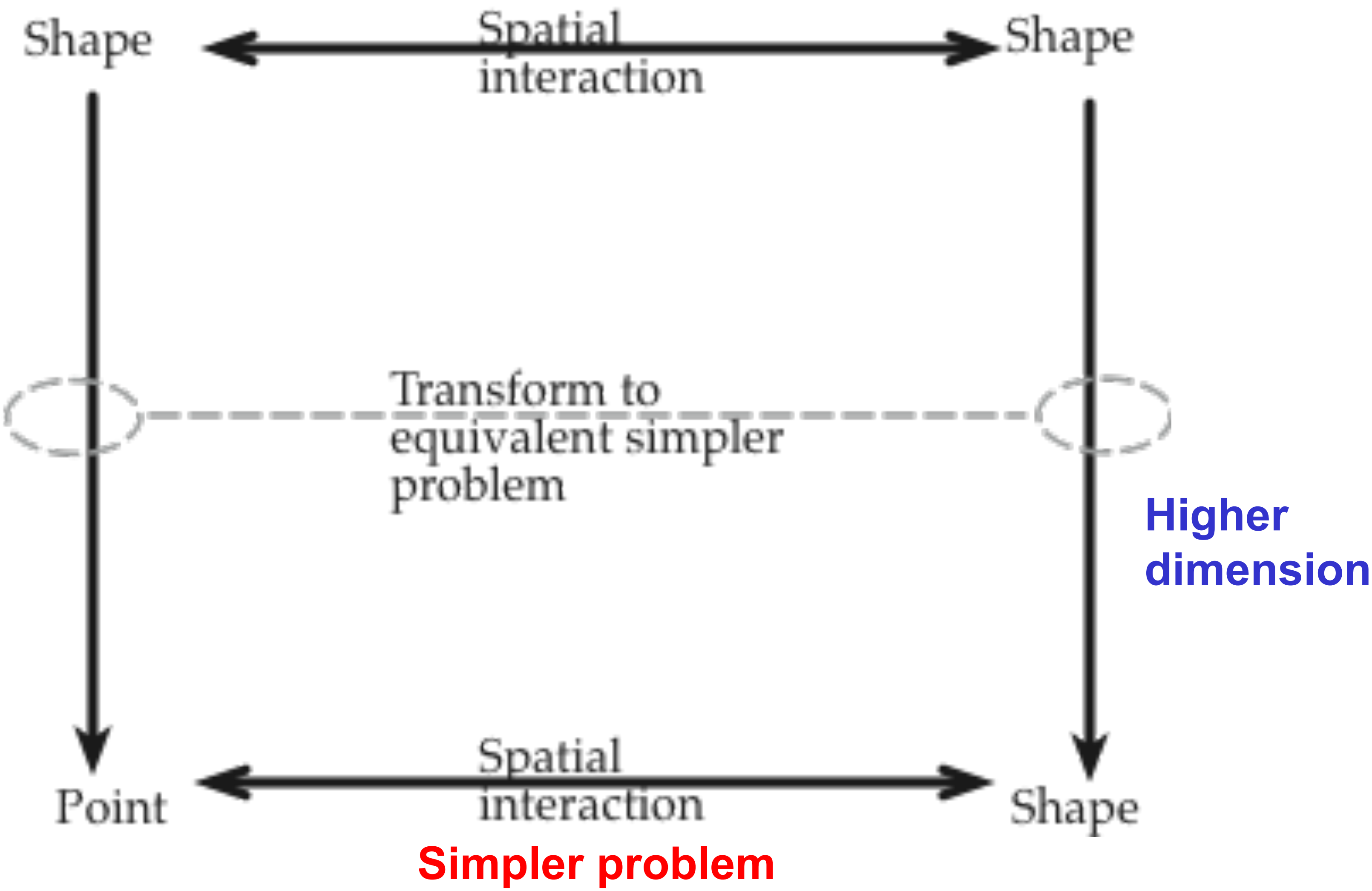
During the last module we saw

- Control architectures: reactive, behavior, deliberative
- Visibility Graphs for Motion Planning
- Started Configuration Space

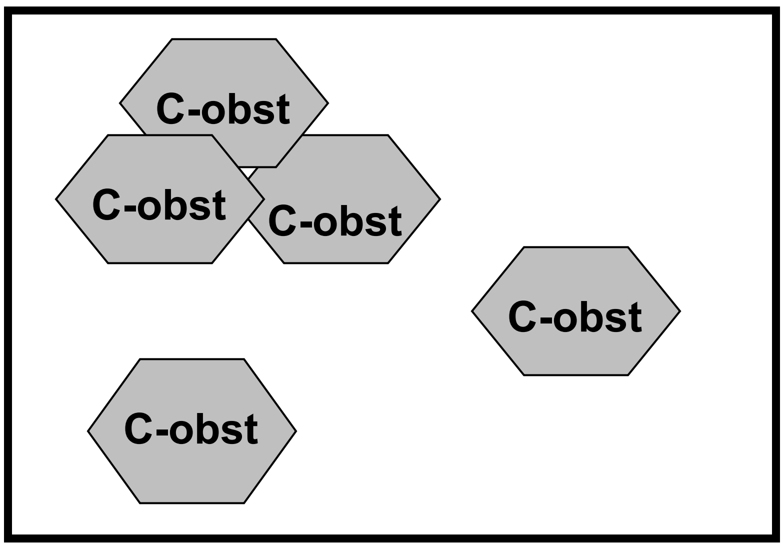
Today

- Understand c-space
- Motion planning with grids
- Probabilistic motion planning

Transforming to C-Space



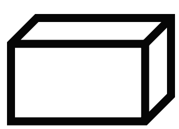
C-space Overview



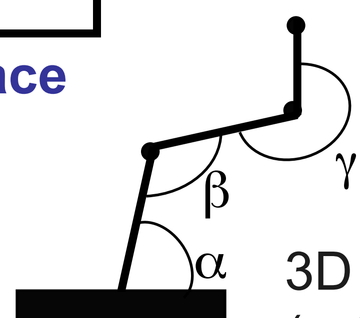
C-Space

- robot maps to a point in higher dimensional space
- parameter for each degree of freedom (dof) of robot
- C-space = set of all robot placements
- C-obstacle = infeasible robot placements

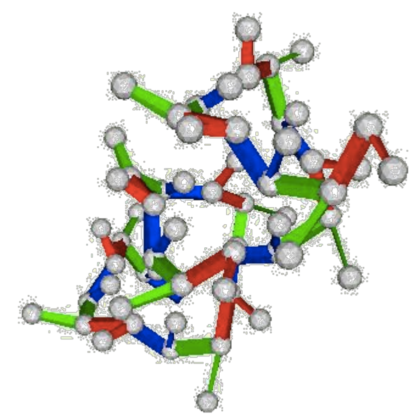
3D C-space
(x,y,z)



6D C-space
(x,y,z,pitch,roll,yaw)

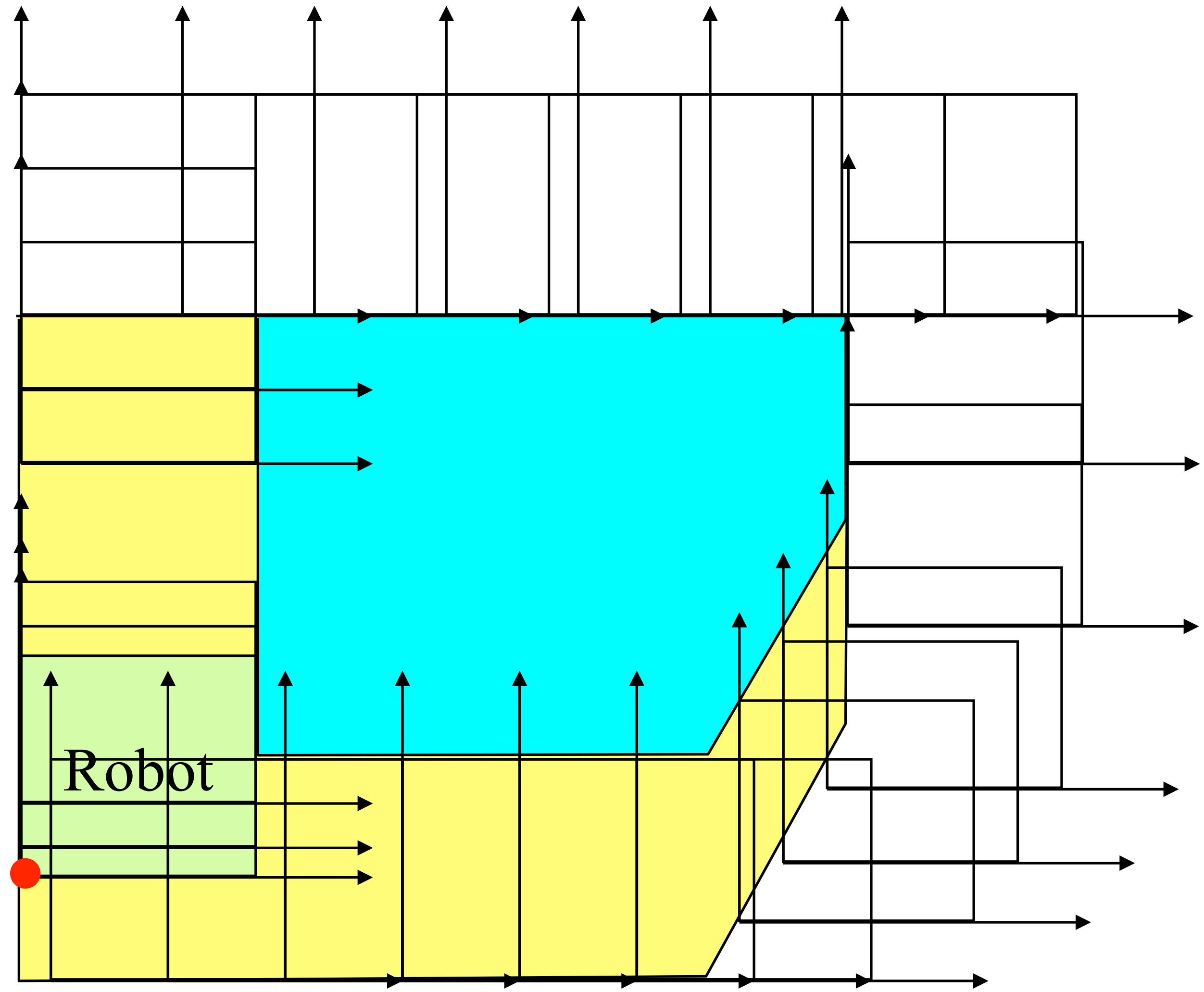


3D C-space
(α,β,γ)



2n-D C-space
$(\phi_1, \psi_1, \phi_2, \psi_2, \ldots, \phi_n, \psi_n)$

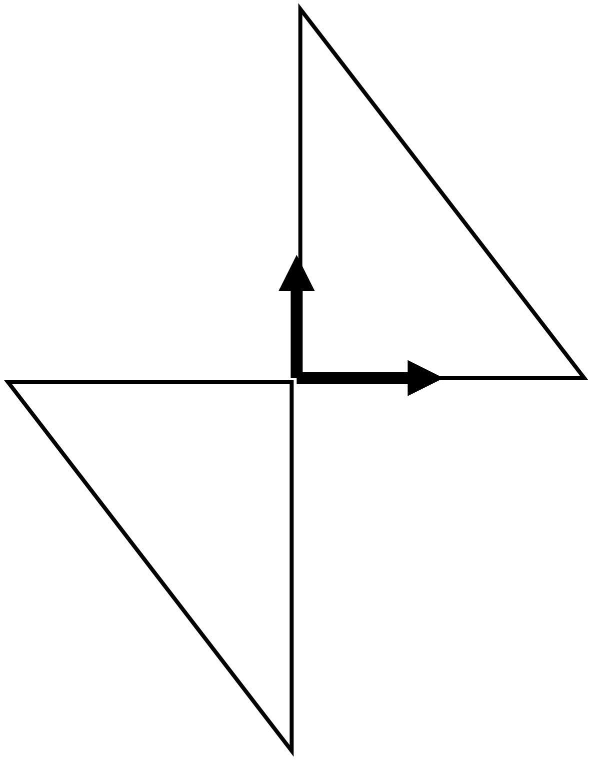
C-obstacle for fixed robot orientation



How do we compute C-space

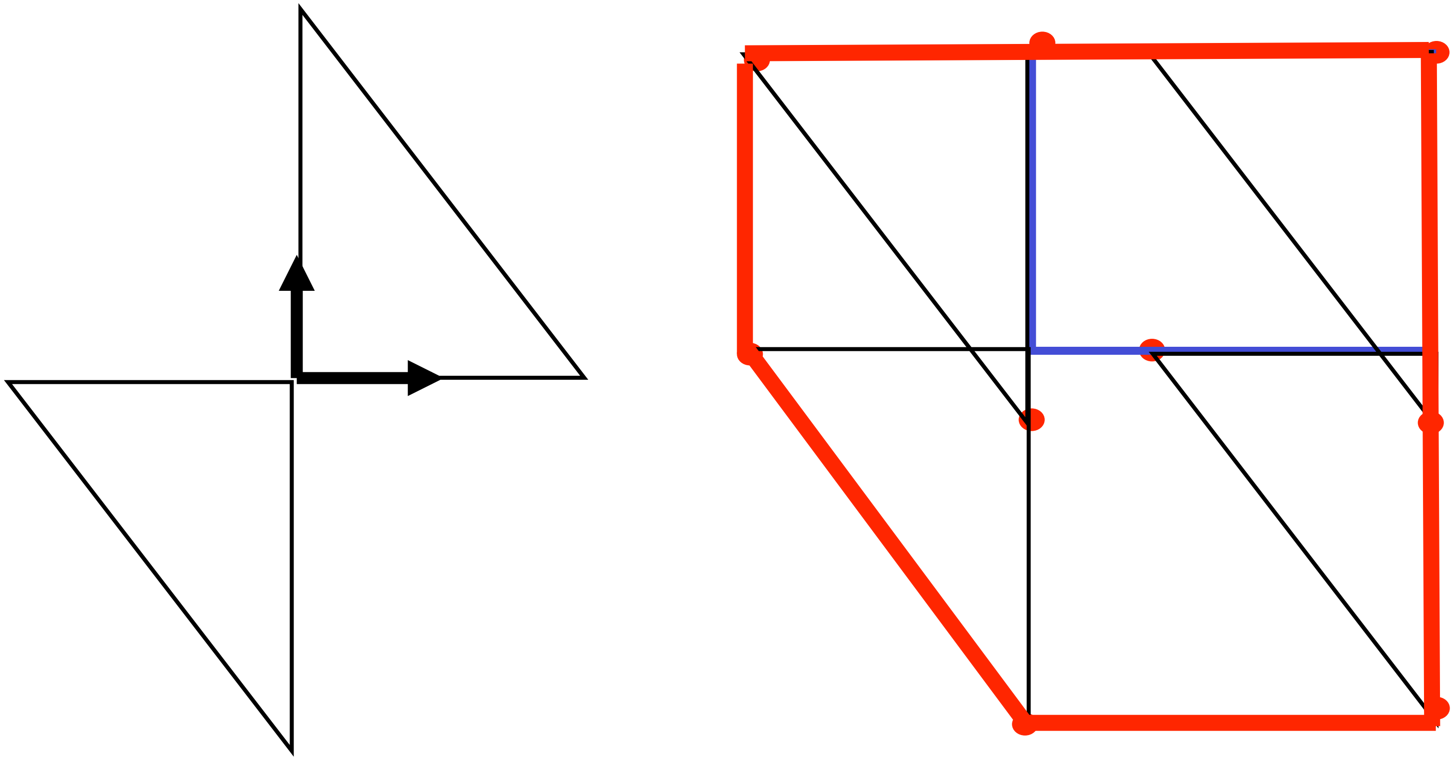
- Identify dimensions
- Compute all c-obstacles

How do we compute c-obstacles?



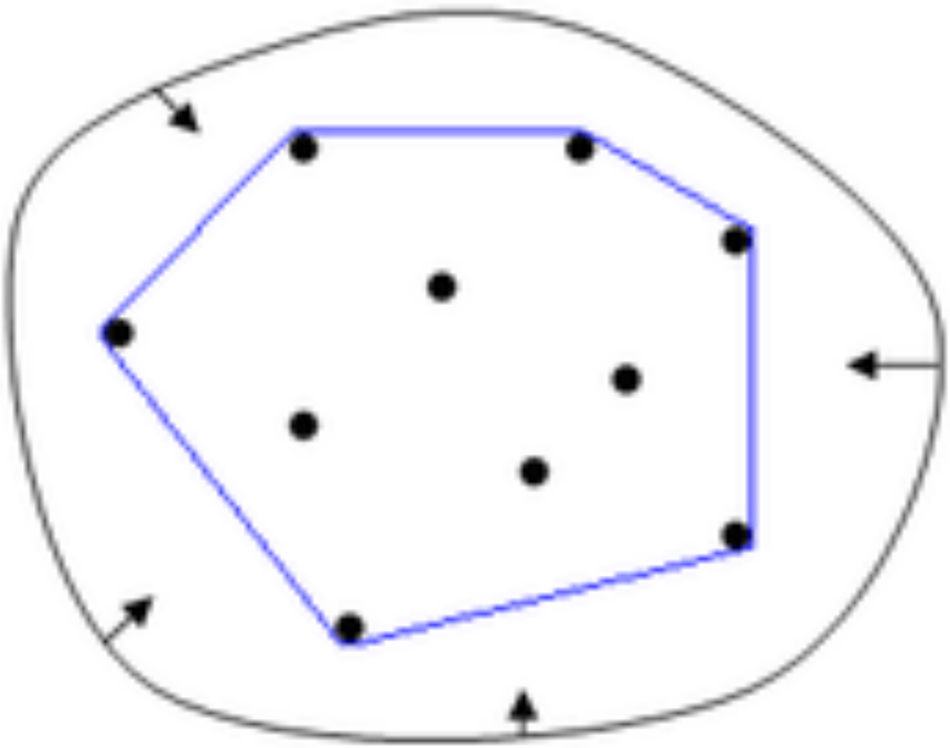
Step 1: Reflect Robot

C-space Algorithm



Step 3: ConvexHull (Vert (- Robot) + Vert (Obstacle))

Convex Hull Algorithm



C-obstacle with Rotations

simple 2D workspace obstacle
=> complicated 3D C-obstacle

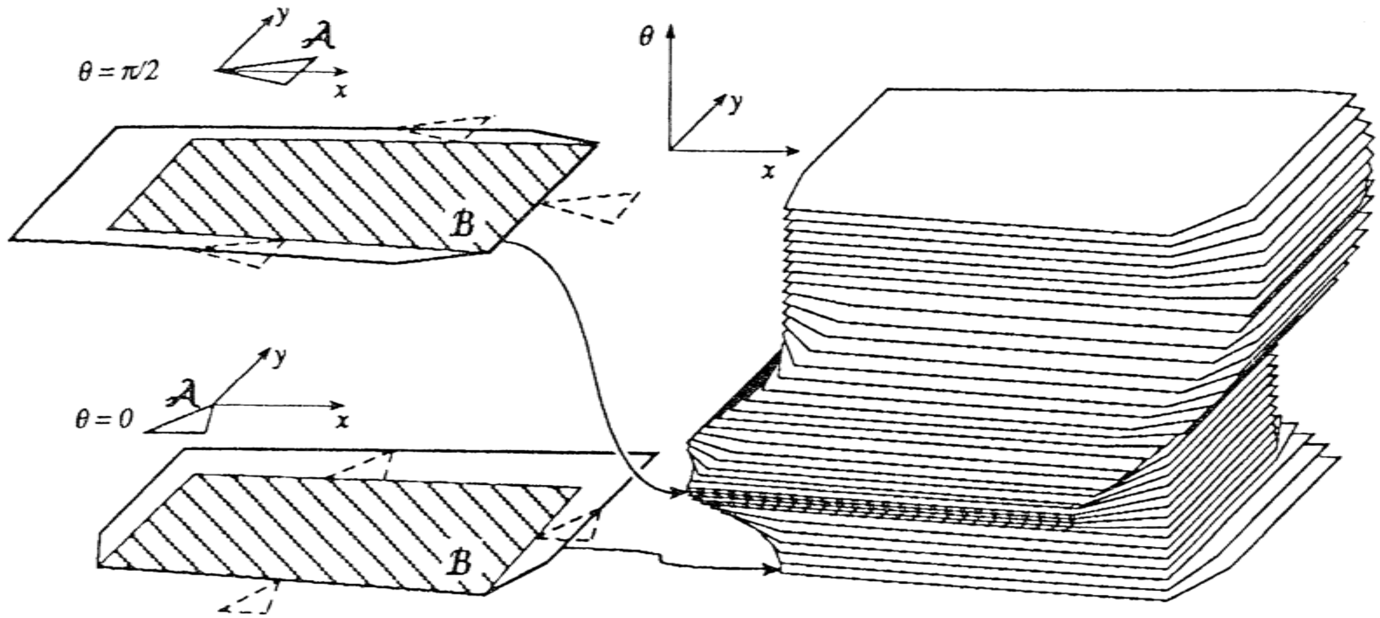


Figure from Latombe'91

Motion Planning Algorithm

(1) Compute c-obstacle for each obstacle

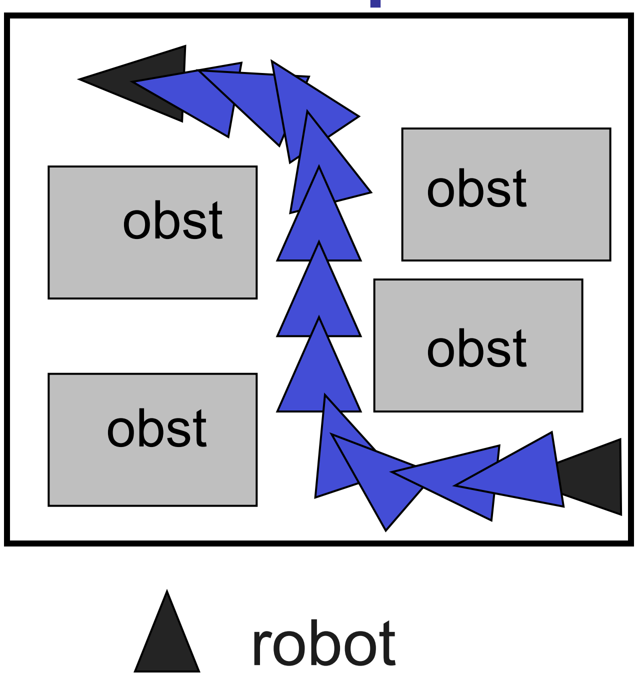
(Reflect points, Minkowsky sums, convex hull)

(2) Find path from start to goal for point robot

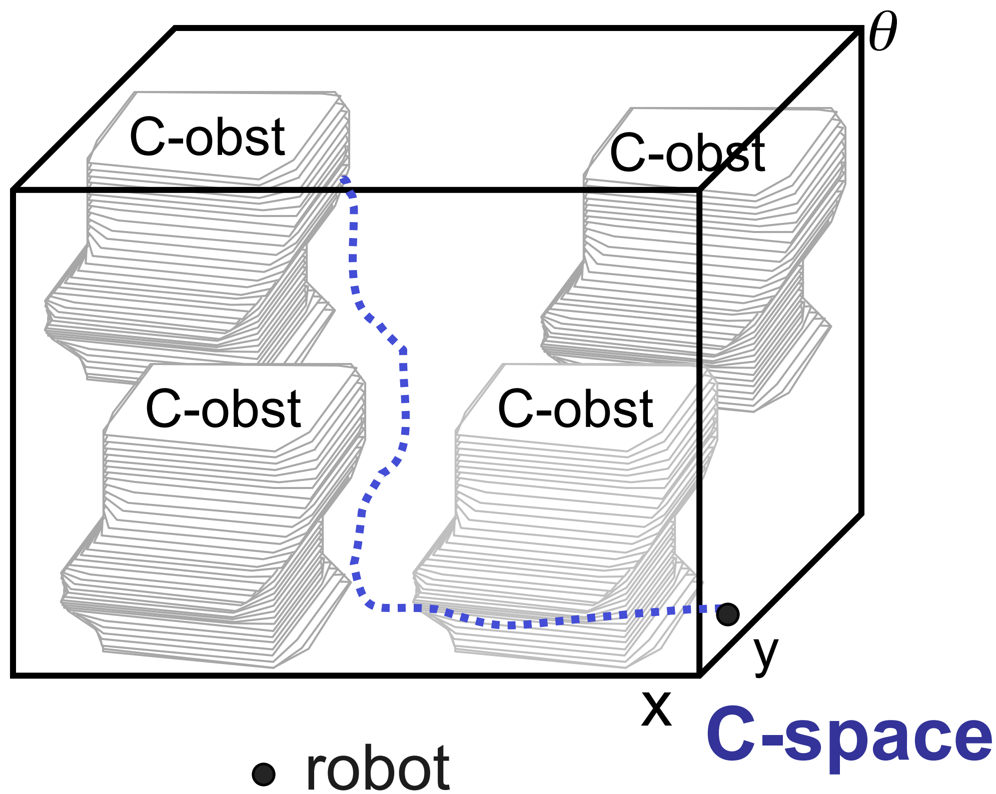
- The robots DOF dictate (1)
- The method for (2) differentiates among motion planning algorithms

Motion Planning Summary

Workspace



Path is swept volume



Path is 1D curve

How do we find the path? Recall Visibility Graphs

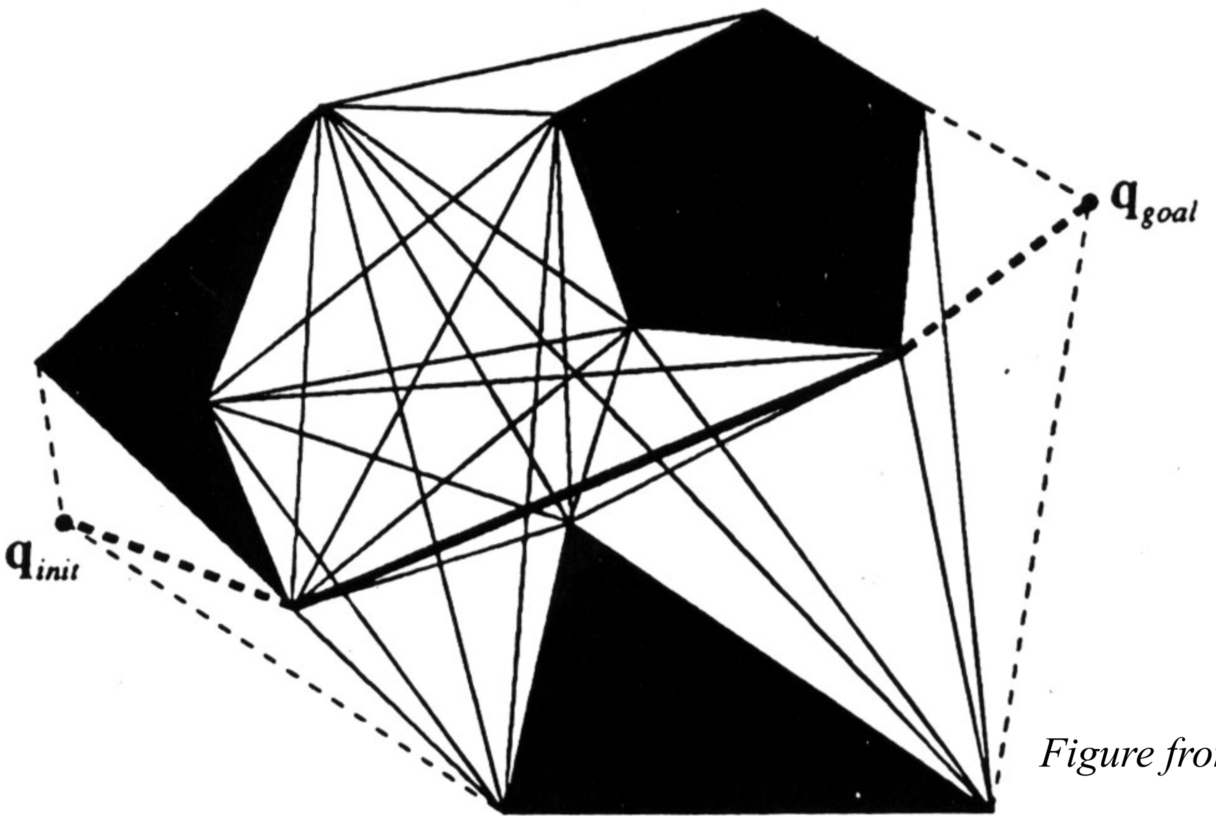


Figure from Latombe'91

In 2D the V-graph method finds the shortest path from S to G
What about 3D?

How hard is this to compute? The Complexity of Motion Plannin

Most motion planning problems are PSPACE-hard
[Reif 79, Hopcroft et al. 84 & 86]

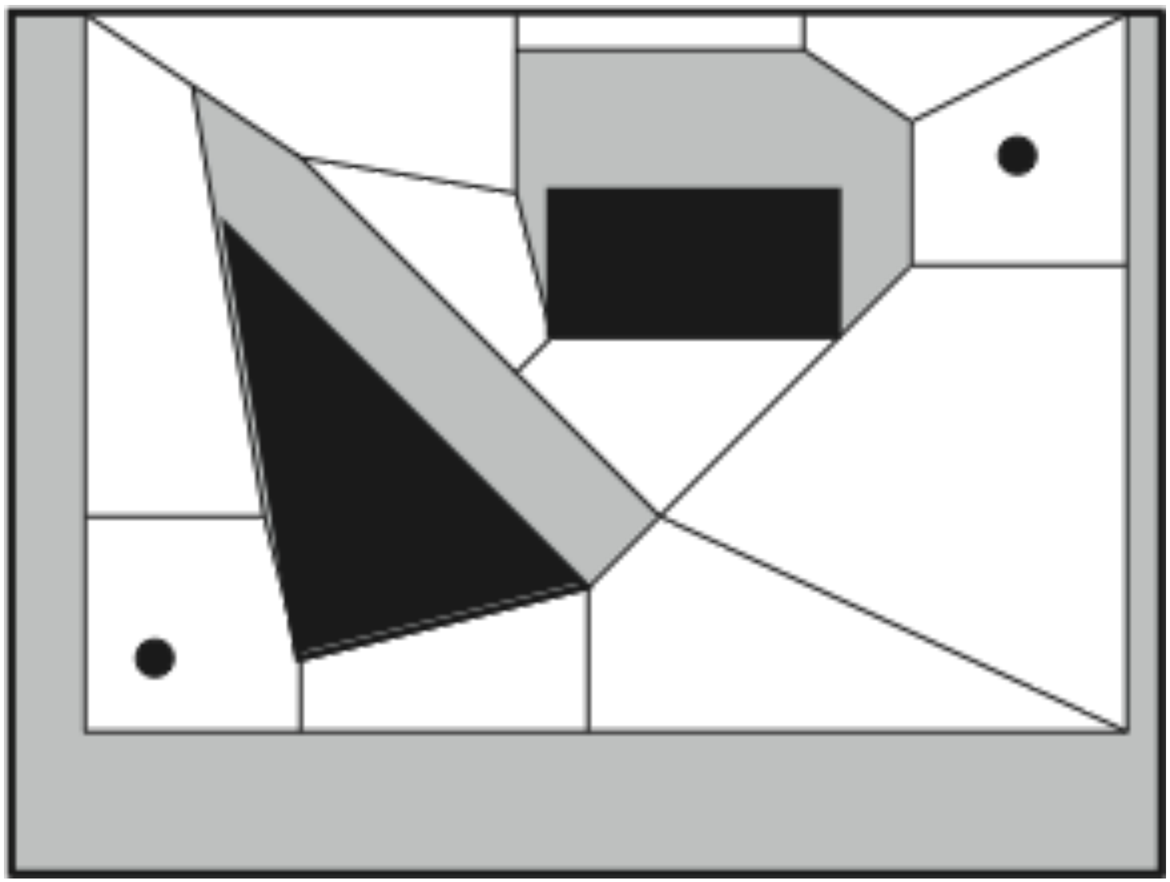
The best deterministic algorithm known has running time that is exponential in the dimension of the robot's C-space **[Canny 86]**

- C-space has high dimension - 6D for rigid body in 3-space
- simple obstacles have complex C-obstacles ➡ impractical to compute explicit representation of freespace for high dof robots

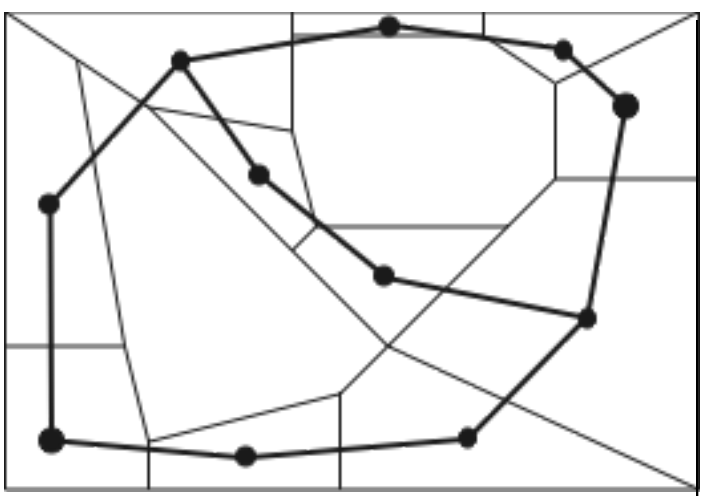
So … attention has turned to <u>*approximation and randomized algorithms*</u> which

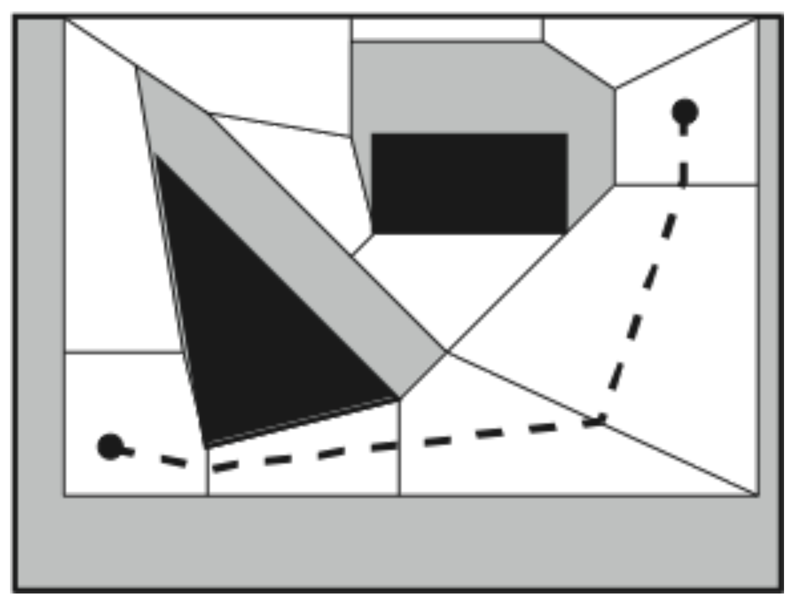
- trade full completeness of the planner
- for a major <u>*gain in efficiency*</u>

Exact Cell Decomposition for finding path



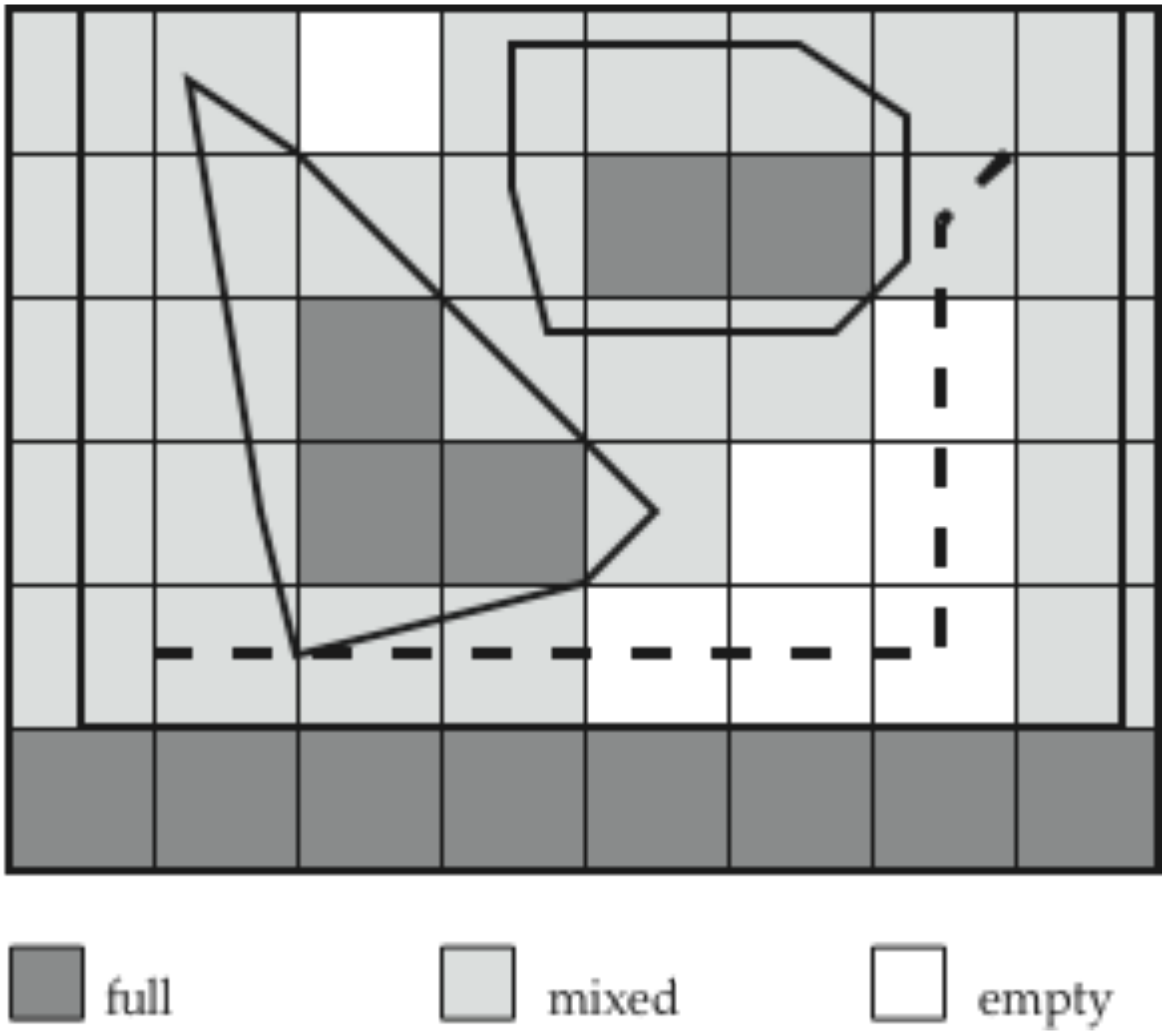
Searching the Convex Cells for finding path



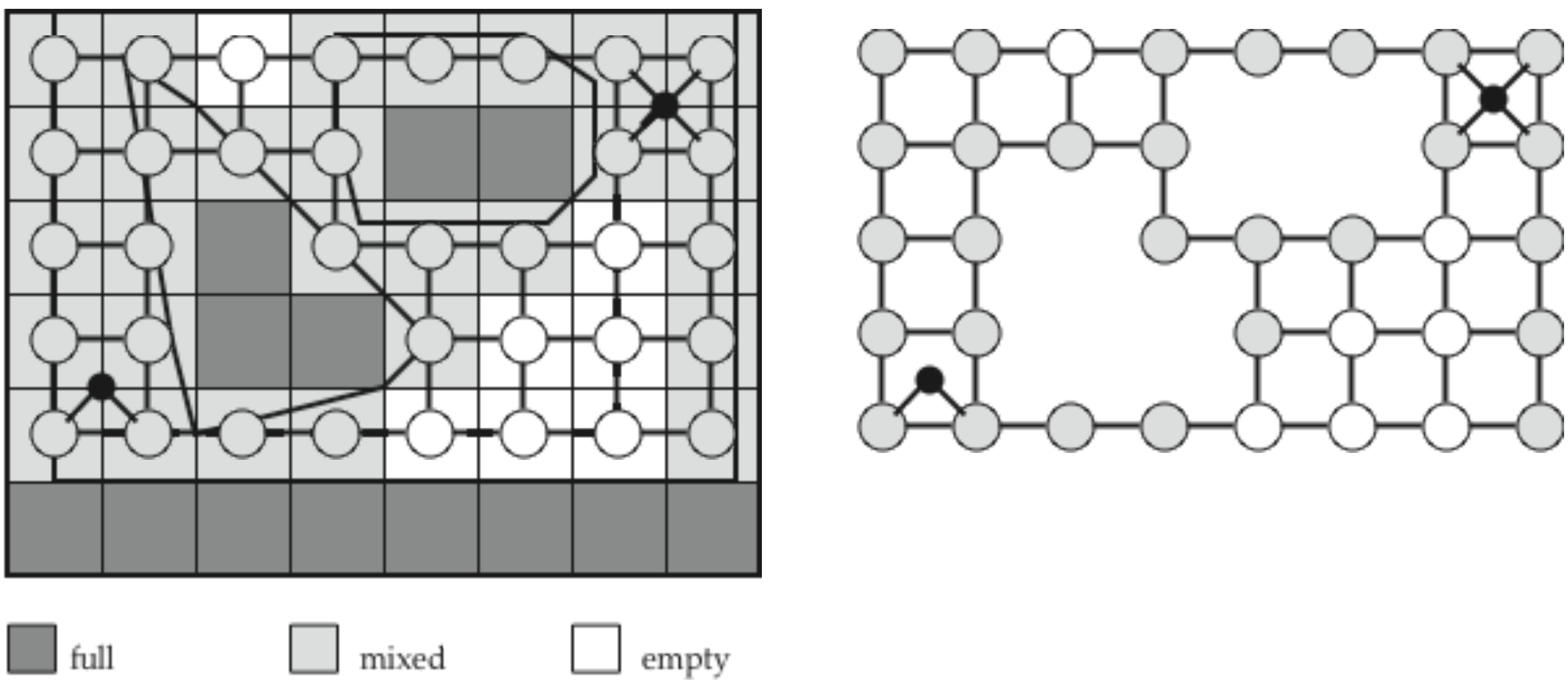


Build graph
Search for path

Approximate Cell Decomposition

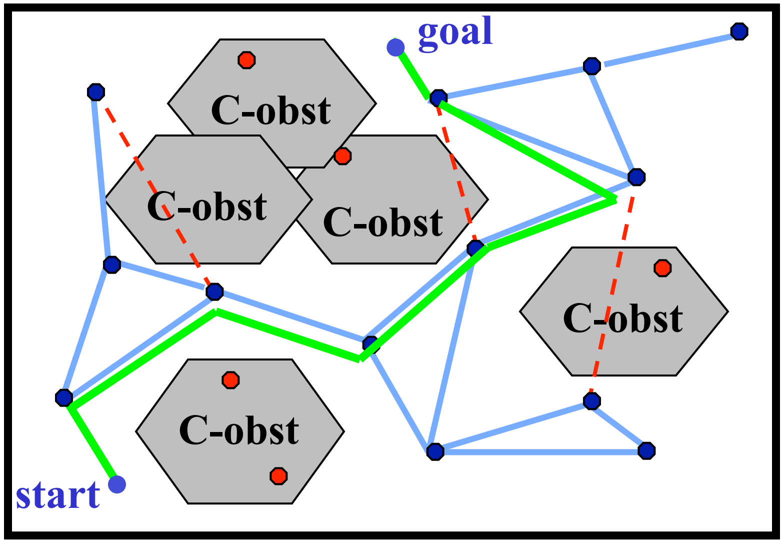


Cell Connectivity Graph



Probabilistic Road Maps (PRM) for finding paths [Kavraki at al 96]

C-space



Roadmap Construction (Pre-processing)

1. Randomly generate robot configurations (nodes)
   - discard nodes that are invalid
2. Connect pairs of nodes to form **roadmap**
   - simple, deterministic *local planner* (e.g., straightline)
   - discard paths that are invalid

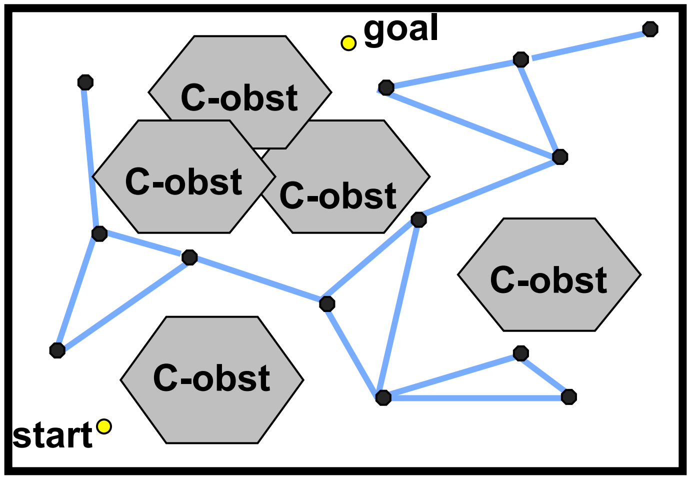
Query processing

1. Connect *start* and *goal* to roadmap
2. Find path in roadmap between *start* and *goal*
   - regenerate plans for edges in roadmap

Primitives Required:

1. Method for Sampling points in C-Space
2. Method for `validating' points in C-Space

More PRMS

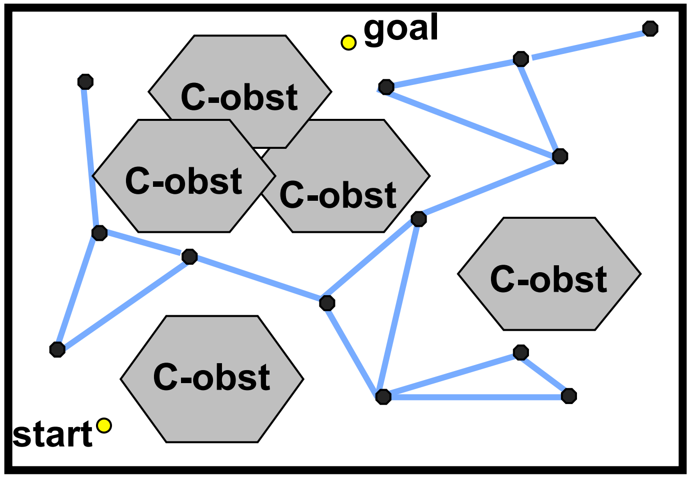


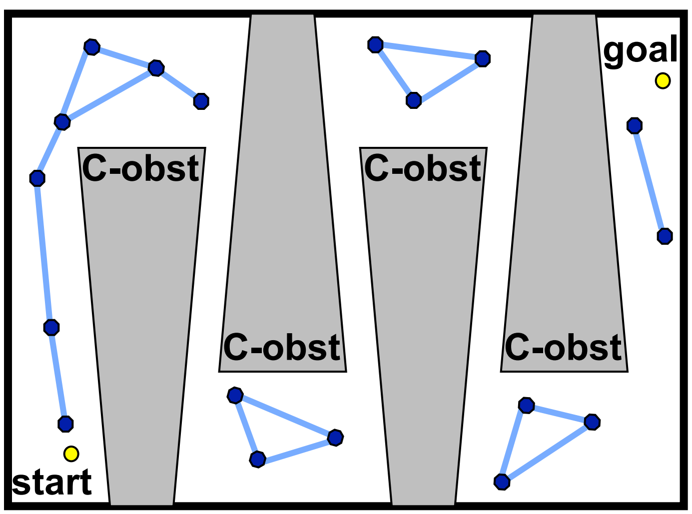
PRMs: Pros

1. PRMs are *probabilistically complete*
2. PRMs apply easily to high-dimensional C-space
3. PRMs support fast queries w/ enough preprocessing

Many success stories where PRMs solve previously unsolved problems

More PRMS





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Many success stories where PRMs solve previously unsolved problems

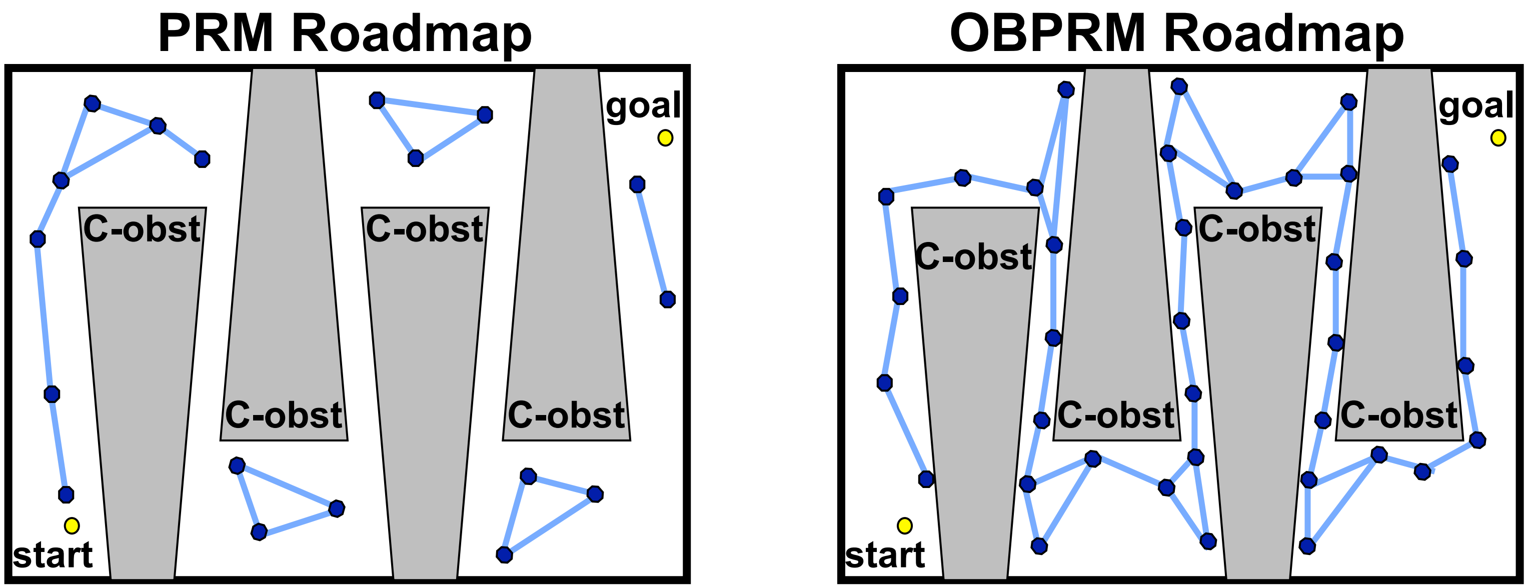
PRMs: Cons

1. PRMs don’t work as well for some problems:
– unlikely to sample nodes in *narrow passages*
– hard to sample/connect nodes on constraint surfaces

Sampling Around Obstacles
[Amato et al 98]

To Navigate Narrow Passages we must sample in them

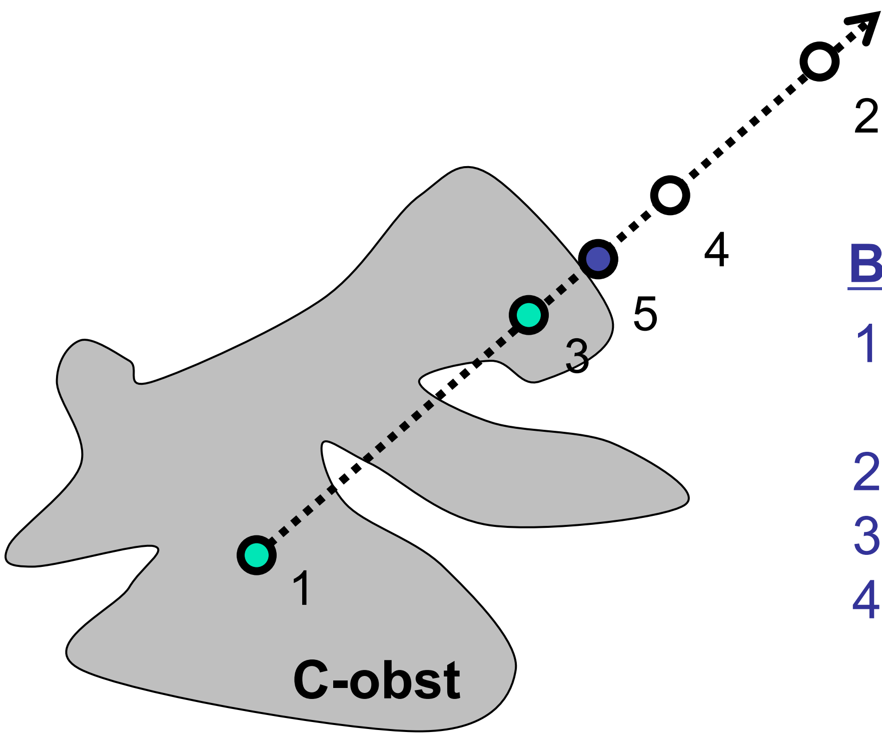
- most PRM nodes are where planning is easy (not needed)



Idea: Can we sample nodes near C-obstacle surfaces?

- we cannot explicitly construct the C-obstacles...
- we do have models of the (workspace) obstacles...

OBPRM: Finding points on C-obstacles



Basic Idea (for workspace obstacle S)

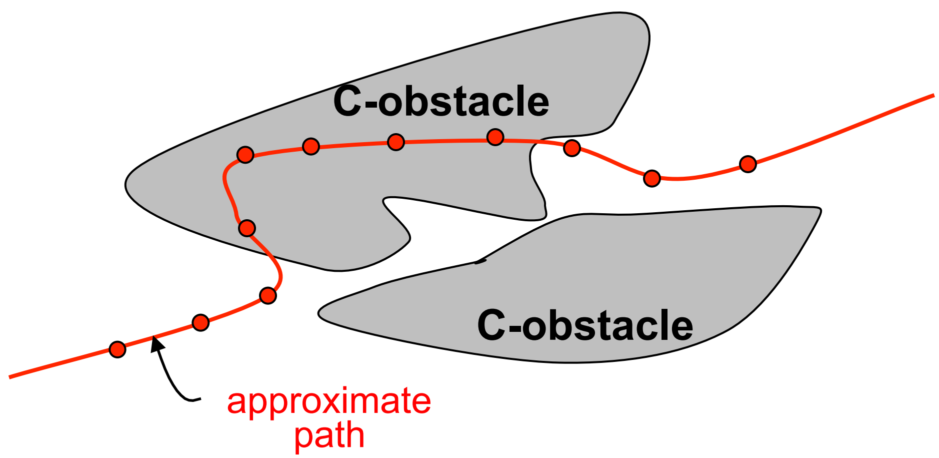
1. Find a point in S's C-obstacle (robot placement colliding with S)
2. Select a random direction in C-space
3. Find a free point in that direction
4. Find boundary point between them using binary search (collision checks)

Note: we can use more sophisticated approaches to try to cover C-obstacle

Repairing Paths [Amato et al]

Even with the best sampling methods, roadmaps may not contain valid solution paths

- may lack points in narrow passages
- may contain approximate paths that are nearly valid



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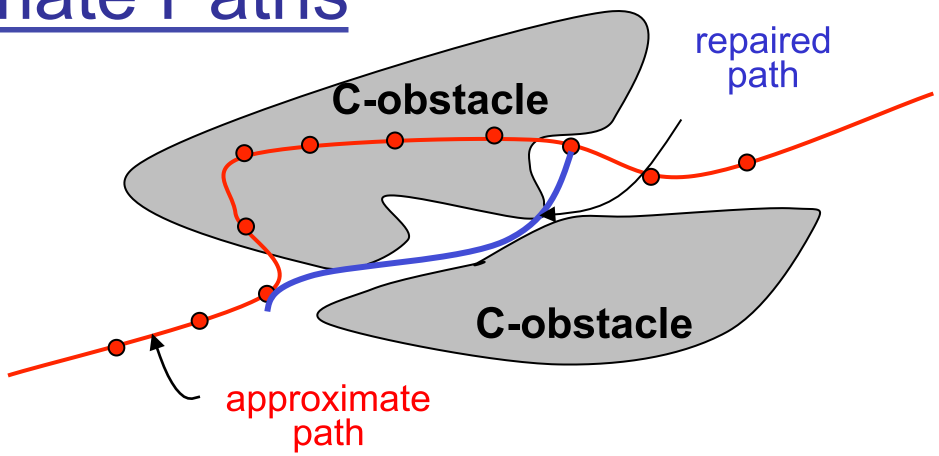
Repairing/Improving Approximate Paths

1. Create initial roadmap
2. Extract *approximate path P*
3. Repair P (push to C-free)
   - Focus search around P
   - Use OBPRM-like techniques



Rapidly-Exploring Random Tree (RRT)

- **[LaValle and Kuffner 2001]**

- Easy to implement
- Quickly finds an answer for a large variety of systems
- Works in high dimensional spaces

RRT Algorithm

- Given
  - Obstacles
  - Start state
  - Goal set, G

1. $t =$ tree rooted at *start*
2. **repeat**
3. $\quad s =$ random configuration
4. $\quad n =$ nearest node to s in t
5. $\quad$ attempt to connect n to s with a path
6. **until** t contains a point in G

RRT Algorithm

- Given
  - Obstacles
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  - Goal set, G

1. t = tree rooted at *start*

2. repeat

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metric function

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RRT Algorithm

- Given
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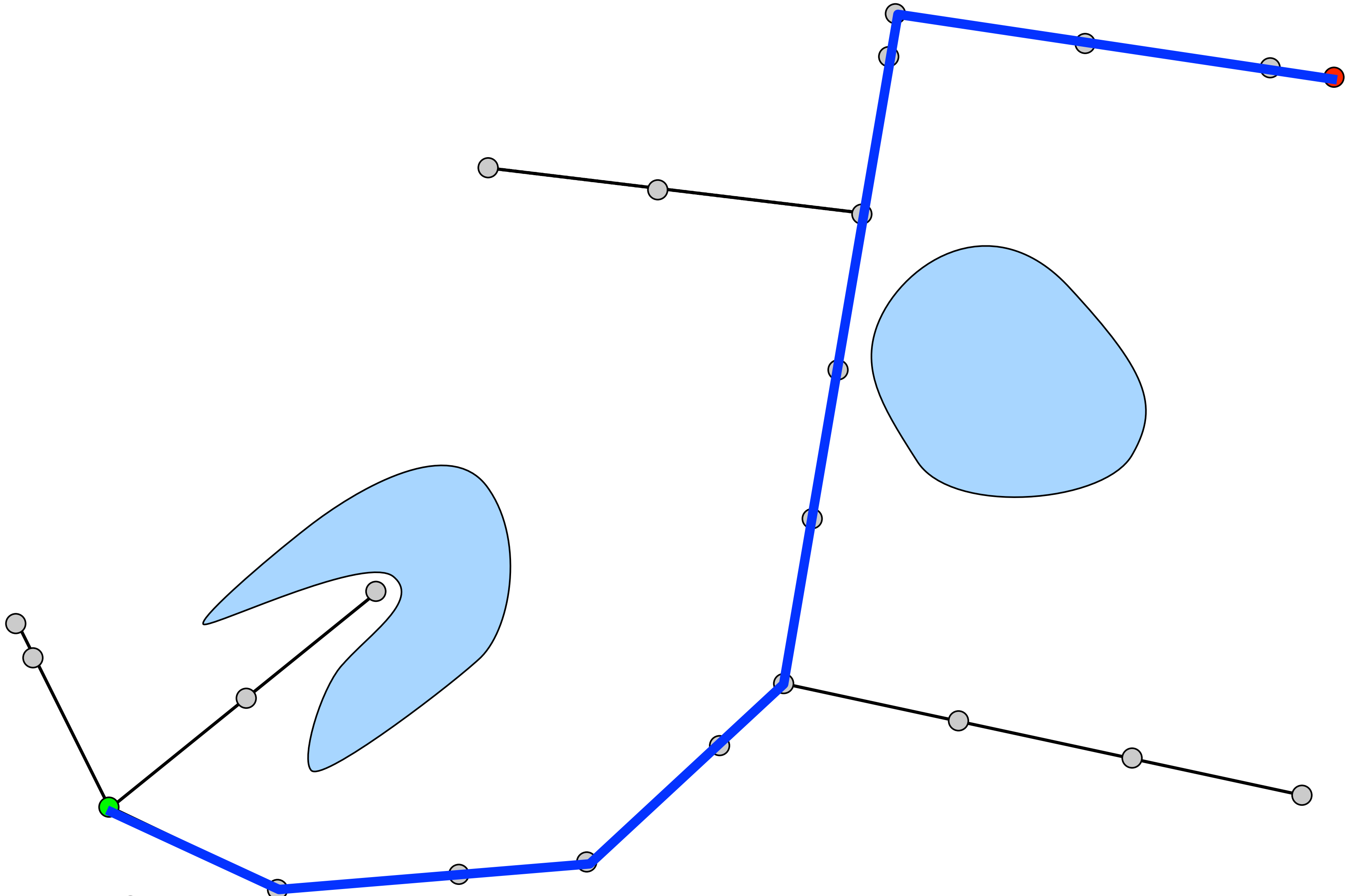
boundary value problem solver

RRT Example



Motion Planning Summary

- Compute c-space for each obstacle
- Compute representation
- Find path from start to goal
  - What should we optimize?