

# 6.141:Robotics systems and science

## Lecture 8: Motion Planning I

control architectures and c-space

Lecture Notes Prepared by Daniela Rus

EECS/MIT

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Thanks to Vijay Kumar

Reading: Chapter 3, and Craig: Robotics

<http://courses.csail.mit.edu/6.141/>

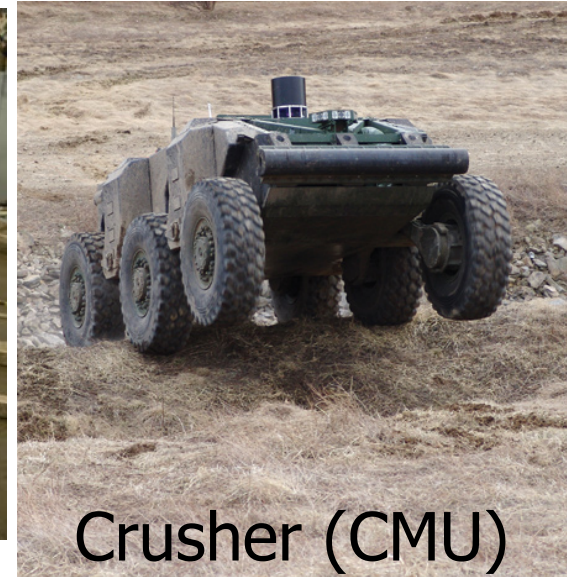
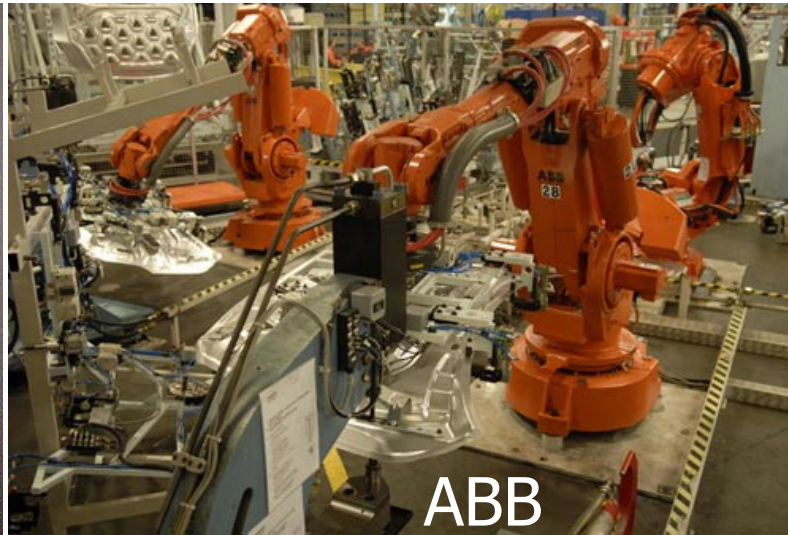
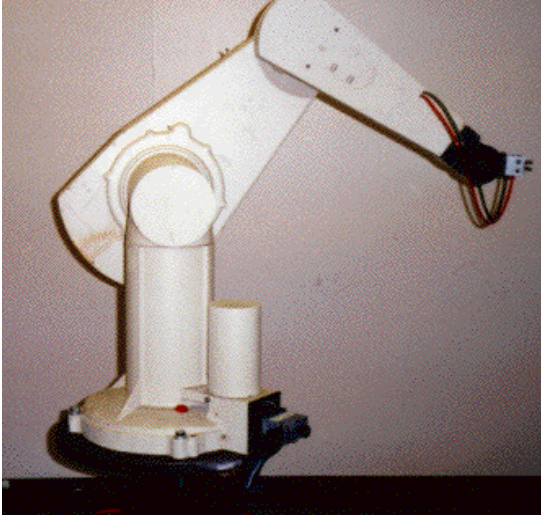
Challenge: Build a Shelter on Mars

# Where are we?

- Last Module:
  - Vision
  - Software engineering
- Today:
  - Robot Control Architectures
  - Deliberative control: motion planning and c-spaces
- Reading: chapter 6

# Motion Planning

Unimation PUMA



Crusher (CMU)

Kawada HRP-2



HERB (CMU)

Willow Garage PR2



KUKA youBot

# What is motion planning?

## Objective:

Find a series of control actions that moves the robot from a start state to a goal condition, while respecting constraints and avoiding collision.

## Task planning:

Start and goal are expressed in terms of environment state rather than robot state

May require symbolic sequential reasoning

# Controlling in the large

- We have seen feedback control
- How do we put together multiple feedback controllers?
  - in what order?
  - with what priority?
- *How do we generate reliable and correct robot behavior?*

# Control Architecture

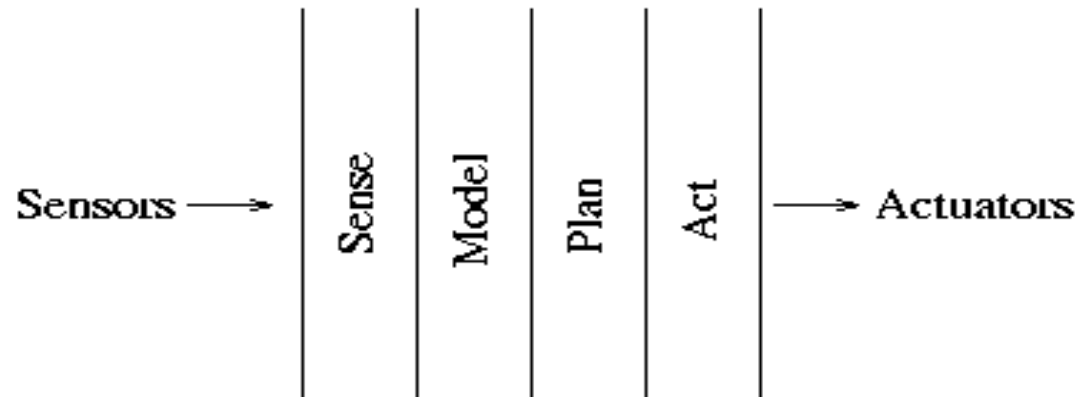
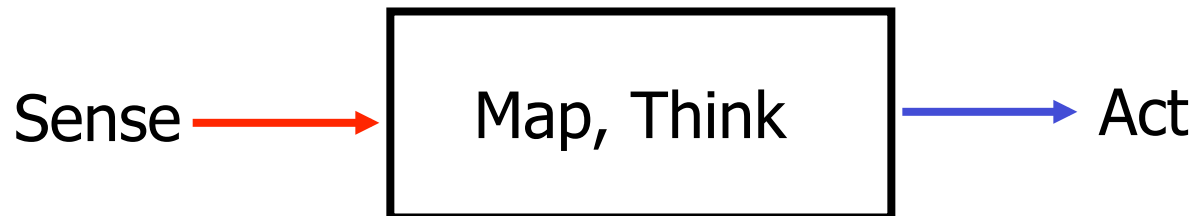
- A control architecture provides a set of principles for organizing a robot (software) control system.
- Like in computer architecture, it specifies building blocks
- It provides:
  - structure
  - constraints

# Control Architecture Types

- Deliberative control
- 
- Reactive control
- Hybrid control
- Behavior-based control

# Deliberative Architecture

- Maps, lots of state
- Look-ahead
- “Think hard, then act”





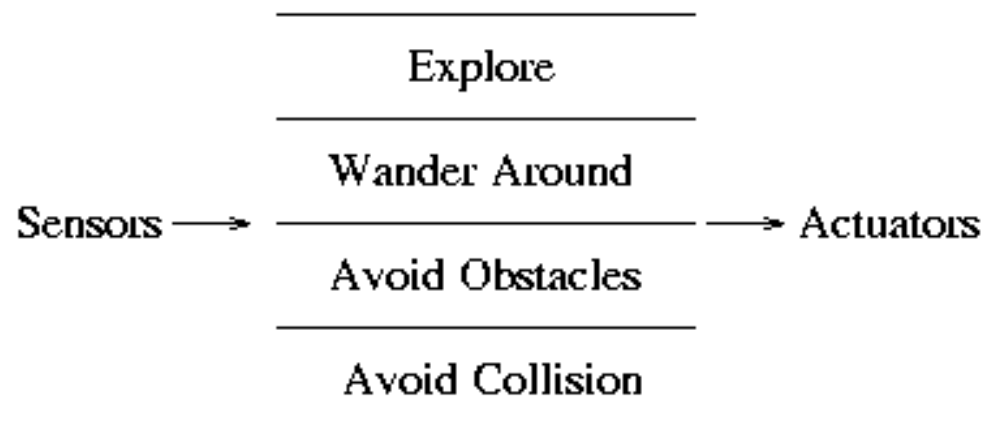
# Reactive Architecture

- No maps, no state
- No look ahead
- “Don’t think, react!”



# Behavior-based Architecture

- Multiple concurrent sense-act processes
- Each behavior uses local sensing to compute its best action
- Robot a combination of behaviors
- “Think the way you act”



# Criteria For Selection

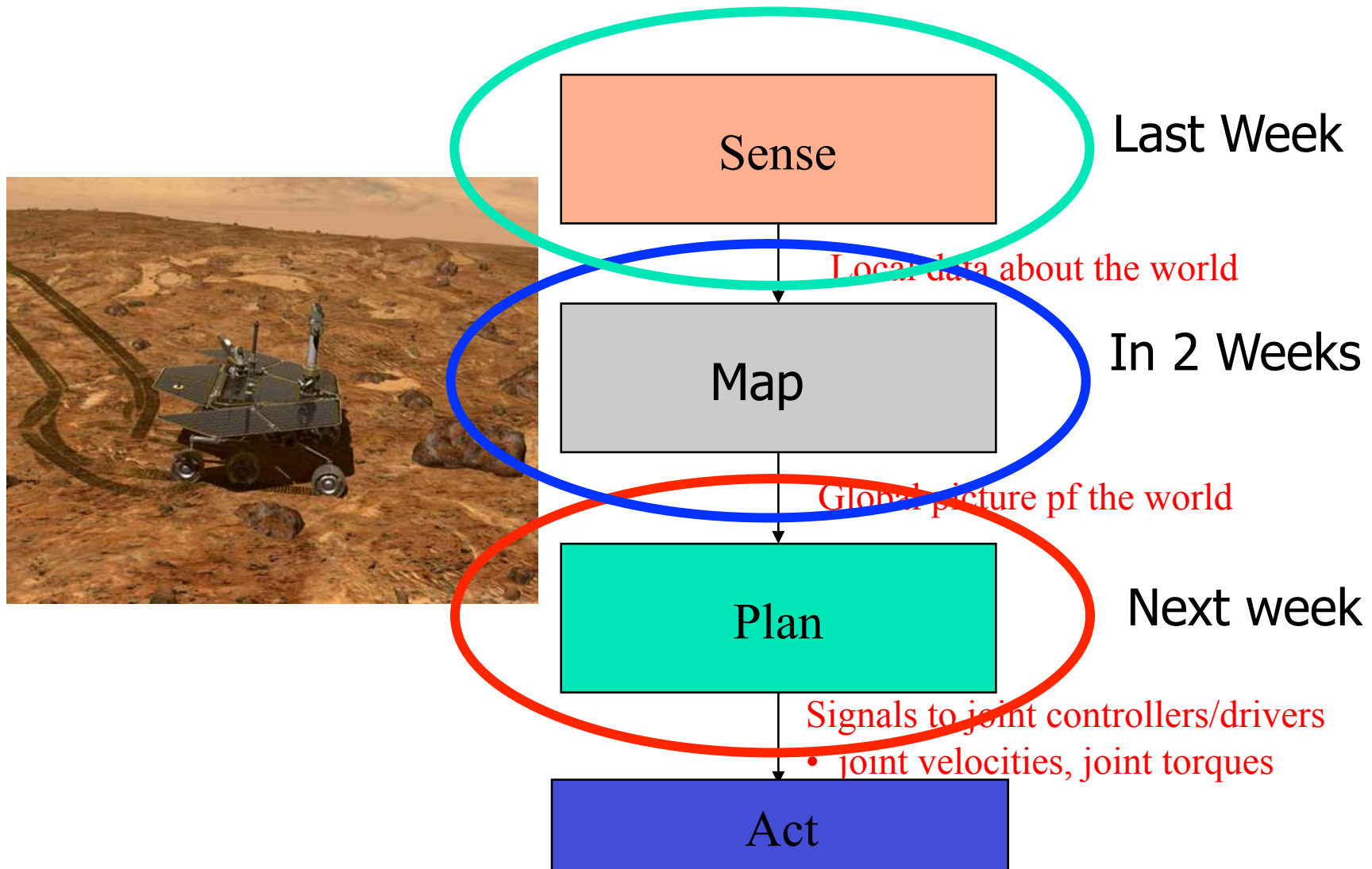
	deliberative	reactive	behavior
Task and environment			
Run-time constraints			
Correctness/ Completeness			
Hardware			

# Motion Planning

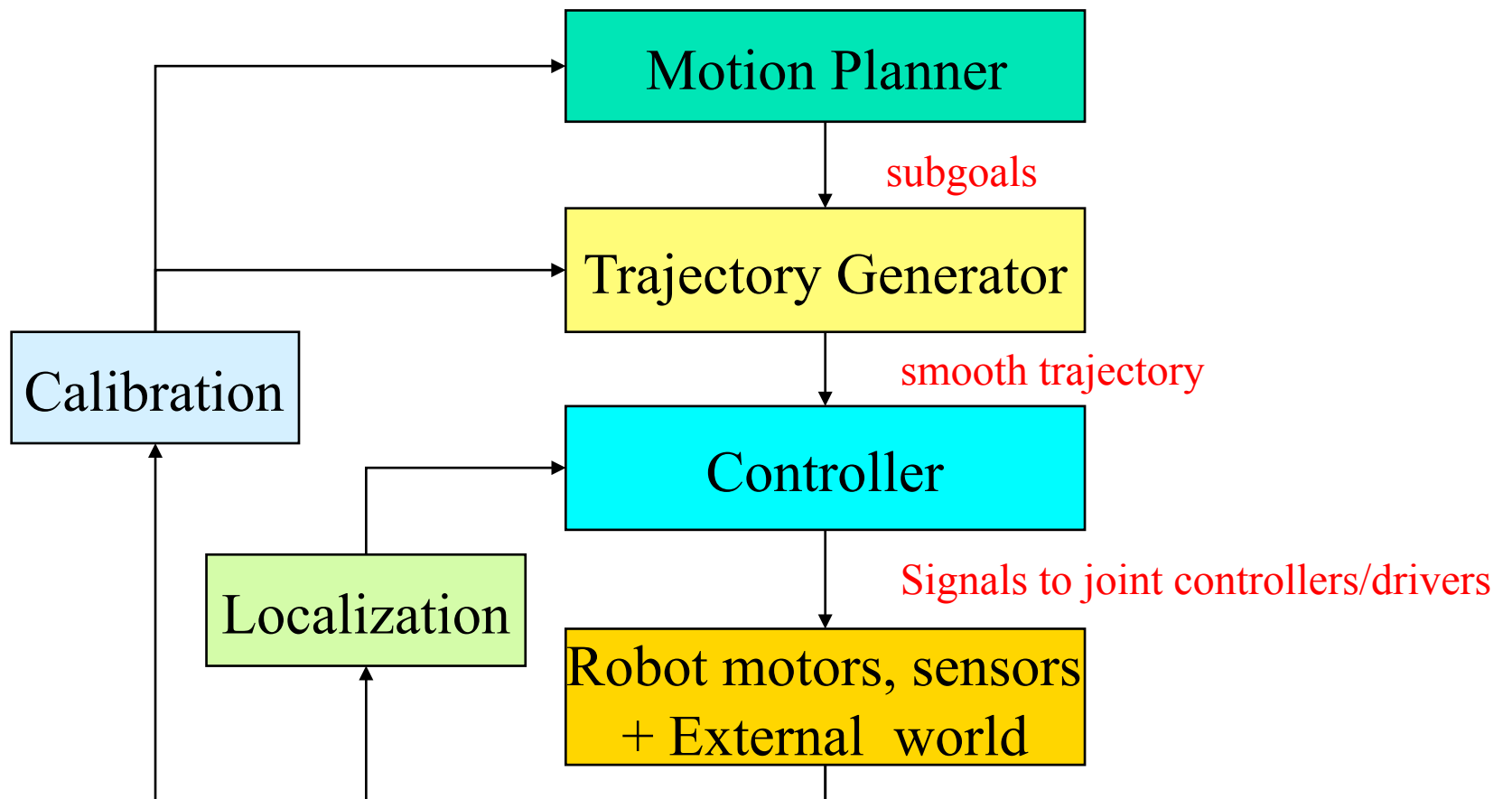
How do we command the robot to move from A to B despite complications?

Complications: error in maps, sensing, control, unexpected obstacles, etc.

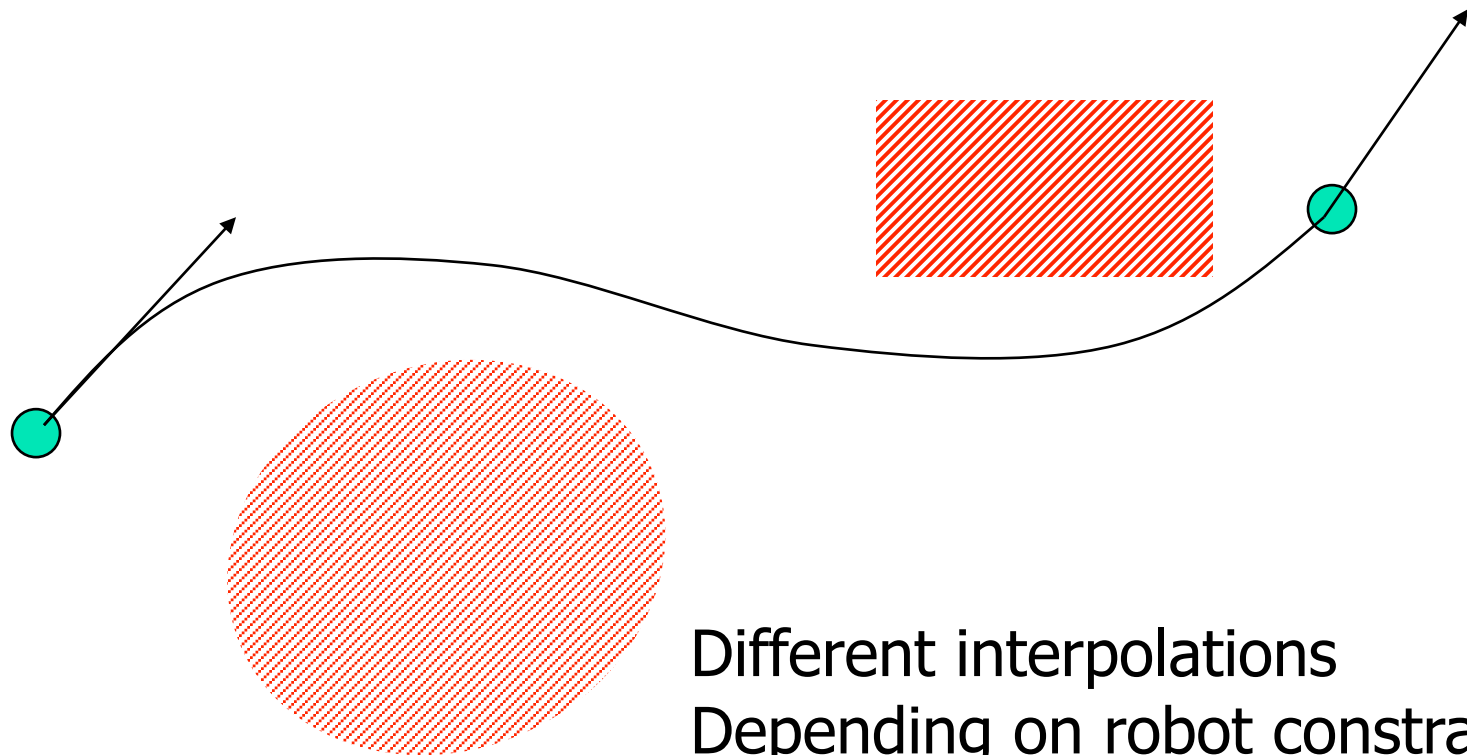
# Deliberative Architecture



# Motion Planning



# Trajectory generation from waypoints



Different interpolations  
Depending on robot constraints

# Motion Planning

- Known Environments (Model)

OFFLINE  
ALGORITHMS

- Unknown Environments (No Model)

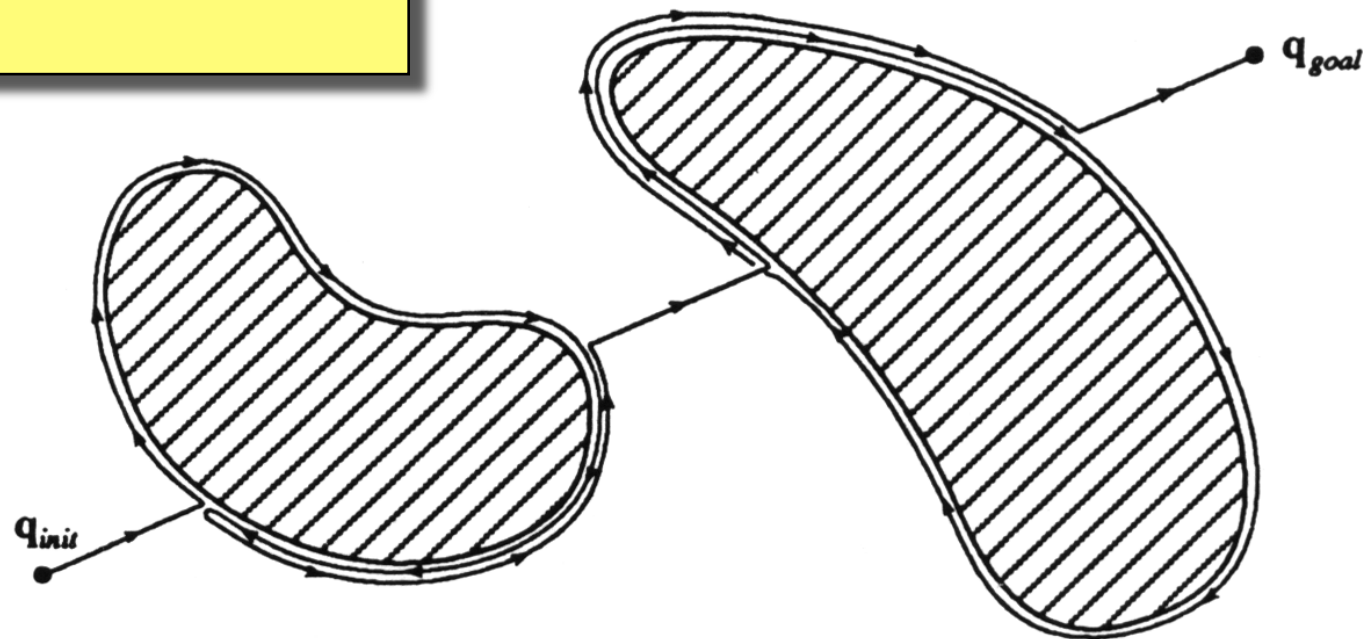
ONLINE  
ALGORITHMS

Example: how do we find a bridge in the fog?

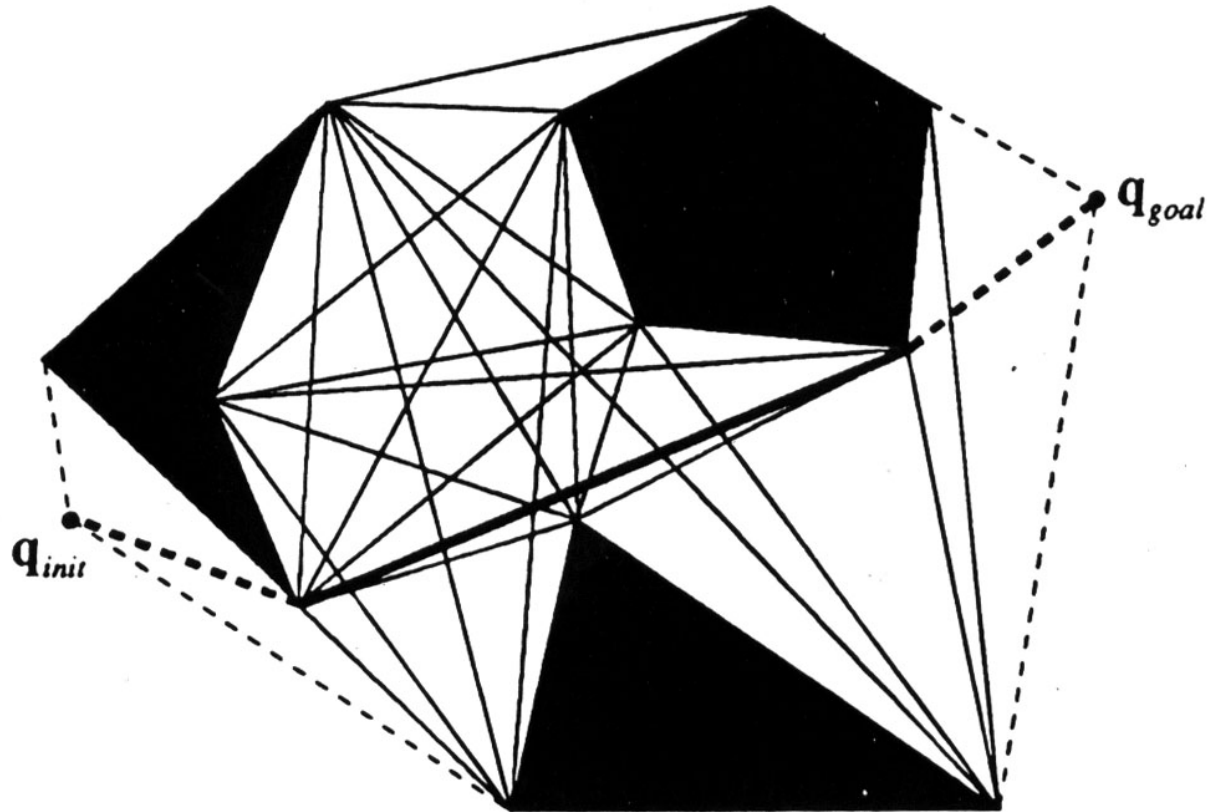


# Online Motion Planning

Always finds a path  
(if it exists)



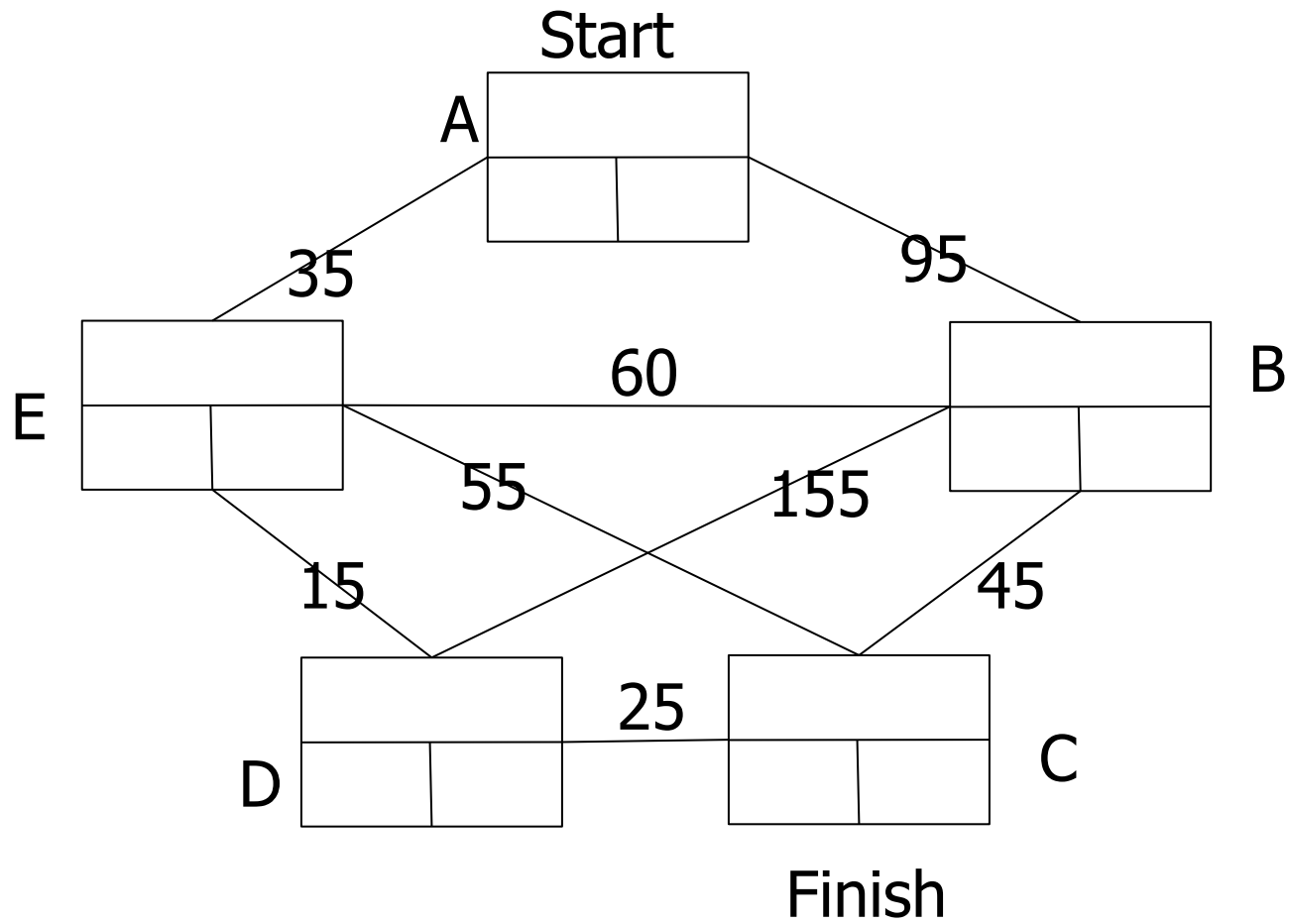
# Visibility Graphs



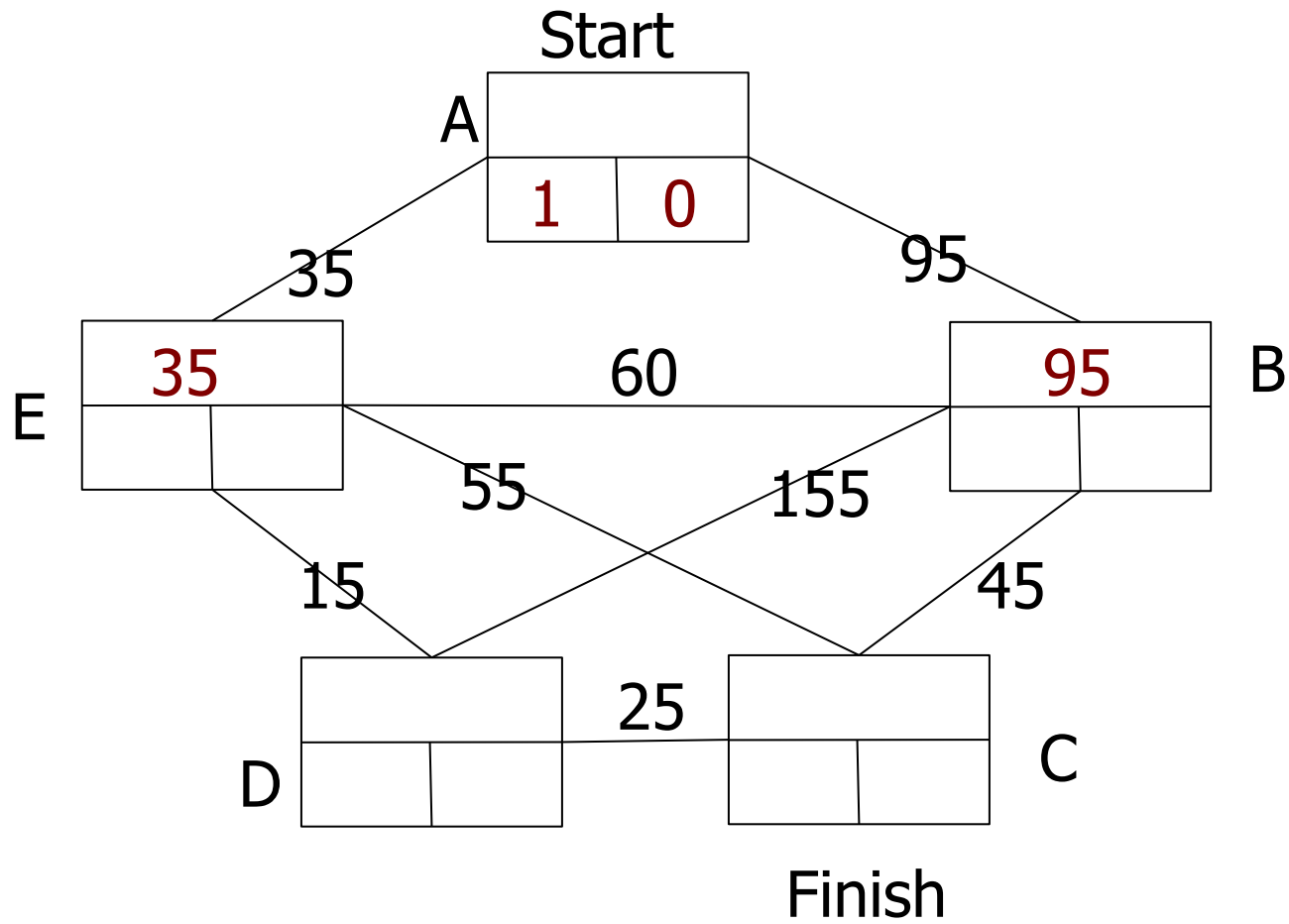
Vertices: Start, Goal,  
obstacle vertices

Edges: all combinations  $(v_i, v_j)$  that do not intersect any obstacle

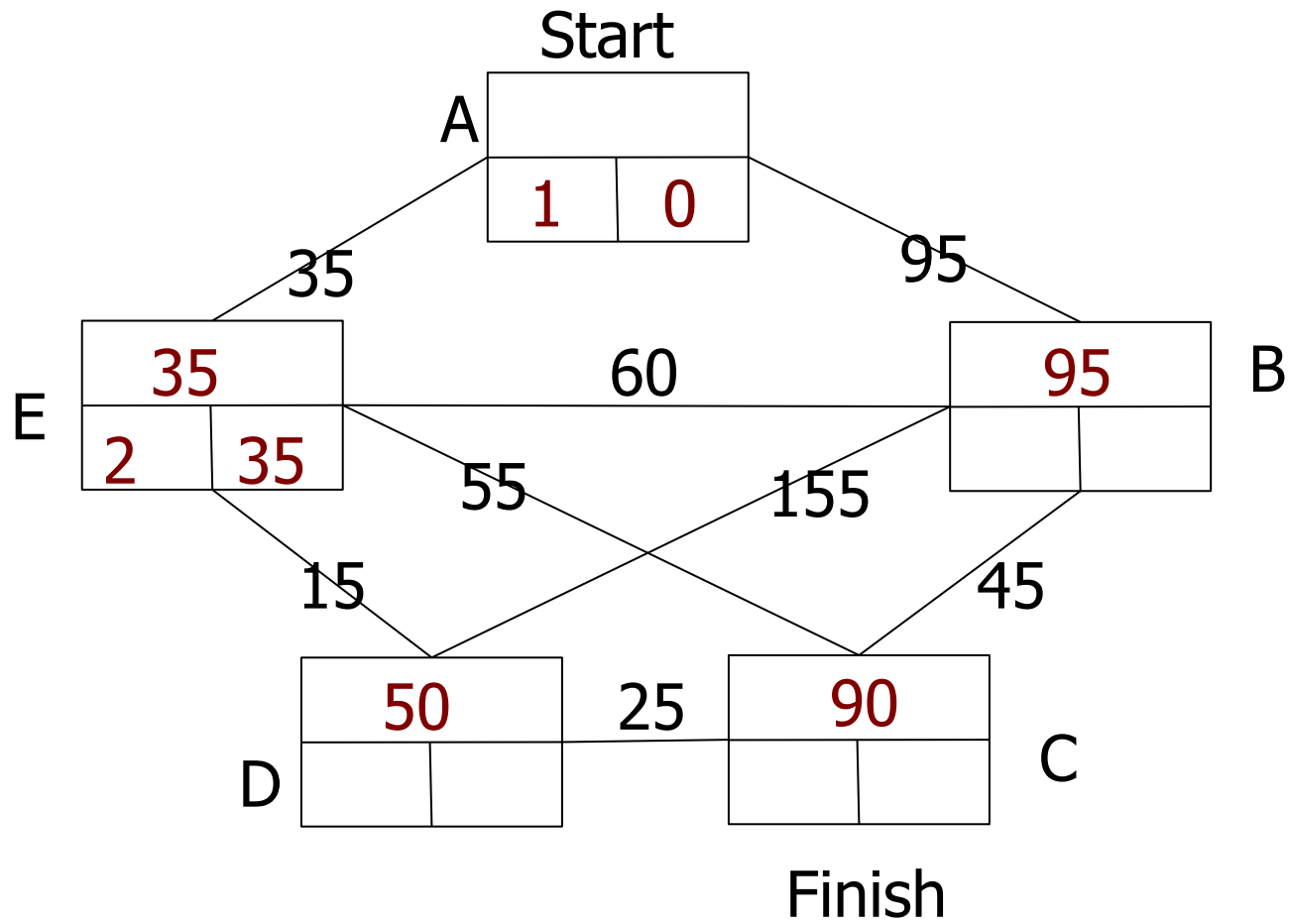
# Shortest path



# Shortest path

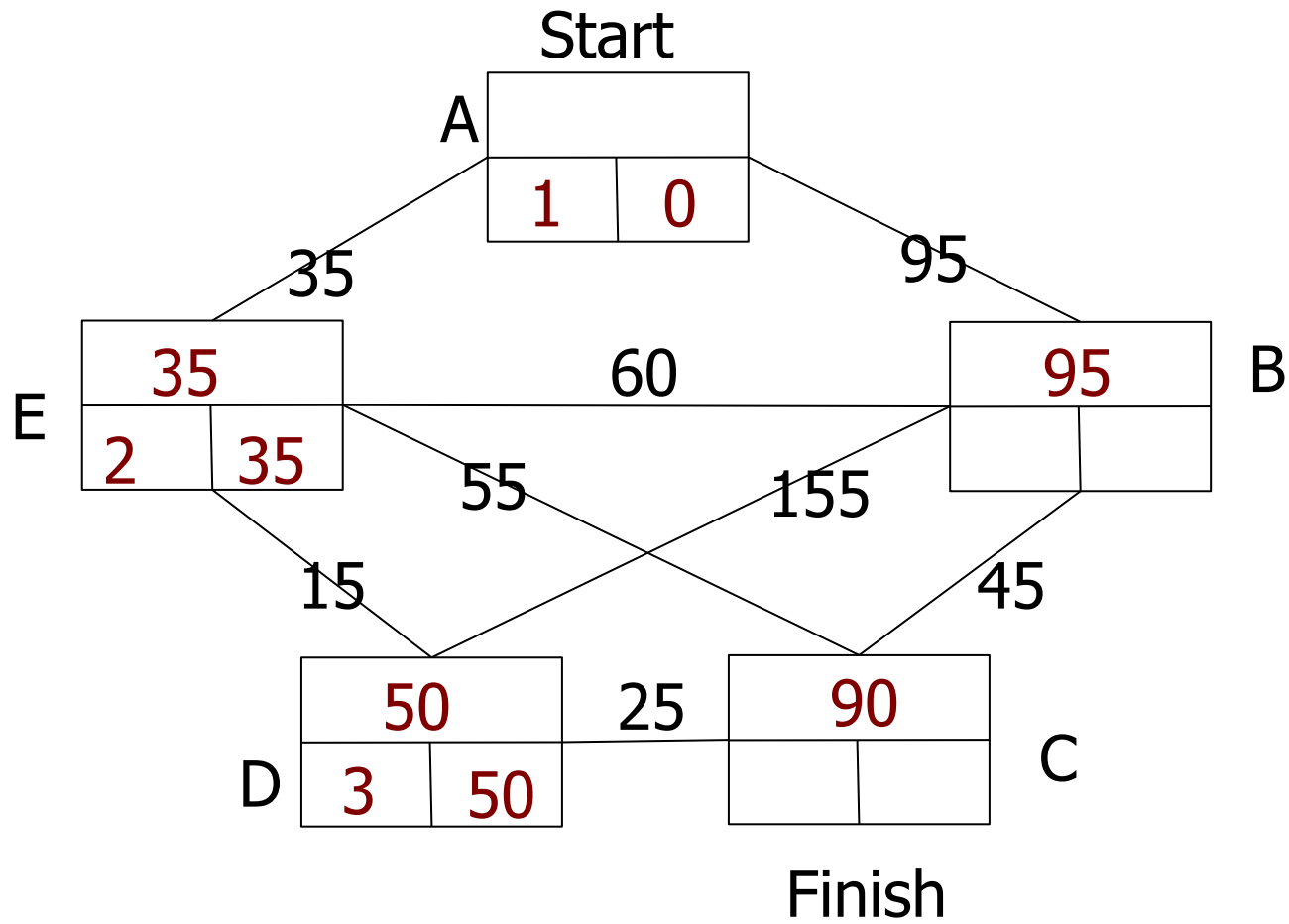


# Shortest path



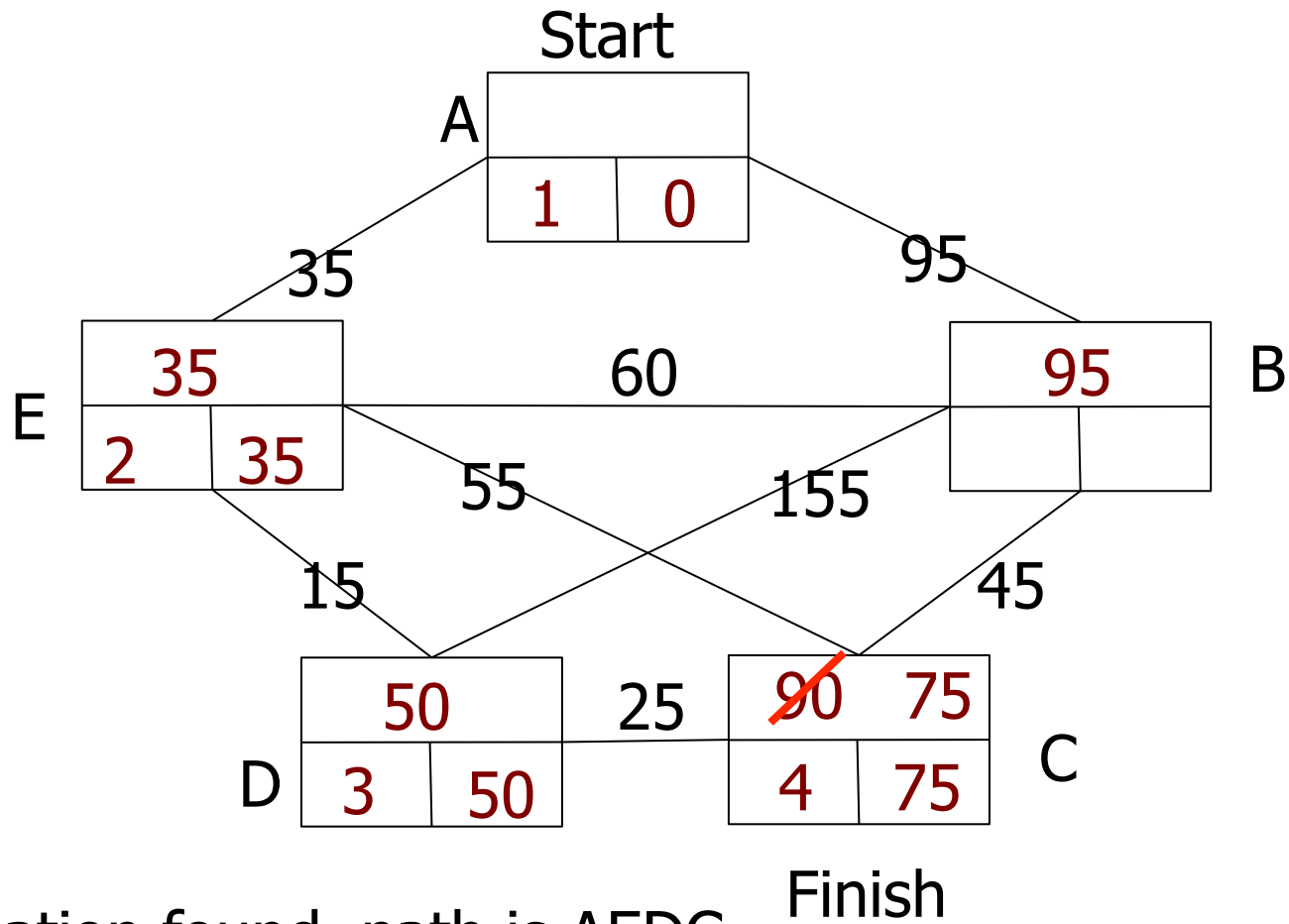
Find node with smallest temporary value; label neighbors

# Shortest path



Find node with smallest temporary value; label neighbors

# Shortest path



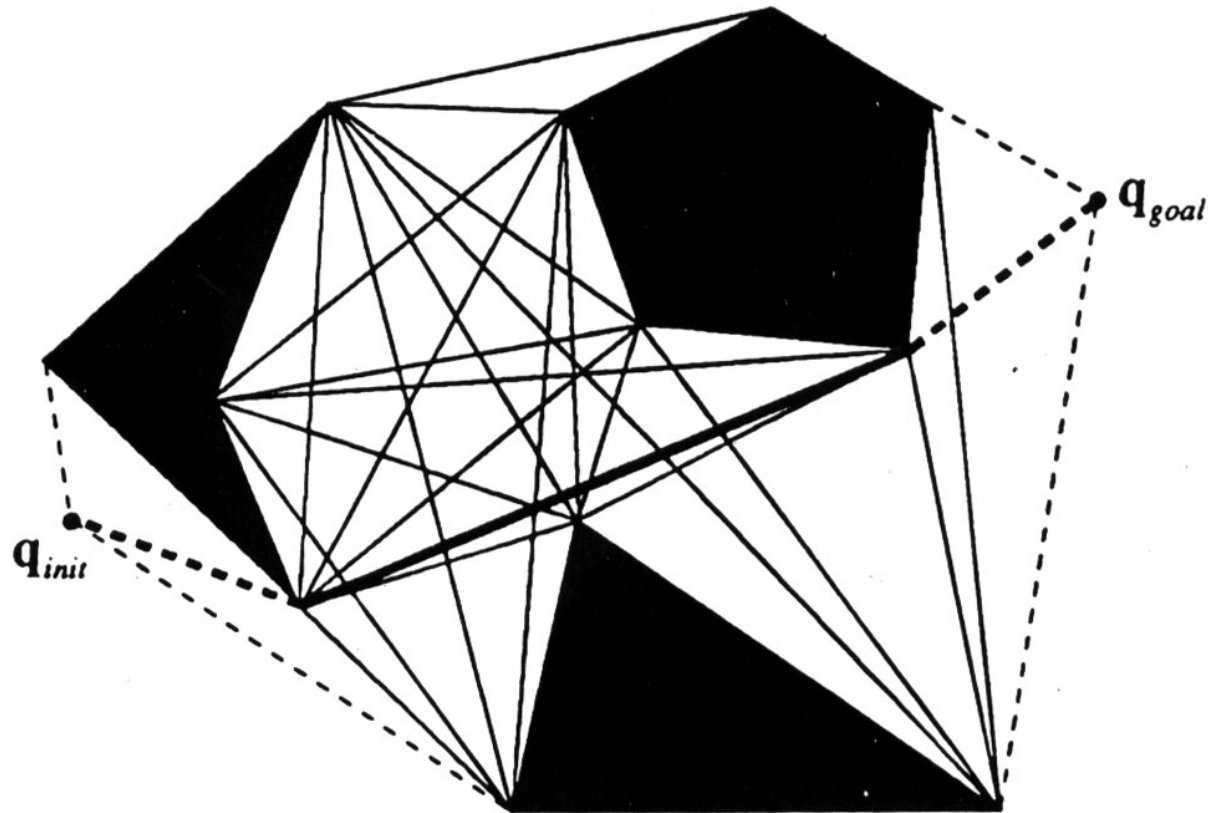
Destination found, path is AEDC

# Search Path: Dijkstra's Algorithm

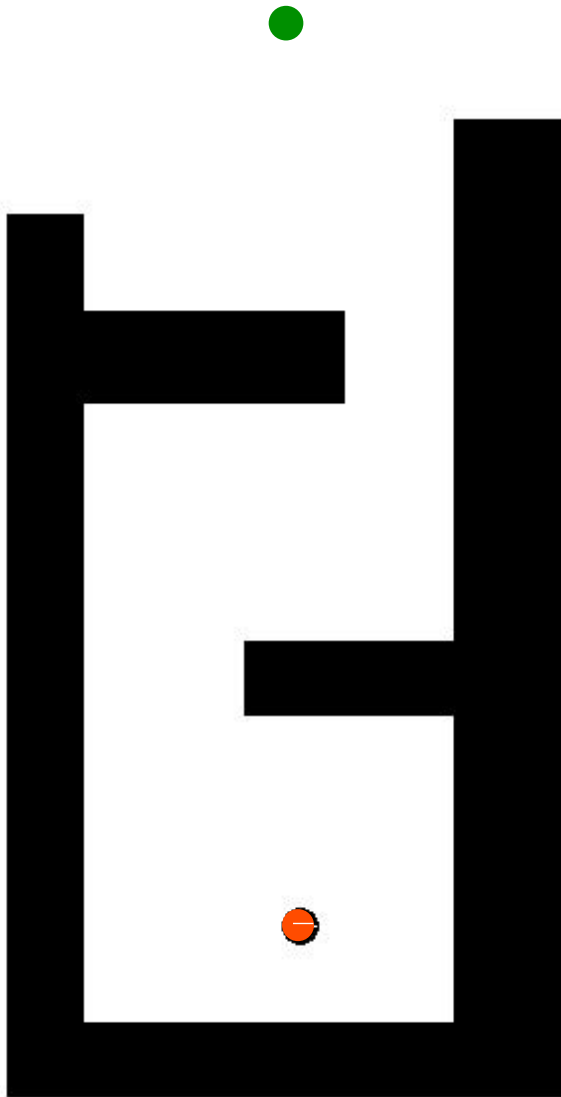
```
1 function Dijkstra(G, w, s)
2   for each vertex v in V[G]           // Initializations
3     d[v] := infinity
4     previous[v] := undefined
5   d[s] := 0
6   S := empty set
7   Q := set of all vertices
8   while Q is not an empty set       // The algorithm itself
9     u := Extract_Min(Q)              // O(n) for linked lists; Fib. Heaps?
10    S := S union {u}
11    for each edge (u,v) outgoing from u
12      if d[v] > d[u] + w(u,v)        // Relax (u,v)
13        d[v] := d[u] + w(u,v)
14        previous[v] := u
```



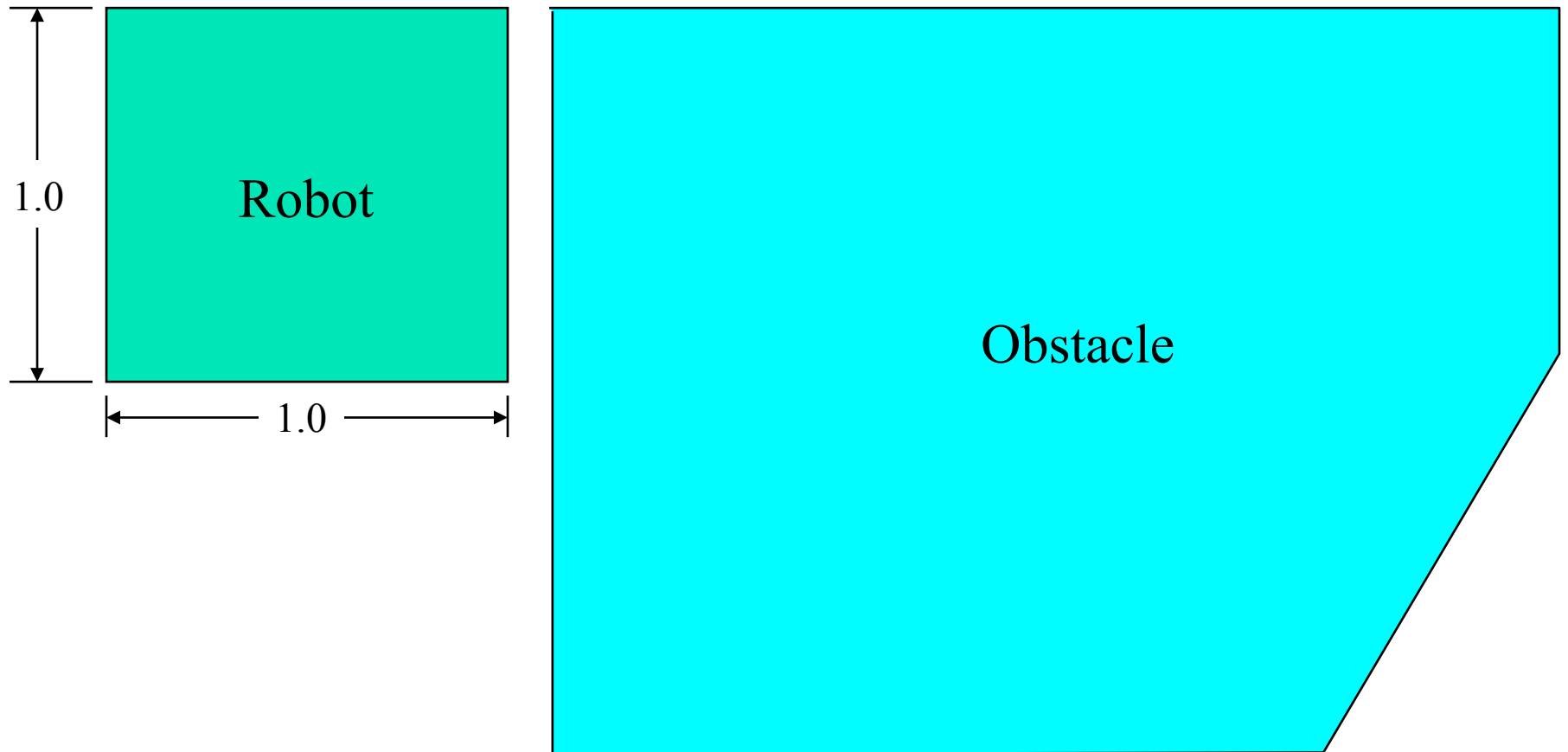
# Visibility Graphs Summary



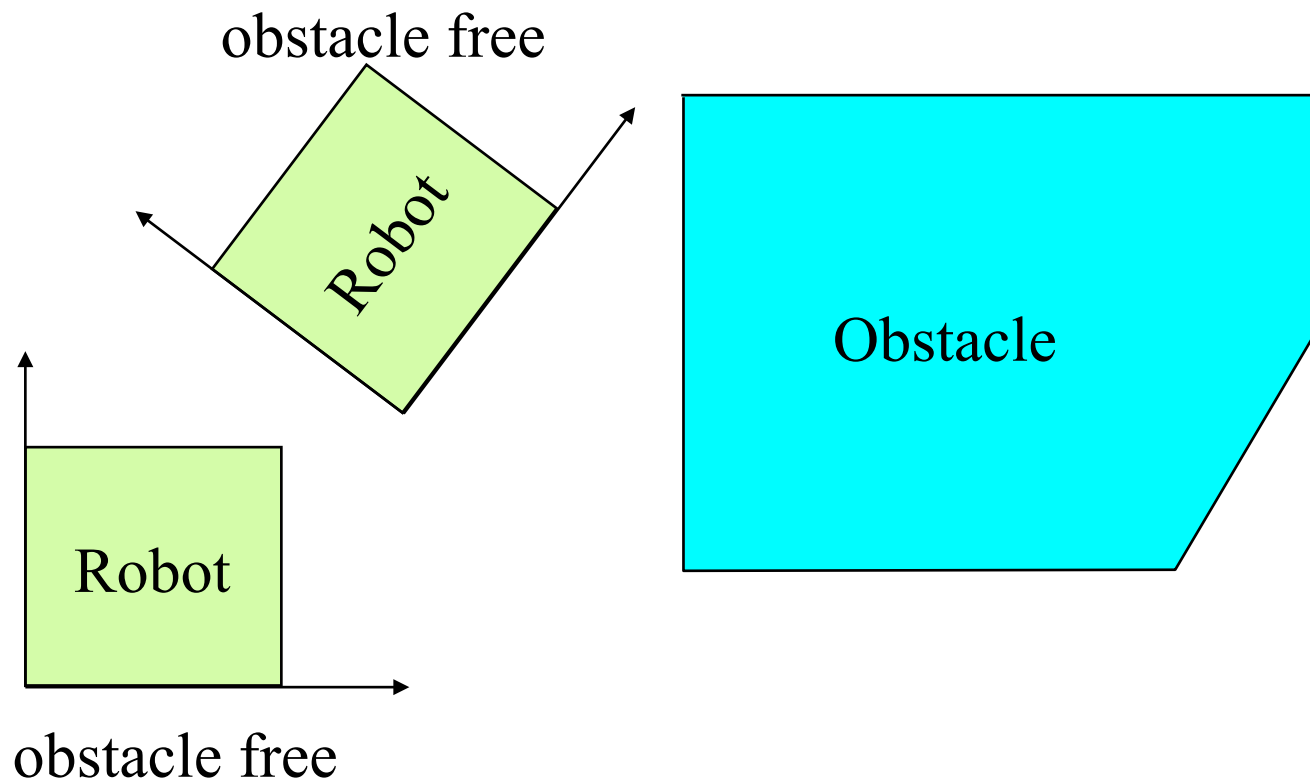
# What if the robot is not a point?



# What if the robot is not a point?

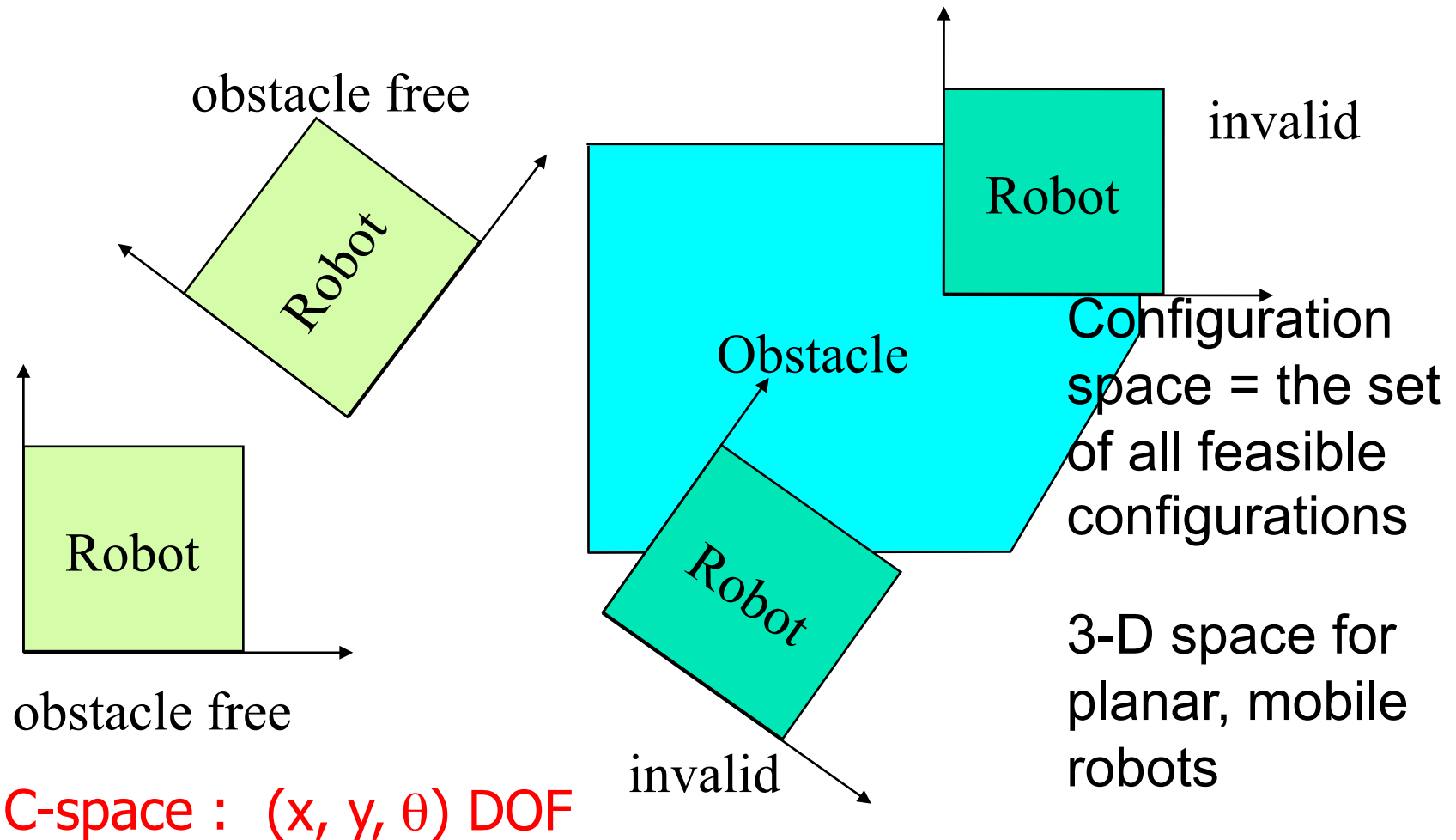


# Configuration space



C-space :  $(x, y, \theta)$  DOF

# Configuration space



# Free Space

## C-OBSTACLE REGION

$B_1, B_2, \dots, B_m$	_____	obstacles
$CB_i$	_____	C-obstacle
$\bigcup_{i=1}^m CB_i$	_____	C-obstacle region

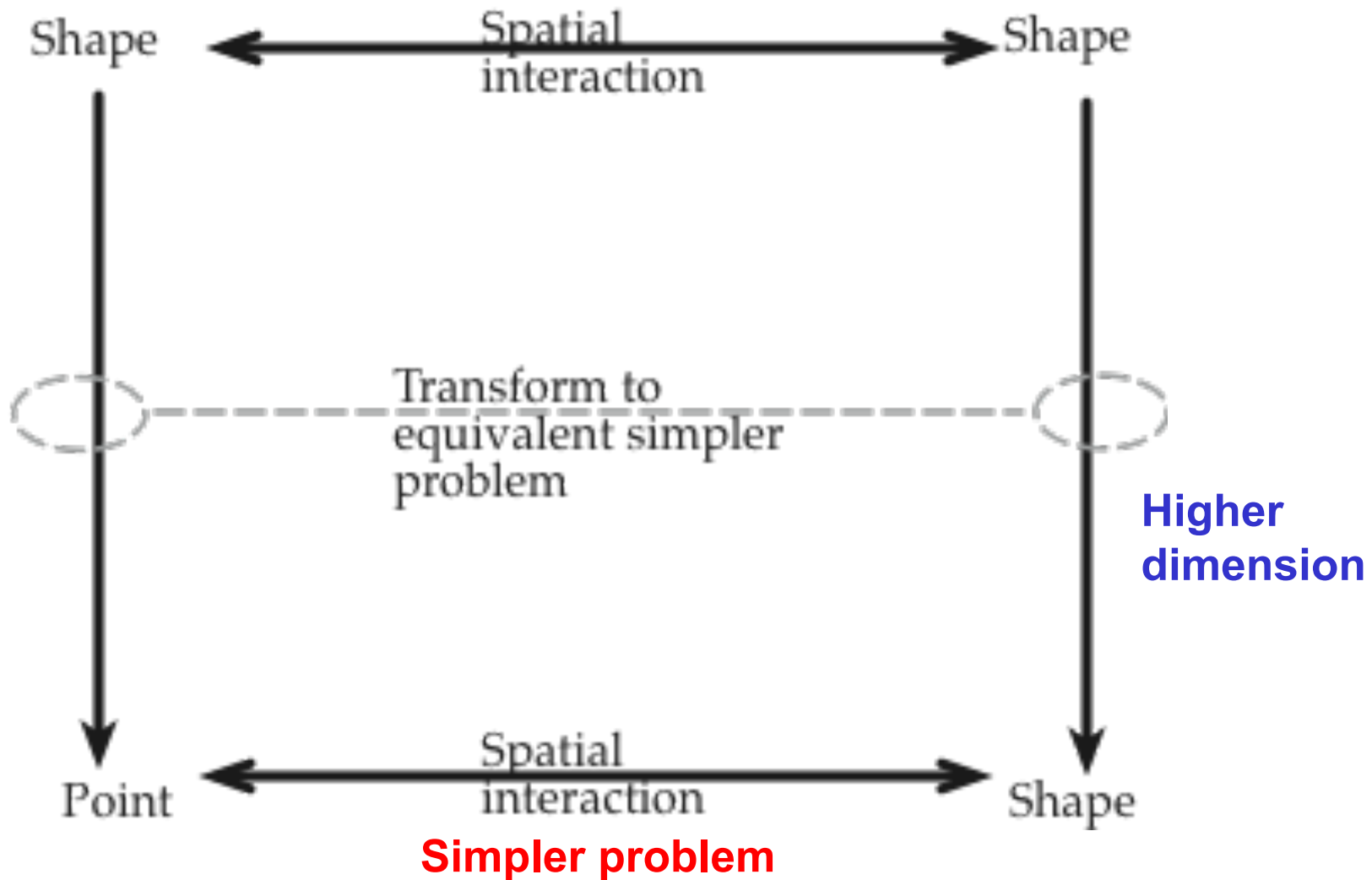
## FREE SPACE

$$C_{\text{free}} = C \setminus \bigcup_{i=1}^m CB_i = \{q \in C : A(q) \cap \bigcup_{i=1}^m CB_i = \phi\}$$

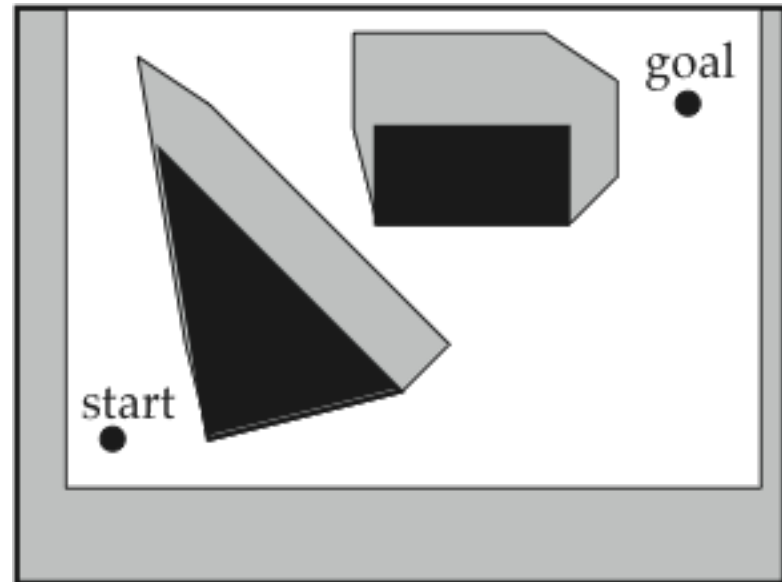
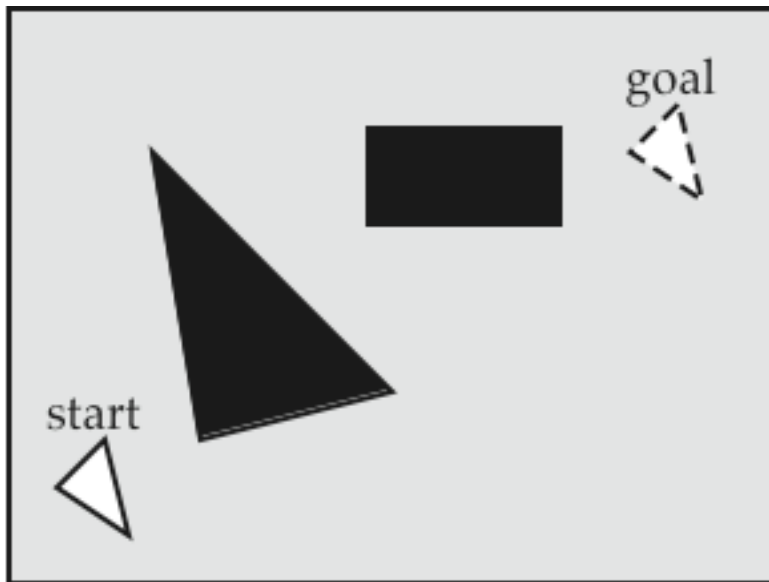
Free configuration  $q$  iff  $q \in C_{\text{free}}$

From  
Robot Motion Planning  
J.C. Latombe

# Transforming to C-Space

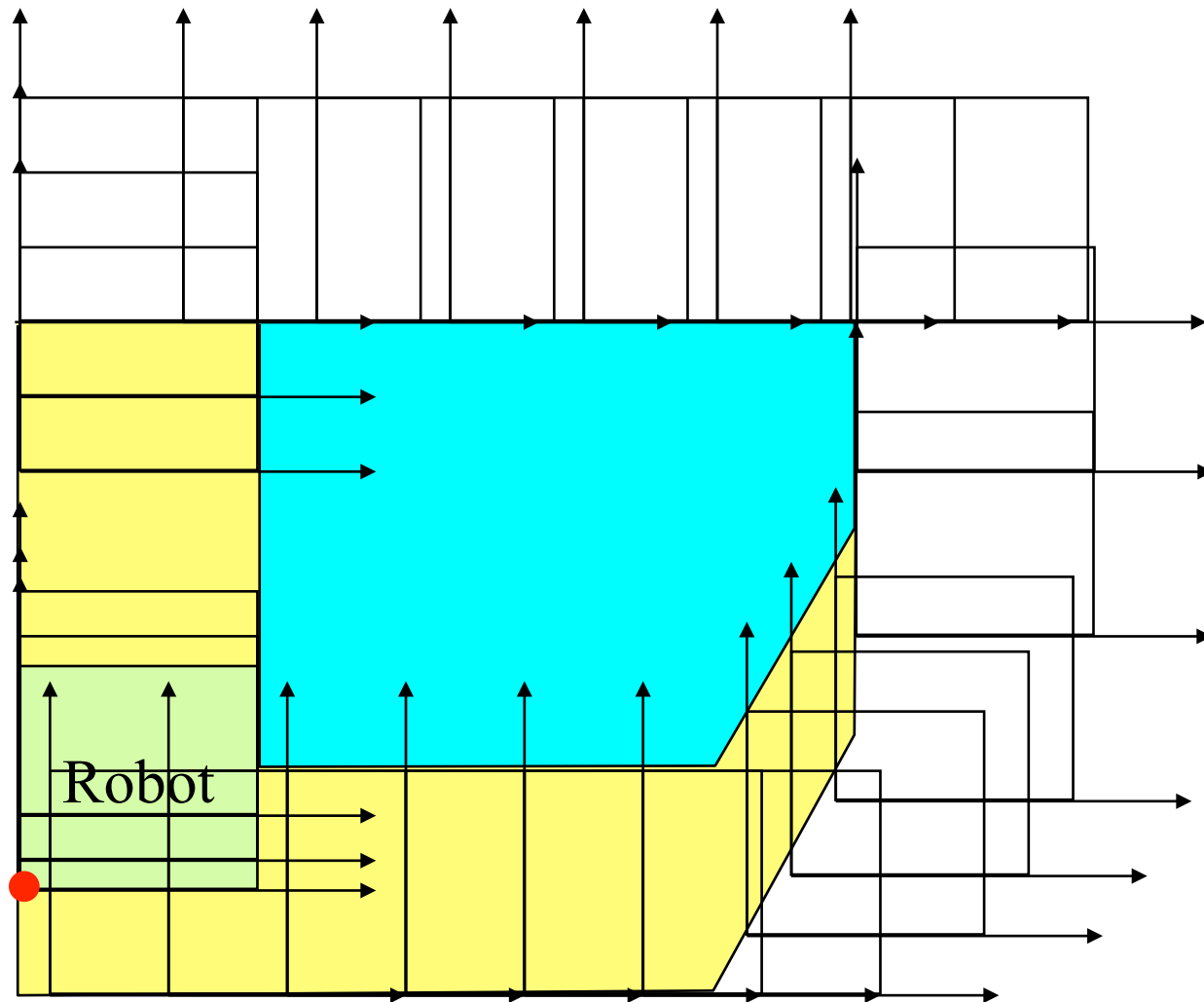


# Transforming to C-Space

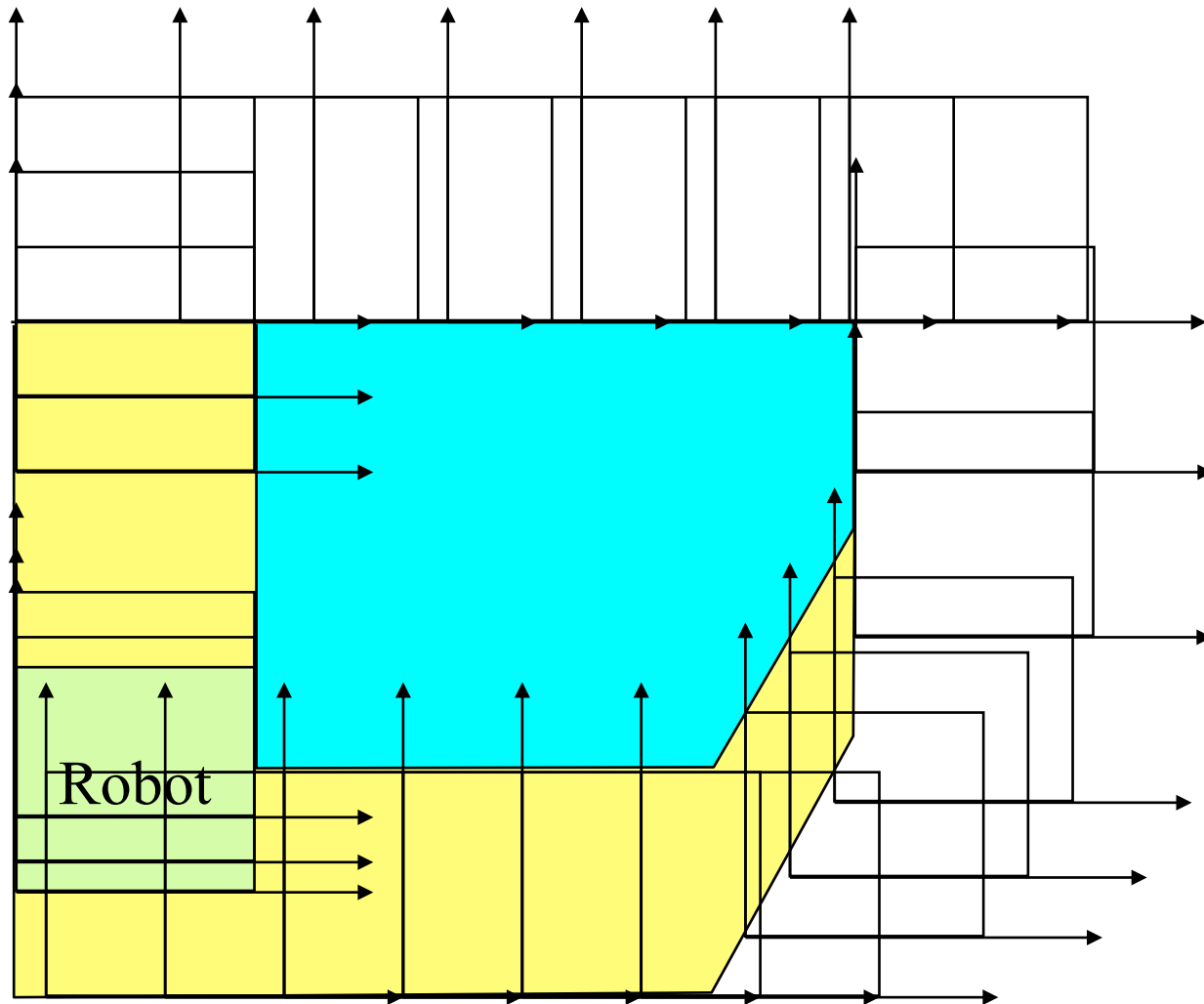




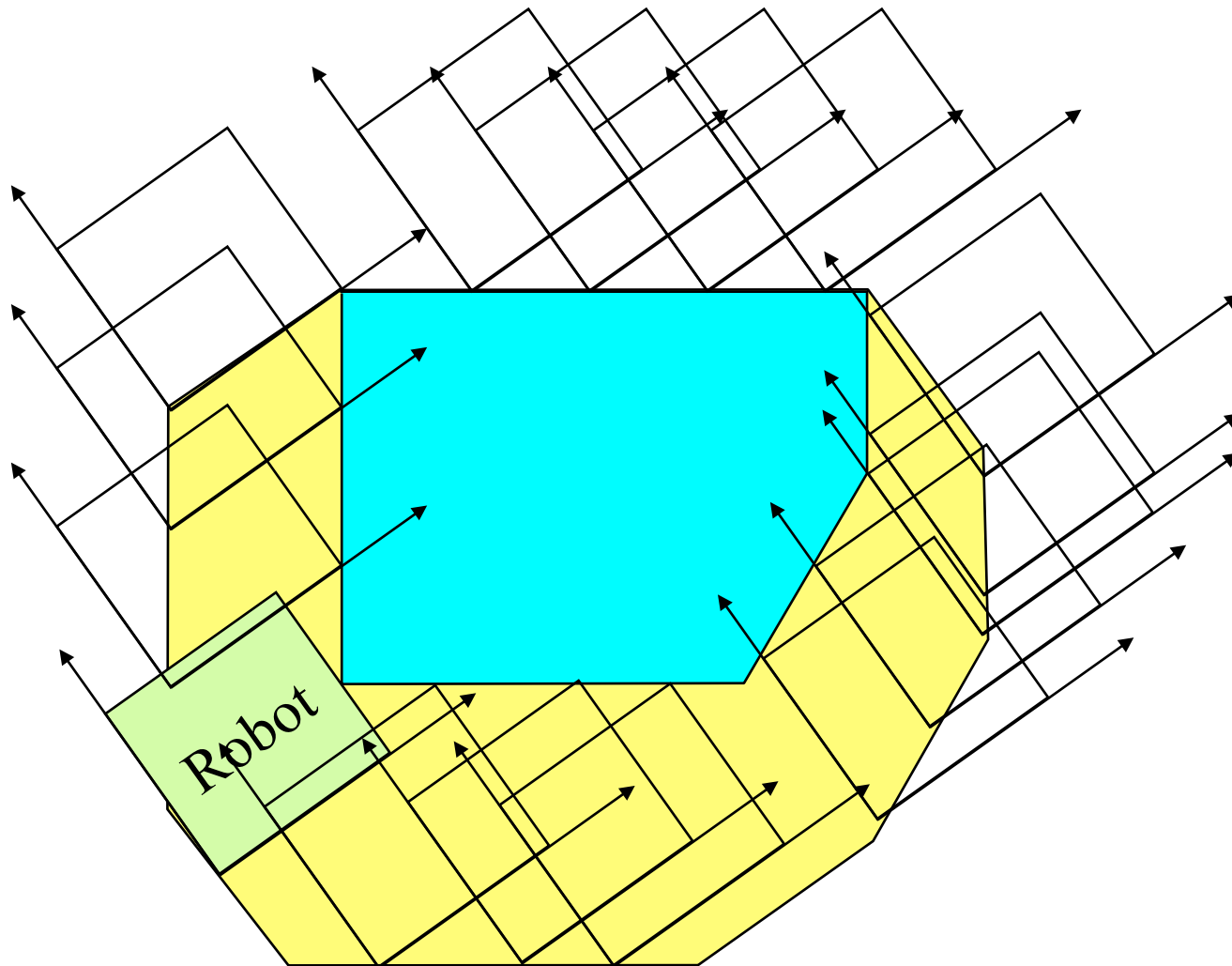
# Allowable Robot positions (no rotations)



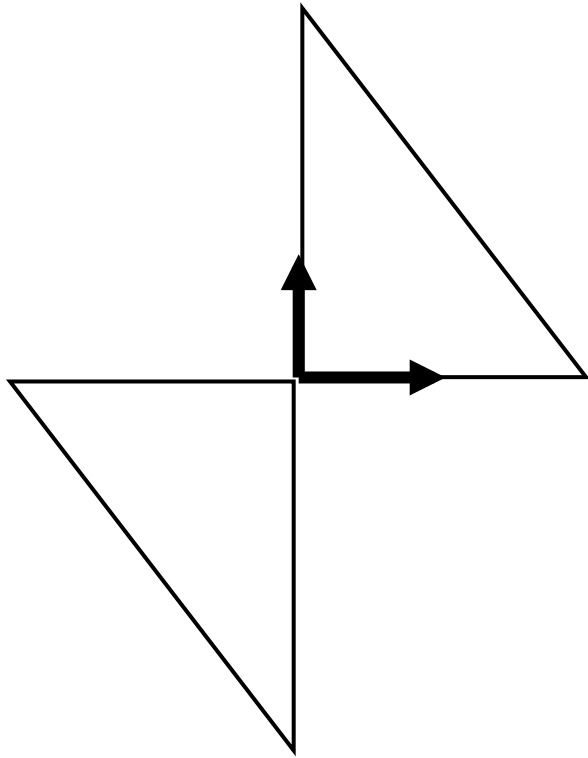
# Allowable Robot positions (no rotations)



# Allowable Robot positions (for some robot rotation)

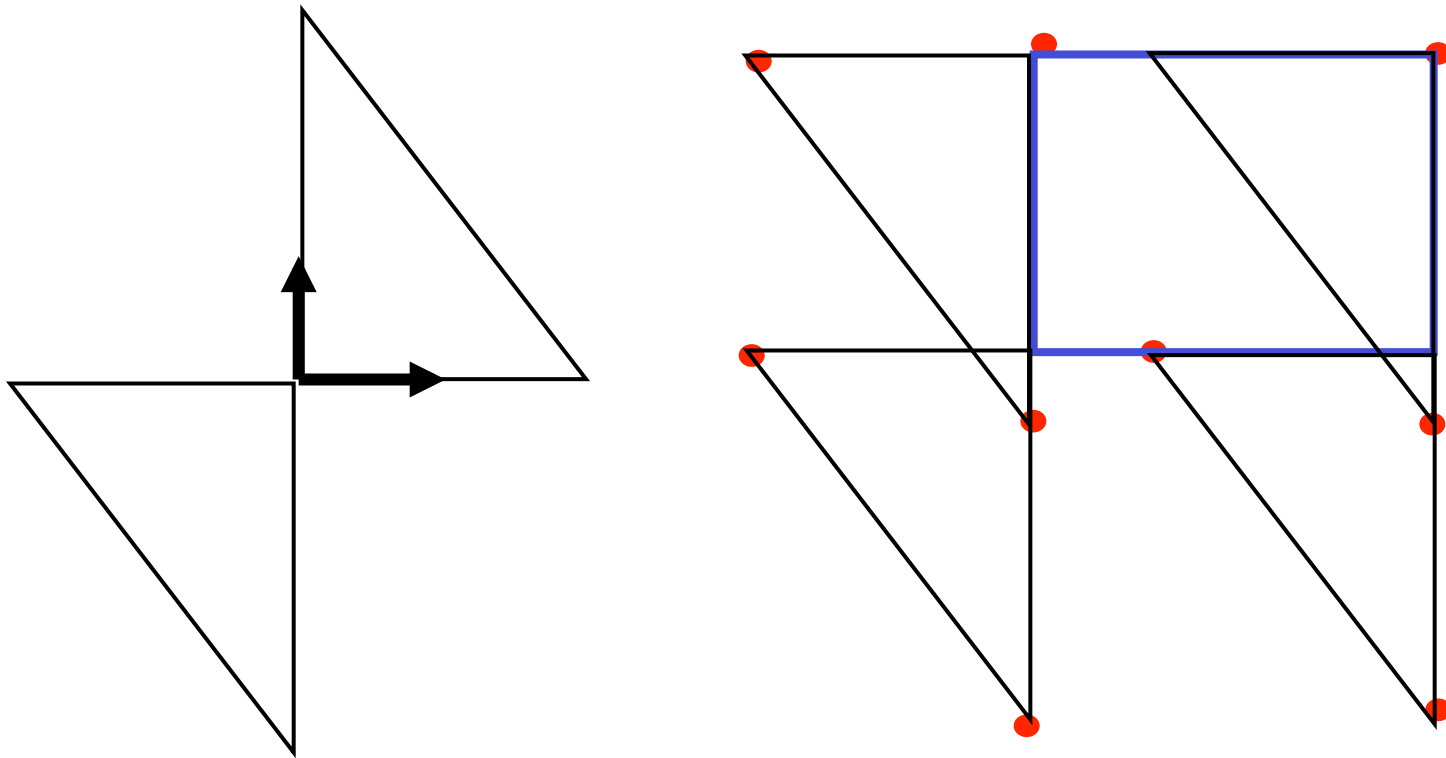


# C-space Algorithm



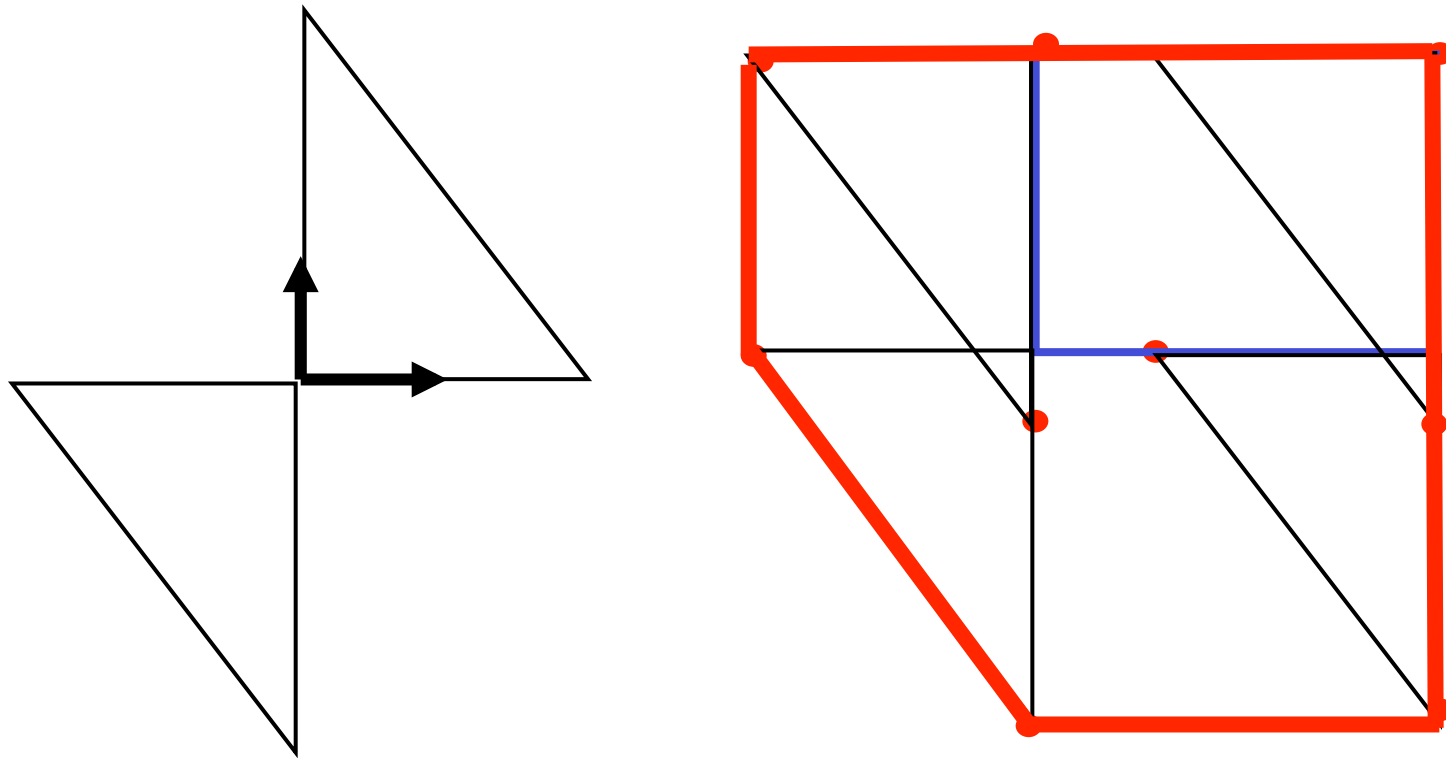
Step 1: Reflect Robot

# C-space Algorithm



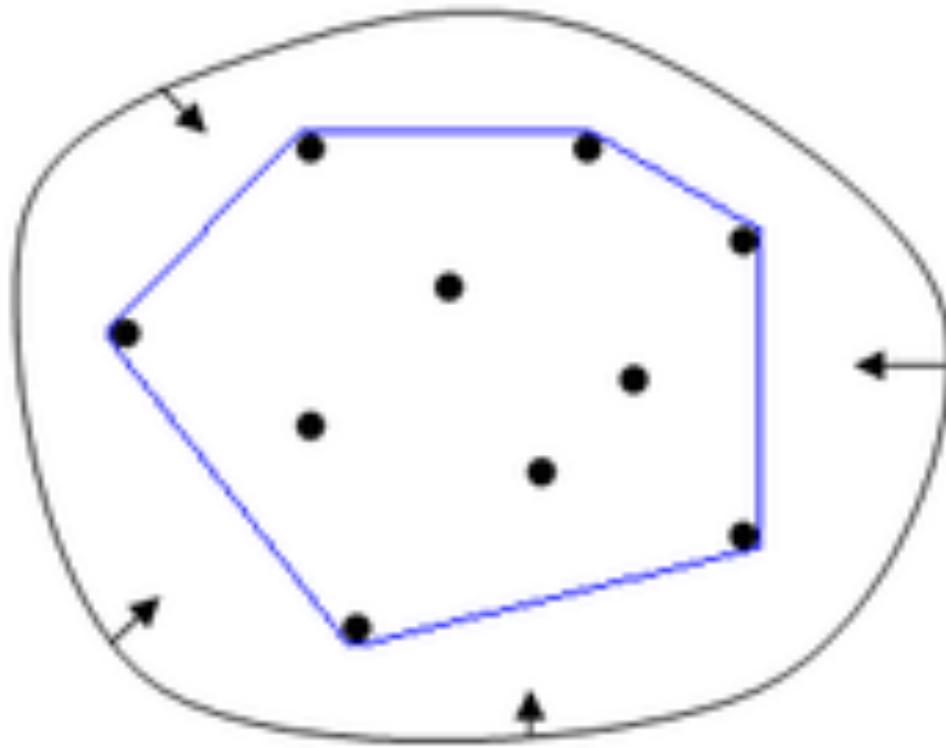
Step 2: Vert ( $\ominus$ Robot)  $\oplus$  Vert (Obstacle)

# C-space Algorithm

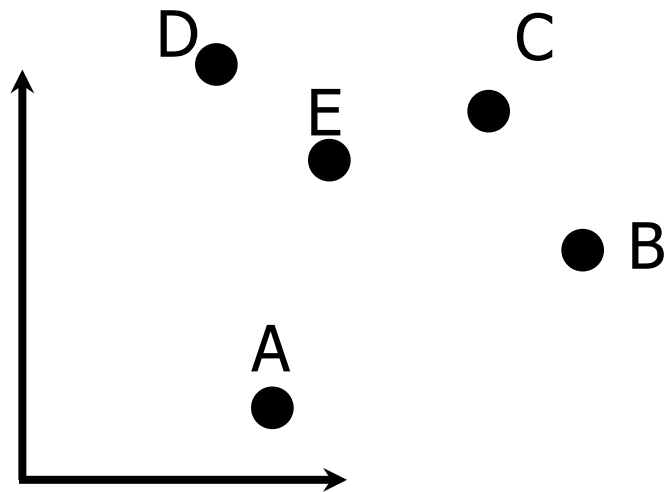


Step 3: ConvexHull (Vert (- Robot) + Vert (Obstacle))

# Convex Hull Algorithm

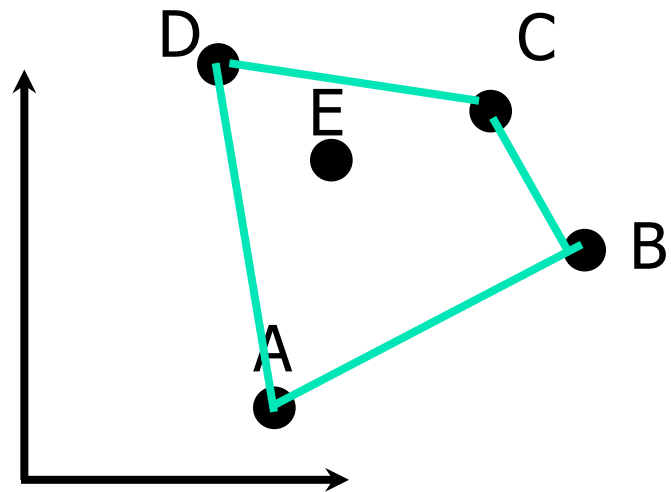


# Convex Hull Algorithm



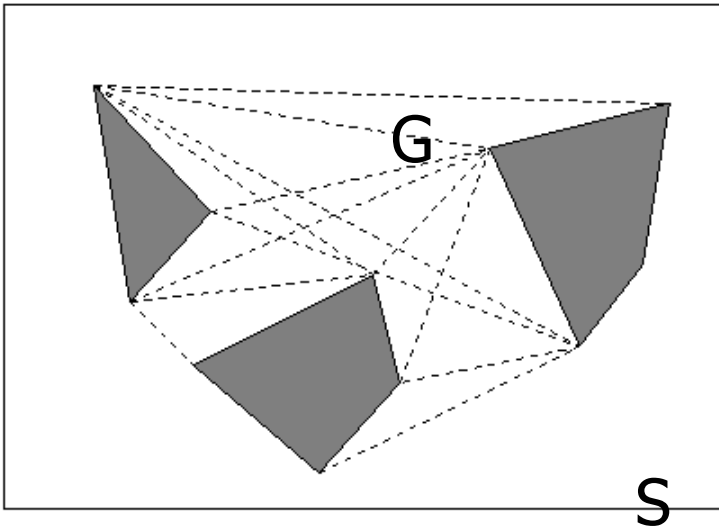


# Convex Hull Algorithm



# Algorithm Summary

- Compute c-space for each obstacle
- Compute v-graph
- Find path from start to goal



V-graph complete; gives optimal shortest path in 2d  
What about 3d? What else can we optimize?