6.141:
Robotics systems and science
Lecture 11: Localization

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EECS/MIT

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Reading: Chapter 3, and Craig: Robotics

http://courses.csail.mit.edu/6.141/

Challenge: Build a Shelter on Mars

Announcements

- Today’s Lab Reports: each team member talks about the technical piece he/she executed

Last week we saw

- Configuration Space
- Motion Planning

Today

- Localization: Where am I?

Navigation Overview

- Where am I?
  - Localization (Today)
  - Assumes perfect map, imperfect sensing
- How can I get there from here?
  - Planning (Last Week)
  - Assumes perfect map, sensing, and actuation
- What have I observed in my travels?
  - Mapping (Later)
  - Assumes perfect localization
- Can I build map *and* localize on-line?
  - Yes; using SLAM (Later is Time Permits)
  - Assumes no prior knowledge of the world

Thought experiment

- Does it make sense to localize in a void
  (an environment containing absolutely nothing)?

  … not very interesting; We conclude that there has to be some kind of “stuff” in environment

- What if the environment is *isotropic*
  (space, fog, water, desert, jungle etc.)?

  … again, not very interesting for robot to move or perform tasks within such an environment

  We conclude that environment must contain *features* that can be sensed (distinguished) by bot

Localization Problem Statement

- Given some representation of the environment, to *localize*, robot must, through sensing, determine its pose *with respect to the specified representation*

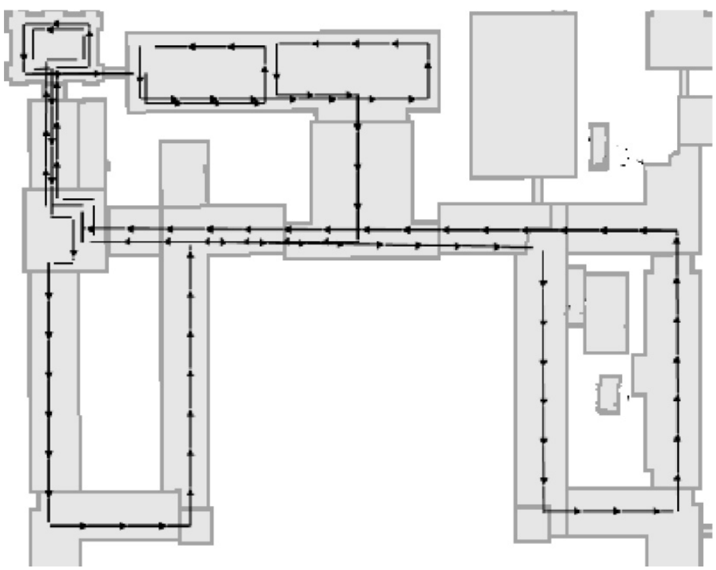
- Defined with respect to some frame or feature set that is *external* to robot:
  - Global coordinate frame
    - E.g., GPS (Earth) coordinates
  - Local coordinate frame
    - Ceiling or floor tiles
    - Mission starting pose
  - Environment features
    - E.g., nearby walls, corners, markings

Basic Localization

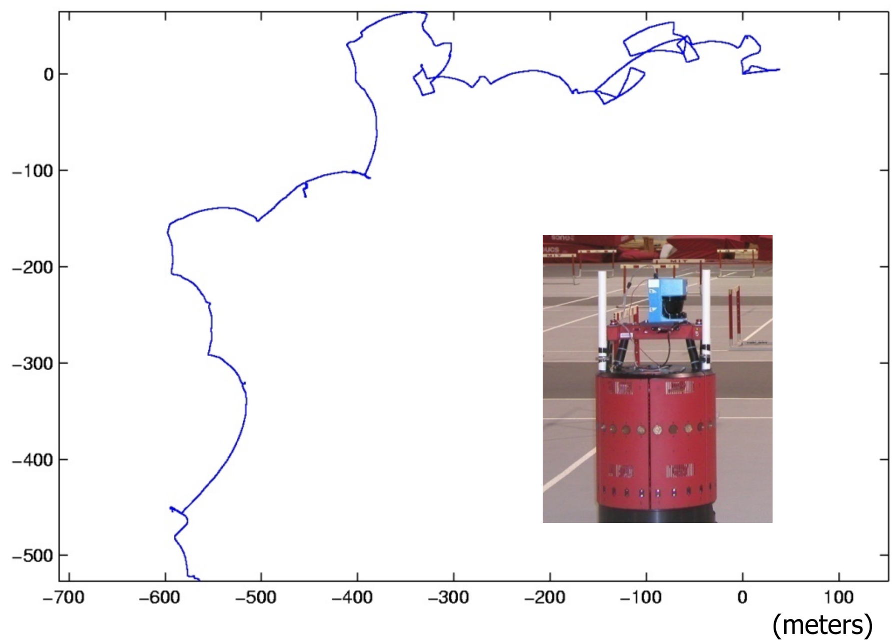
- Open-loop pose estimation:
  - Maintain pose estimate based on expected results of motion commands (no sensing)
- Dead reckoning:
  - Use proprioception (odometry, inertial) to estimate pose w.r.t. *initial* coordinate frame
  - Multiple error sources:
    - Wheel slip, gear backlash
    - Noise (e.g. from encoders)
    - Sensor, processor quantization errors
  - Pose error accumulates with time and motion
  - Typically ~ a few percent of distance traveled

Dead Reckoning Error

- Two hours of slow, rolling motion through MIT main campus corridors at third-floor level
  - Bosse, Leonard, Newman, Teller (IJRR 2004)
- High-precision inertial sensors exist… do they solve problem?



True path topology
(manually drawn and overlaid)



Integrated odometry
(Nomadics B21)

Landmark Attributes

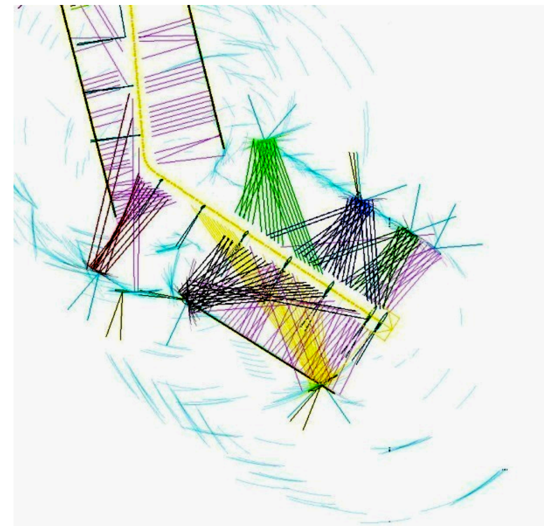
- Is landmark *passive* or *active*?
  - Must sensor emit energy to sense landmark?
- Is landmark *natural* or *artificial*?
  - If placed in env't, how are locations chosen?
- Which sensor(s) can detect it?
  - Vision, sonar, radio, tactile, chemical, …
- What are landmark's geometric properties?
  - Plane, line, segment, point, diffuse source, …
- What is *discriminability* of landmark?
  - (Will discuss this in detail in a minute)

Landmark Types

	Passive	Active
Natural	Wall corner Texture patch River bend Earth’s surface	Sun, North star Magnetic dipole Pressure gradient Mineral vent
Artificial	Surveyor’s mark Retro-reflector Lighthouse (day) Trail blaze Buoy, channel marker	Chemical marker Radio beacon Lighthouse (night) LORAN GPS

Types of Measurements

- Range to surface patch, corner
  - Sonar return
- Bearing (absolute, relative, differential)
  - Compass; vision (calibrated camera)
- Range to point
  - RSS, TOF from RF/acoustic beacon
  - Cricket (TDoA of acoustic & RF pulse)



- Range and (body-relative) bearing to object
  - Radar return
  - Laser range scanner return
  - Vision (stereo camera rig)
- Distance to sea surface, floor
  - Pressure (depth), bathymetry (depth, altitude)

Discriminability Challenges

- Landmark *Detection*
  - Is landmark distinguishable from *background*?
- Landmark *Measurement, Data Fusion*
  - Sensor gives a noisy, quantized measurement of landmark geometry (bearing and/or range)
  - How accurately can a measurement localize a landmark?
  - How can multiple corrupted measurements be combined into one accurate localization estimate of a landmark?
- Landmark *Identification*
  - To which element of *representation* (i.e., map) does the detected and measured landmark correspond?
  - To which *previously-observed landmark* (if any) does the currently observed landmark correspond?
  - Also known as the "data association" or "feature correspondence" or "matching" problem

Localization Degrees of Freedom

- Model robot/vehicle as a single rigid body
- Aerial, orbital, underwater navigation
  - 6 DOFs: three position + three orientation
- Terrestrial operation (rolling, walking)
  - 3 DOFs: two position + one orientation
  - Used for planar, mildly non-planar terrain
- Underwater surveying (high C. O. B.)
  - 4 DOFs: three position + one orientation

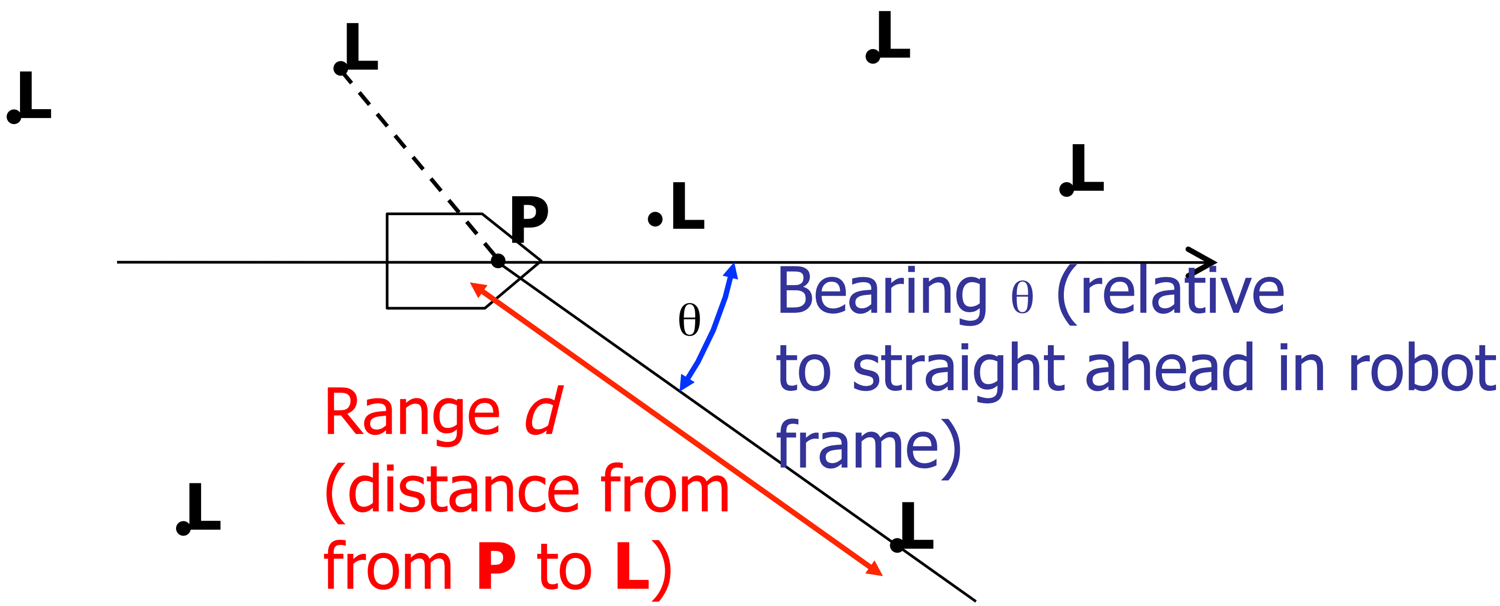
WHOI AUV, Hanu Singh (Aug. 2004)

Localization Scenarios

- Estimating location in 2D
  - From measured *ranges* (distances)
  - From measured *bearings* (directions)
  - We'll look at noiseless, noisy cases

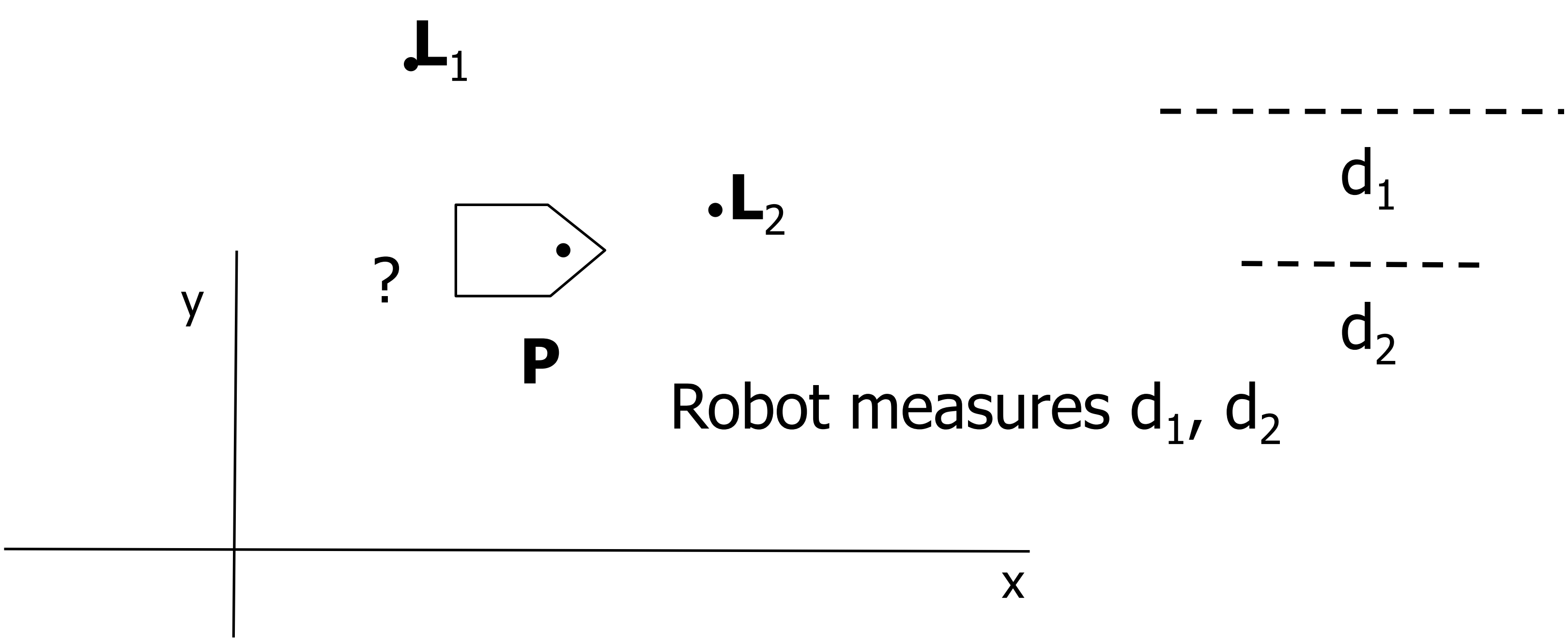
Triangulation

- Natural geometry for 2D localization
  - Simplest framework combining range, bearing
  - Used by Egyptians, Romans for engineering



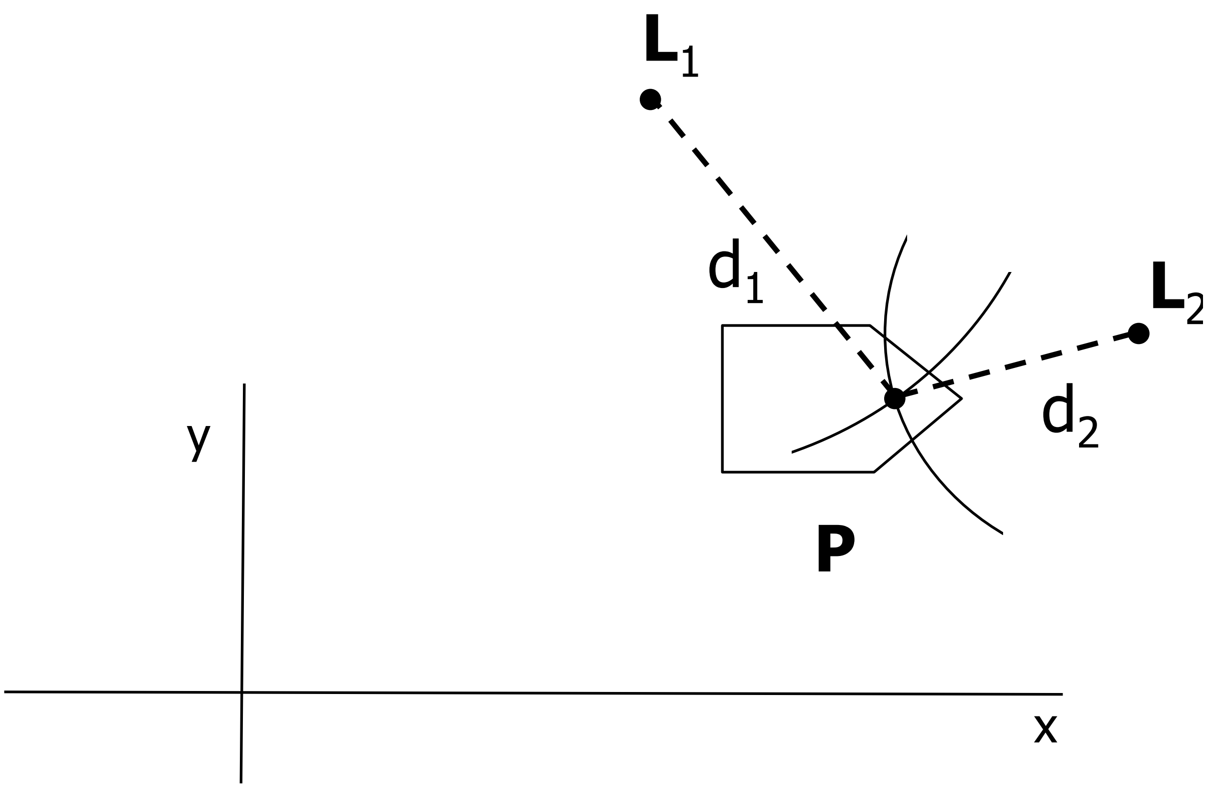
Triangulation from range data

- Robot at unknown position **P** measures distances d_1, d_2 to *known* landmarks $\mathbf{L}_1$, $\mathbf{L}_2$
- Given d_1, d_2, what are possible values of **P**?



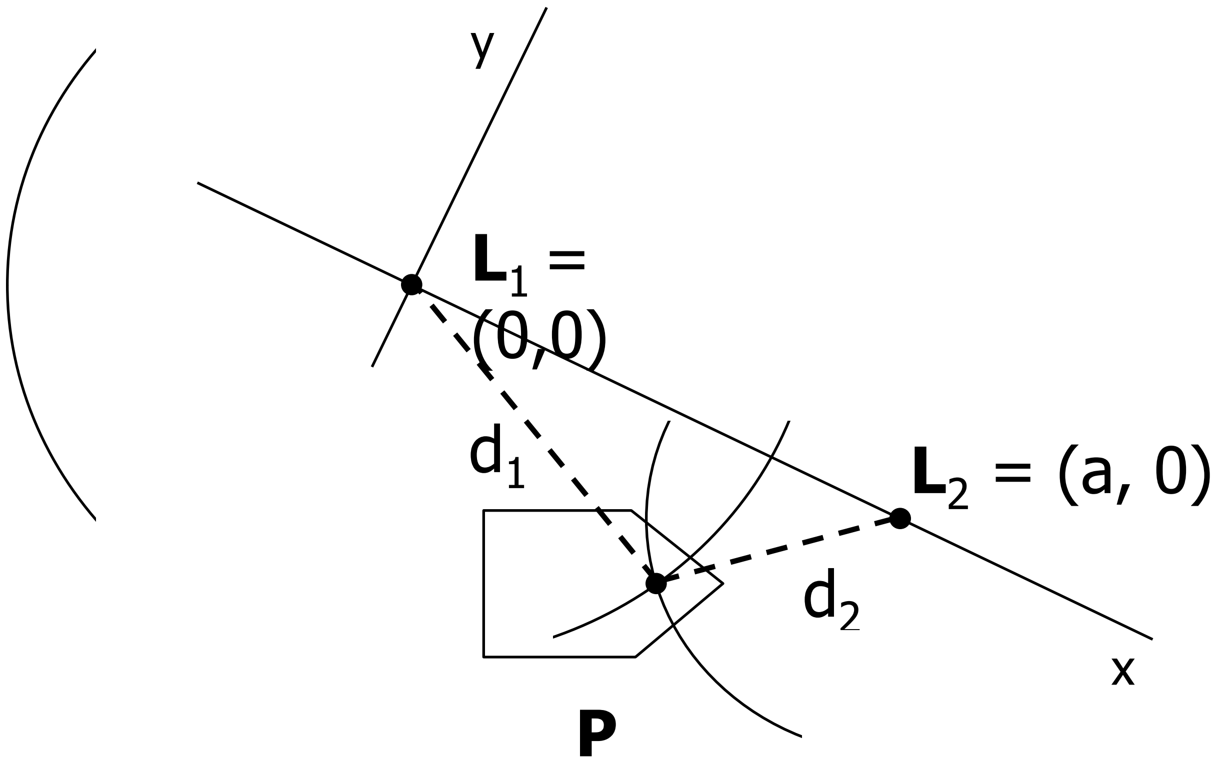
Triangulation from range data

- Robot must lie on circles of radius d_1, d_2 centered at $\mathbf{L}_1$, $\mathbf{L}_2$ respectively



Triangulation from range data

- Change basis: put **L**$_1$ at origin, **L**$_2$ at (a,0)



$$x = (a^2 + d_1^2 - d_2^2)/2a$$

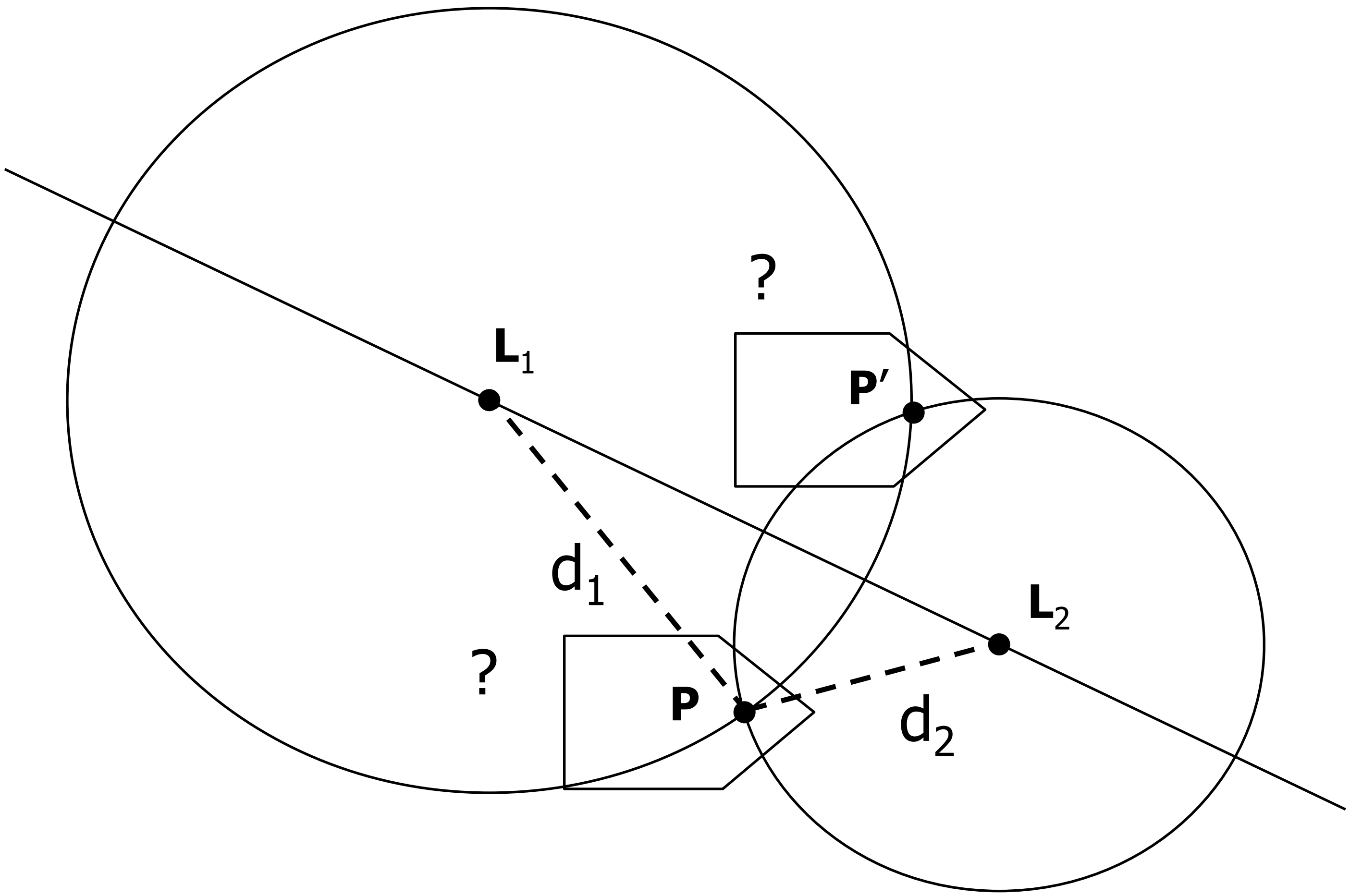
$$y = \pm\sqrt{(d_1^2 - x^2)}$$

(Try e.g. setting d_1 = a, d_2 = 0)

Are we done?

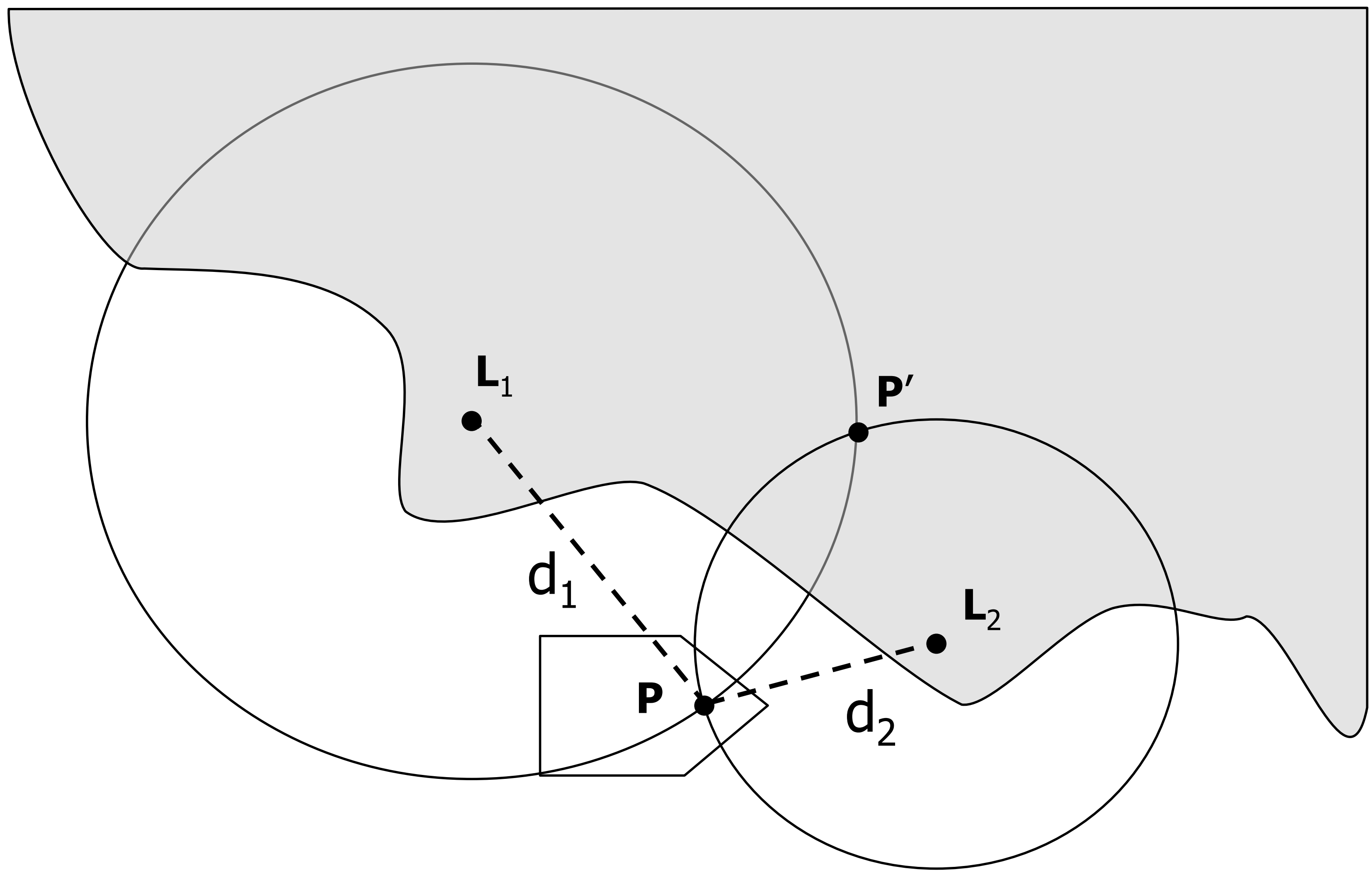
Triangulation from range data

- Two solutions in general, P and P′
- How to select the correct solution?



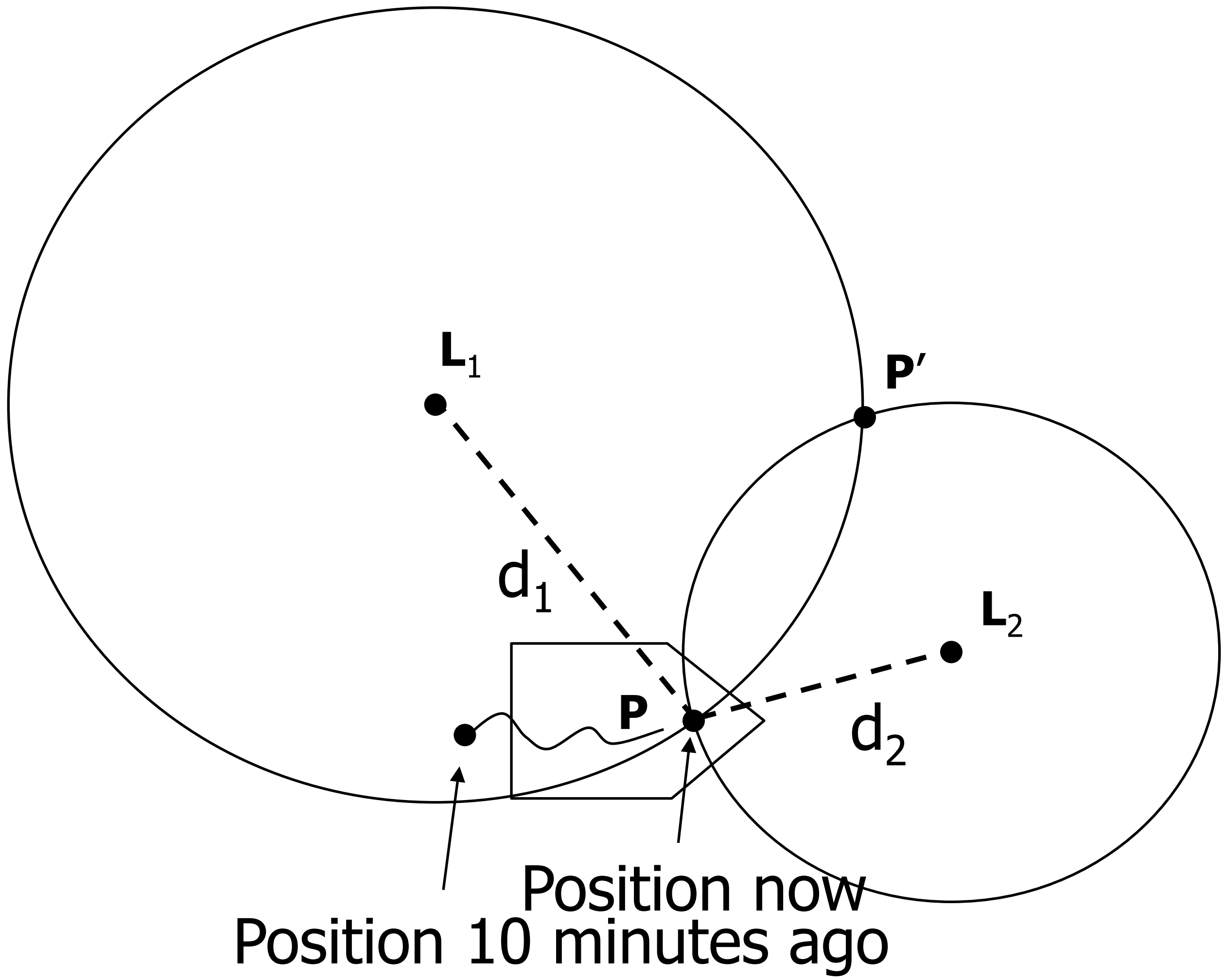
Disambiguating solutions

- *A priori* information (richer map)



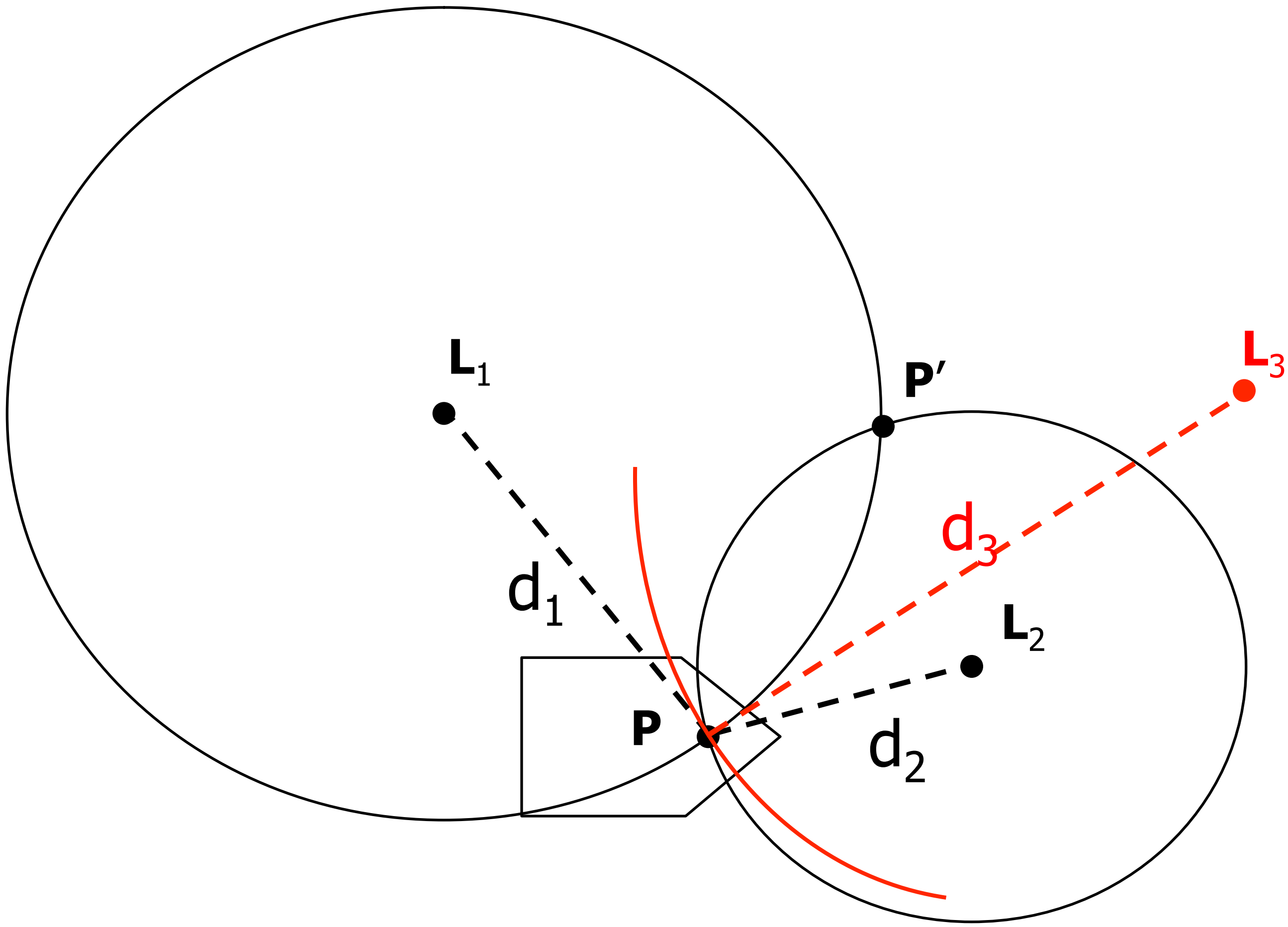
Disambiguating solutions

- Continuity (i.e., spatiotemporal information)



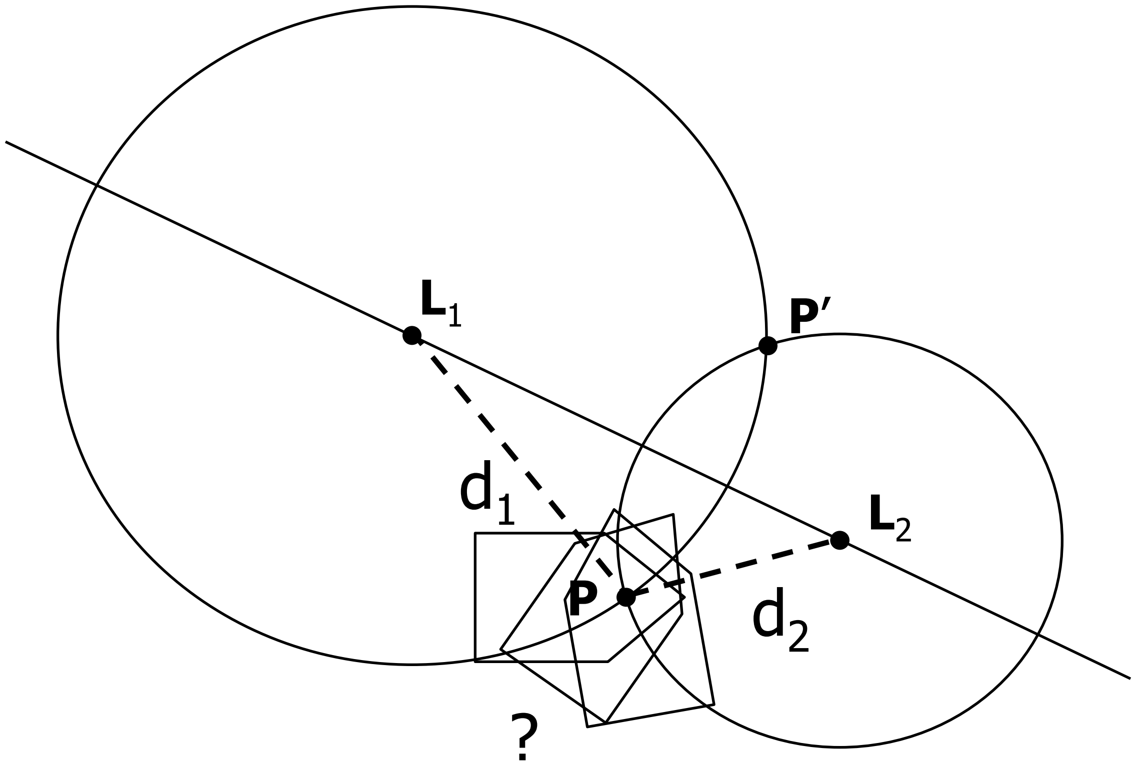
Disambiguating solutions

- Additional landmarks (redundancy)



Triangulation from range data

- Are we done yet, i.e., is pose fully determined?
- No: absolute heading is *not determined*

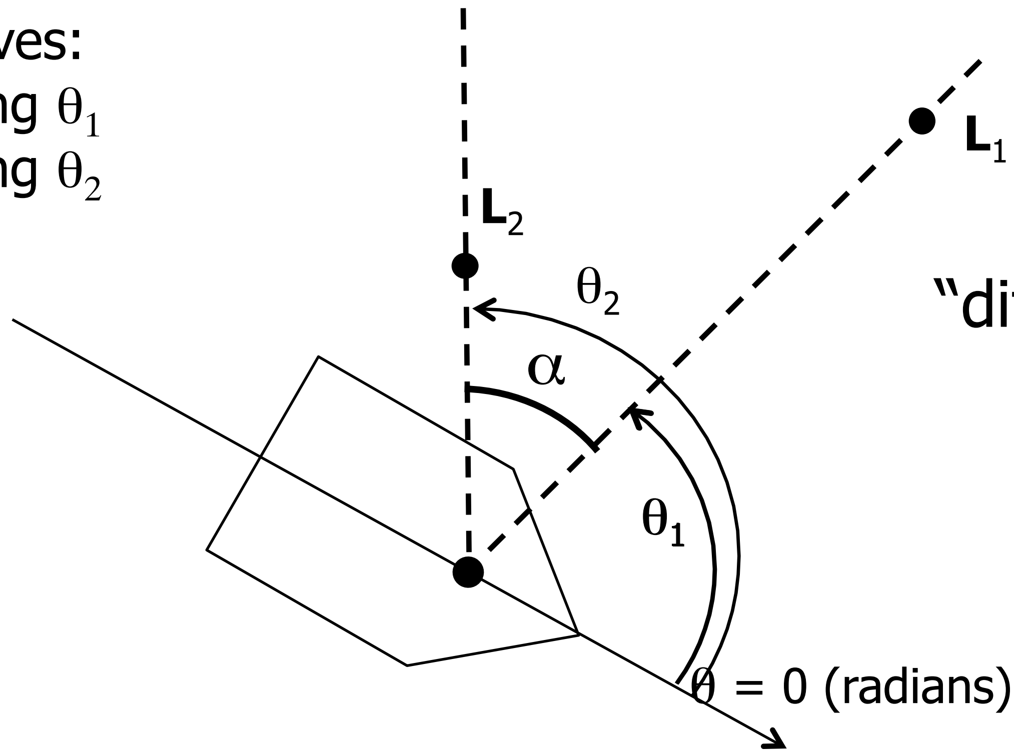


- How to get heading?
  - Motion (difference of positions inferred across *time*)
  - Extent (using two ranges measured over ship *baseline*)

Triangulation from bearing data

- *Body-relative* bearings to two landmarks
  - Bearings measured relative to “straight ahead”

Robot observes:
L_1 at bearing θ_1
L_2 at bearing θ_2

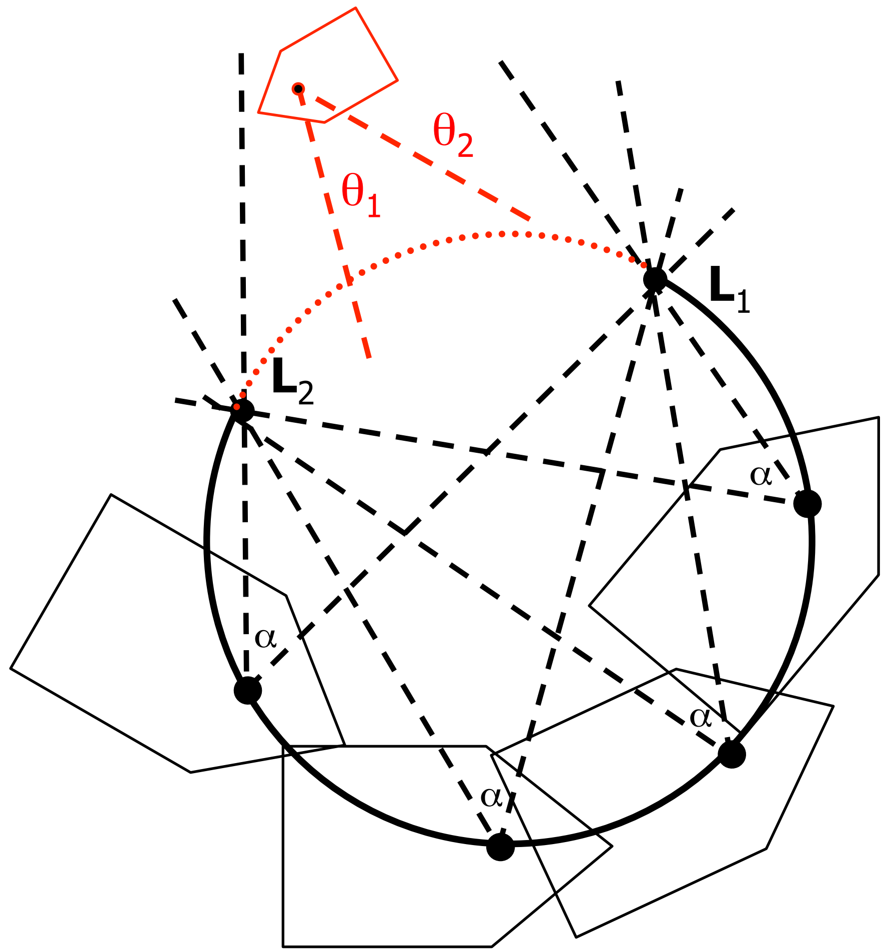


“differential bearing”

$$\alpha = \theta_2 - \theta_1$$

… are two bearings enough for unique localization?

Triangulation from two bearings

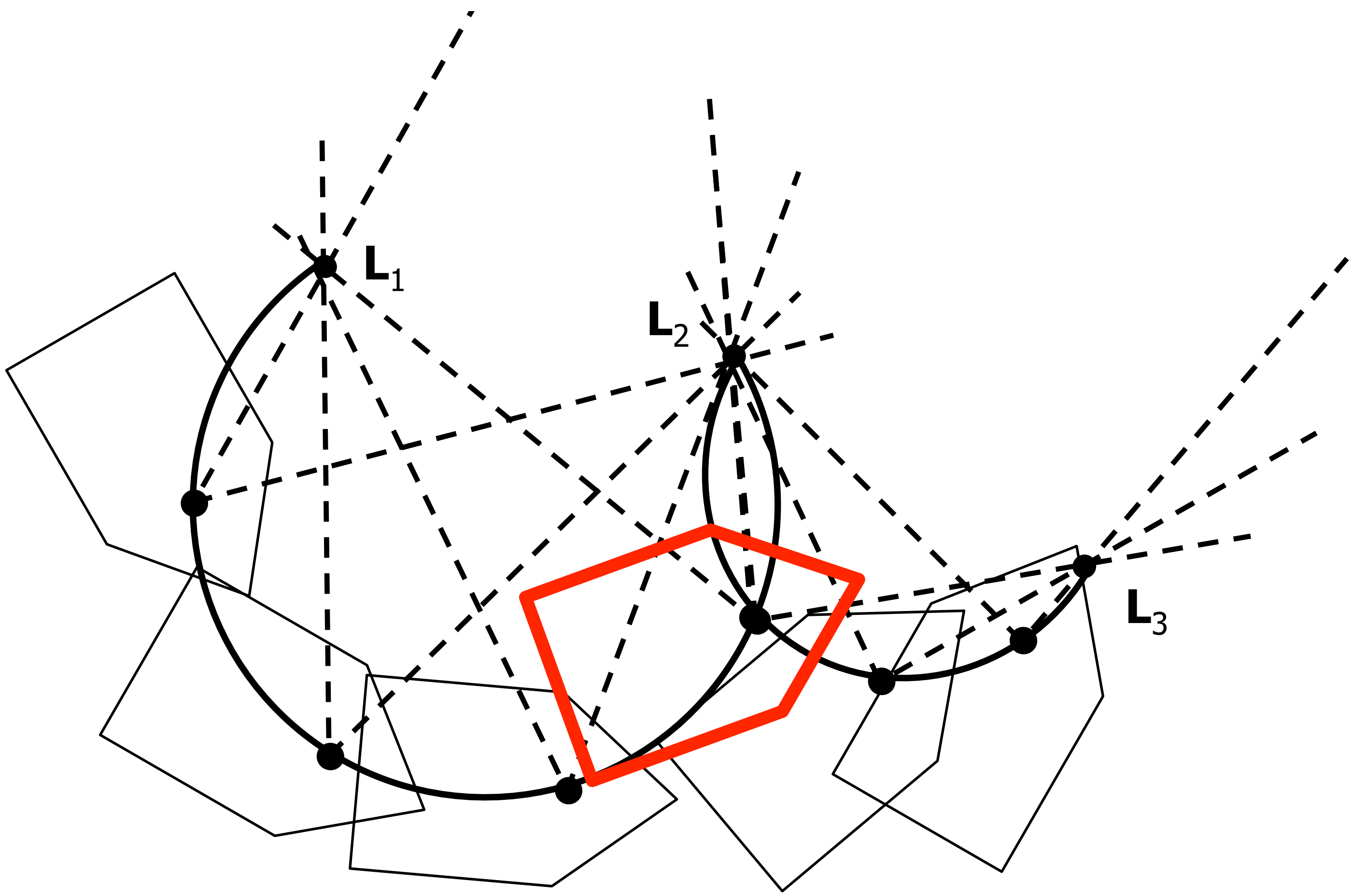


- Robot somewhere on circular arc shown
  - Can it be *anywhere* on circle?

(No; ordering constraint)

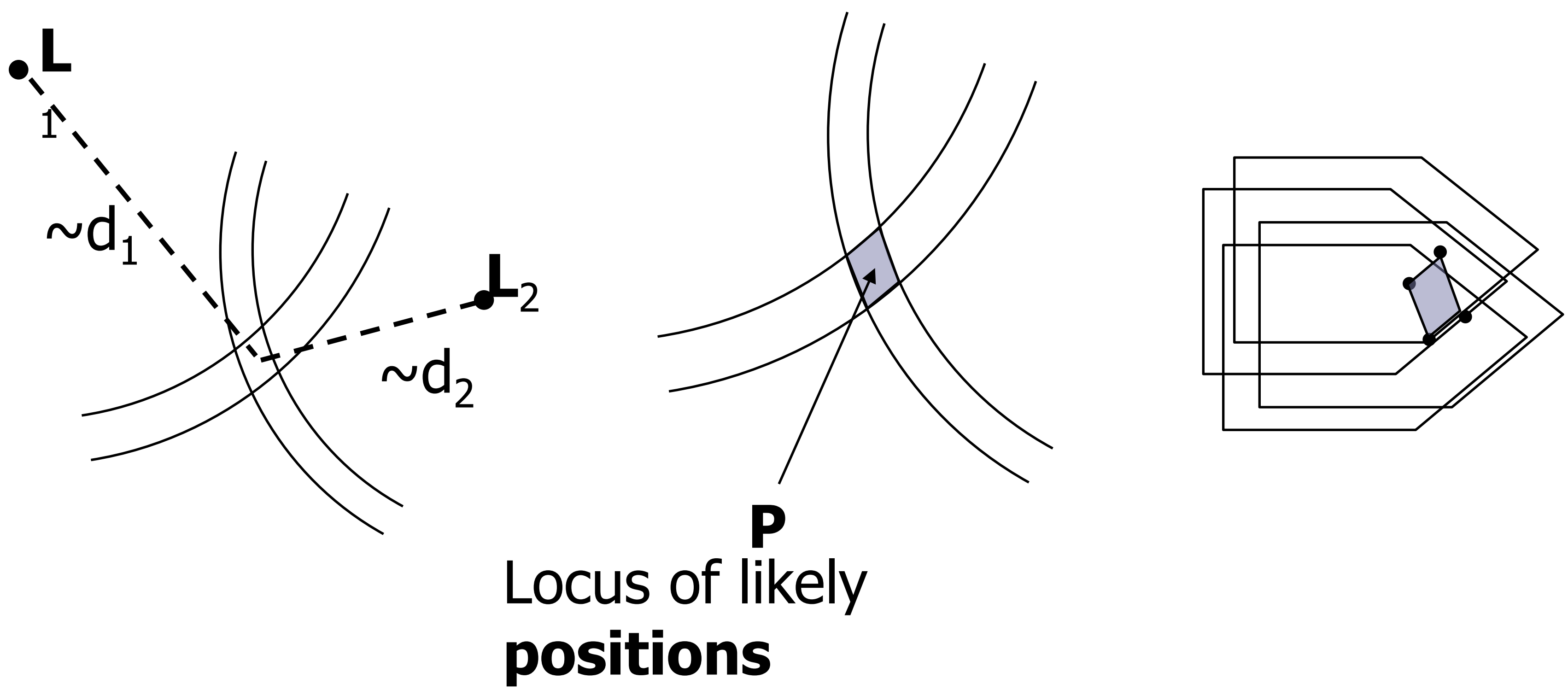
Triangulation from bearing data

- Measure bearing to third landmark
  - Yields robot position *and* orientation
  - Also called robot *pose* (in this case, 3 DoFs)



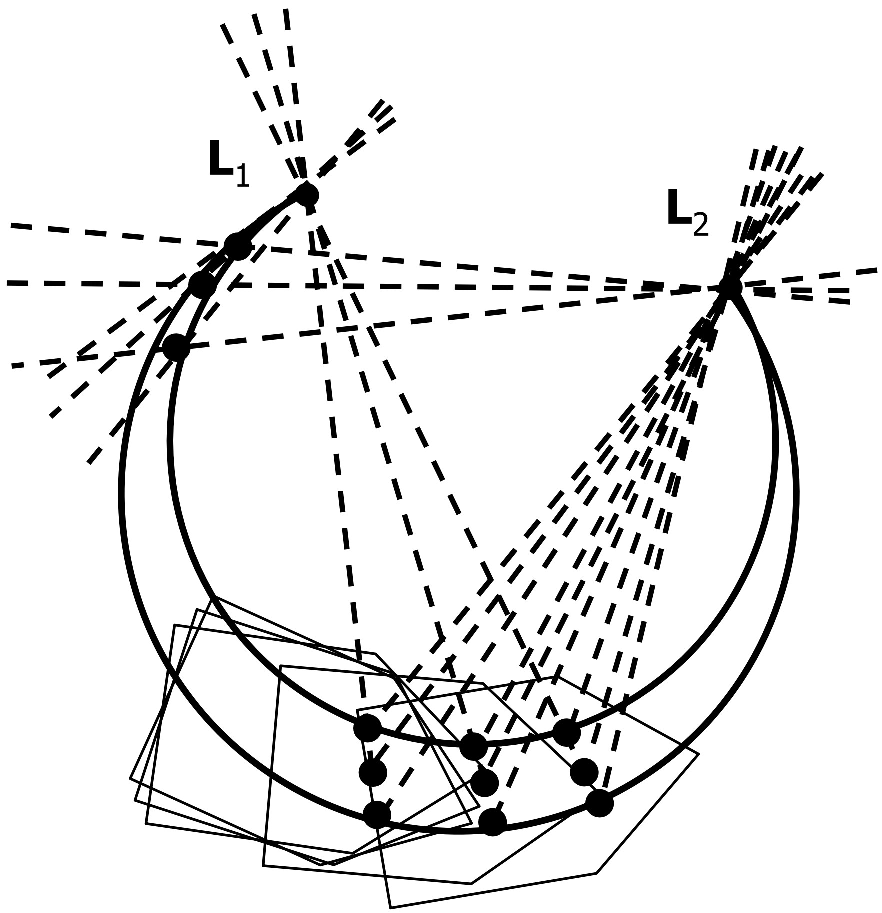
Measurement Uncertainty

- Ranges, bearings are typically *imprecise*
- Range case (estimated ranges $\sim d_1$, $\sim d_2$)



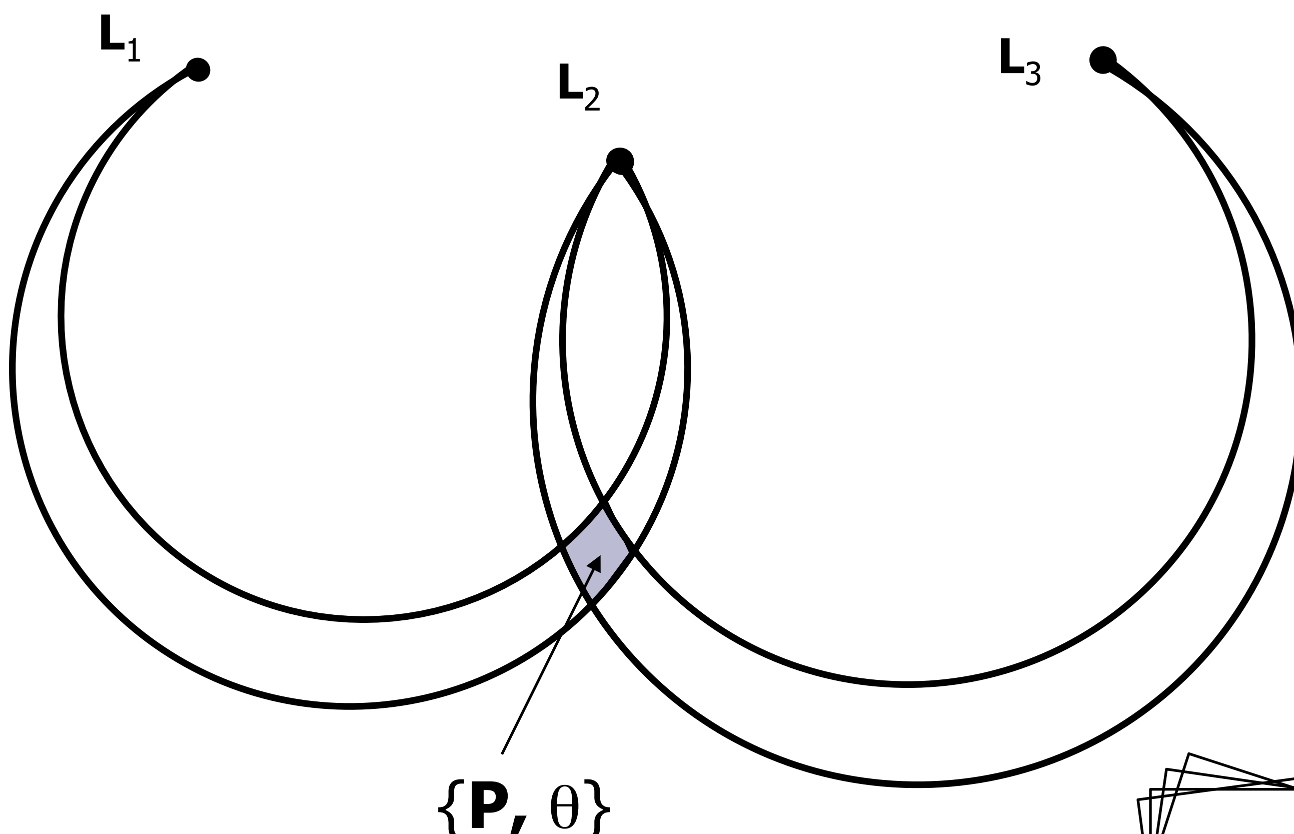
Measurement Uncertainty

- Two-bearing case (estimated bearings $\sim\theta_1$, $\sim\theta_2$)
- What is *locus* of recovered vehicle poses?
- Solve in closed form? Is there an alternative?

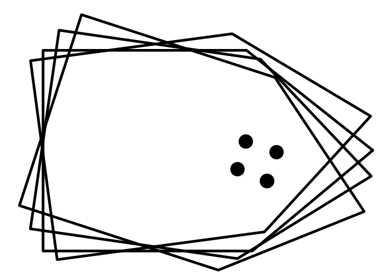


Measurement Uncertainty

- Bearing case (measurements $\sim\theta_1$, $\sim\theta_2$, $\sim\theta_3$)



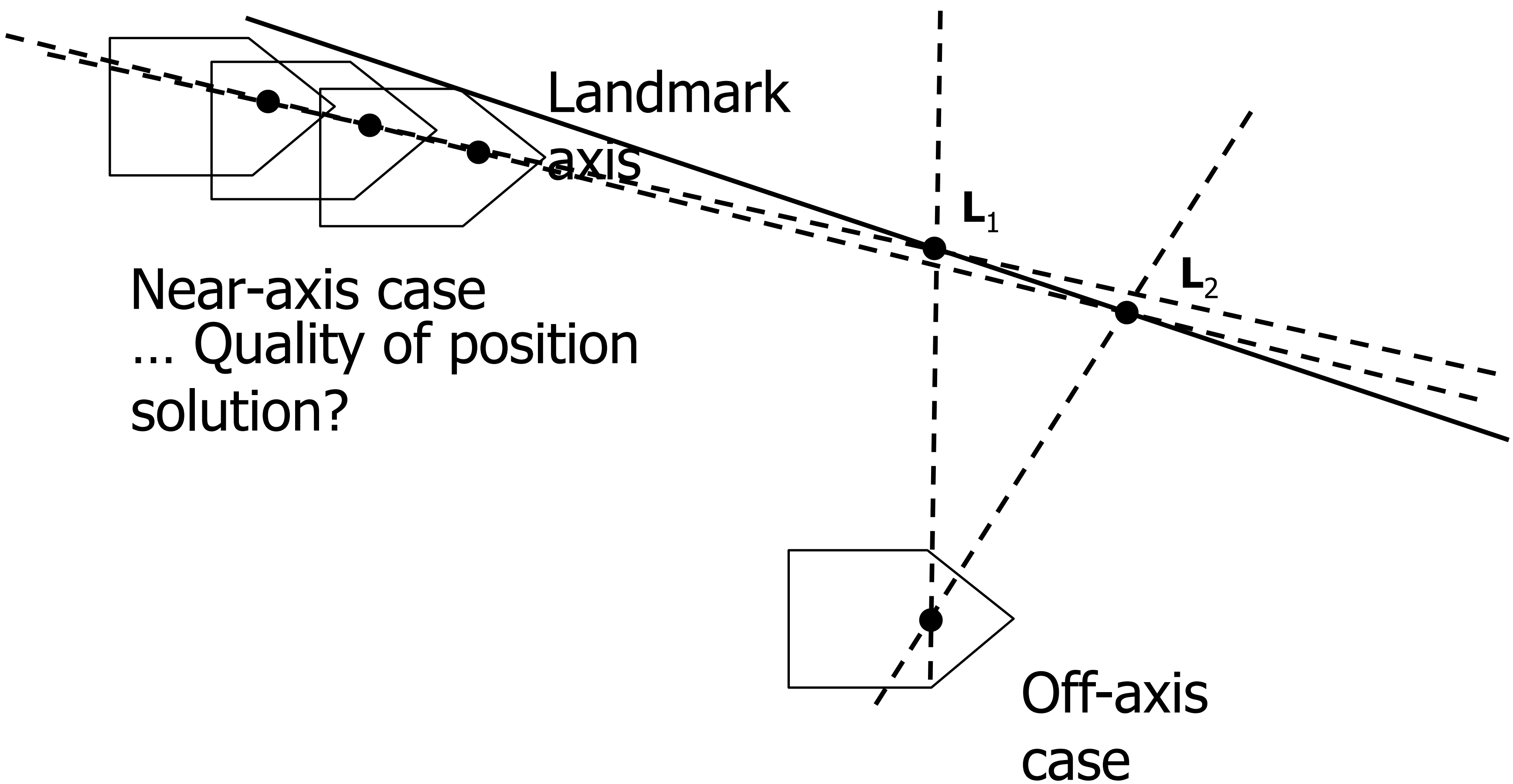
Locus of likely **poses**



- … is this always a satisfactory pose bound?

Landmark, sensor geometry

- Consider off-axis and near-axis bearing measurements to two known landmarks
  (simplification: assume absolute heading is known)



Dilution of Precision

- General phenomenon that sensor, landmark, and motion geometry can *degrade* solution quality, even for a *fixed set* of observed landmarks

- Geometric DOP = GDOP
  - Also Vertical DOP, Horizontal DOP etc.

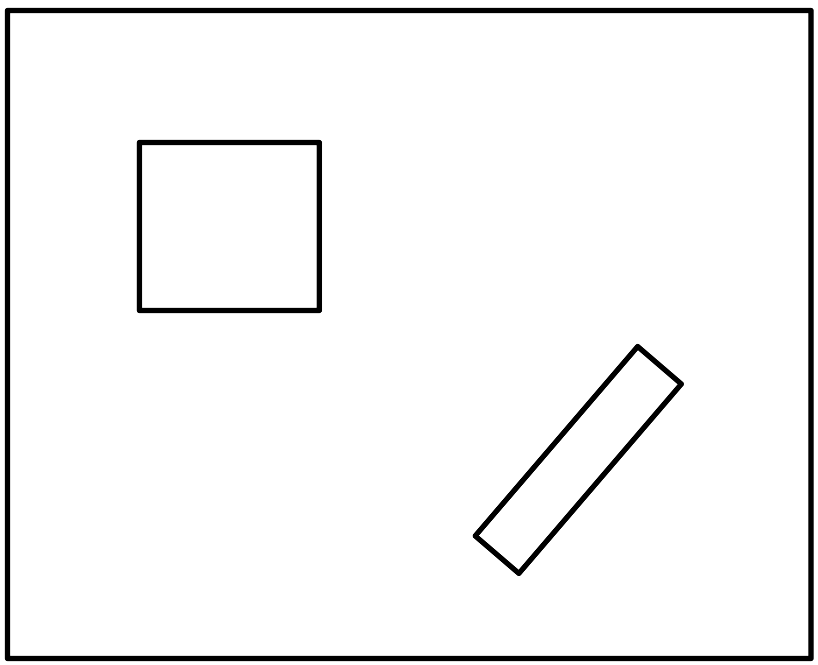
- How to take GDOP into account?
  - If sufficiently many landmarks are available, *select* those with minimal GDOP
  - Decouple pose, solve separately, recombine

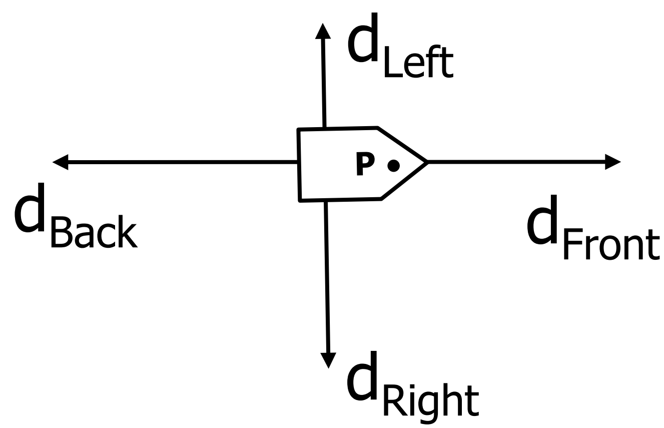
To Think About: RSS Challenge

- Will your challenge solution rely on localizing within the provided map?
  - Can solve challenge with or without localization
  - Decide early, as choice has significant implications
- Source 1: colored blocks
  - Placed at known map locations, but ID may not be available
- Source 2: colored balls
  - Placed at known map locations, in unique color combinations
- Source 3: sonar returns
  - Range data from 2 (or 4, if you choose) sonars on chassis

To Think About: Localization

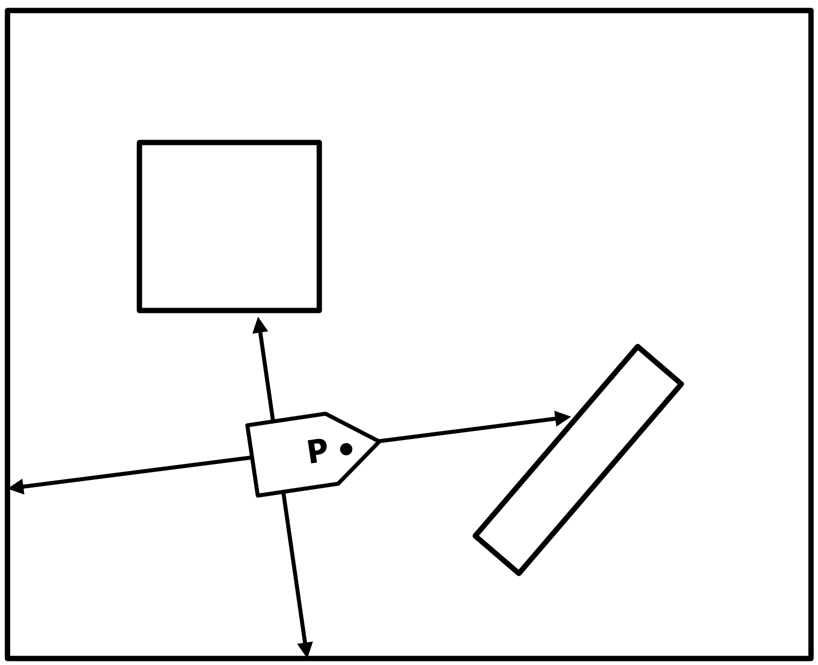
- Suppose robot sonars return four (noisy) range measurements $\{d_{F,B,L,R}\}$ as shown
- What robot *poses* are consistent with data?
- How might you identify them *efficiently*?

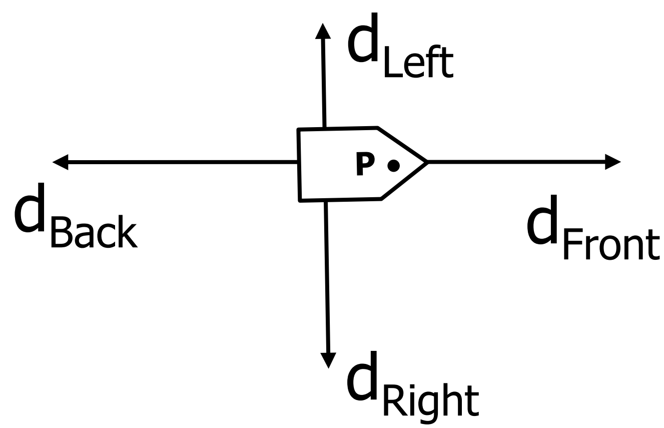




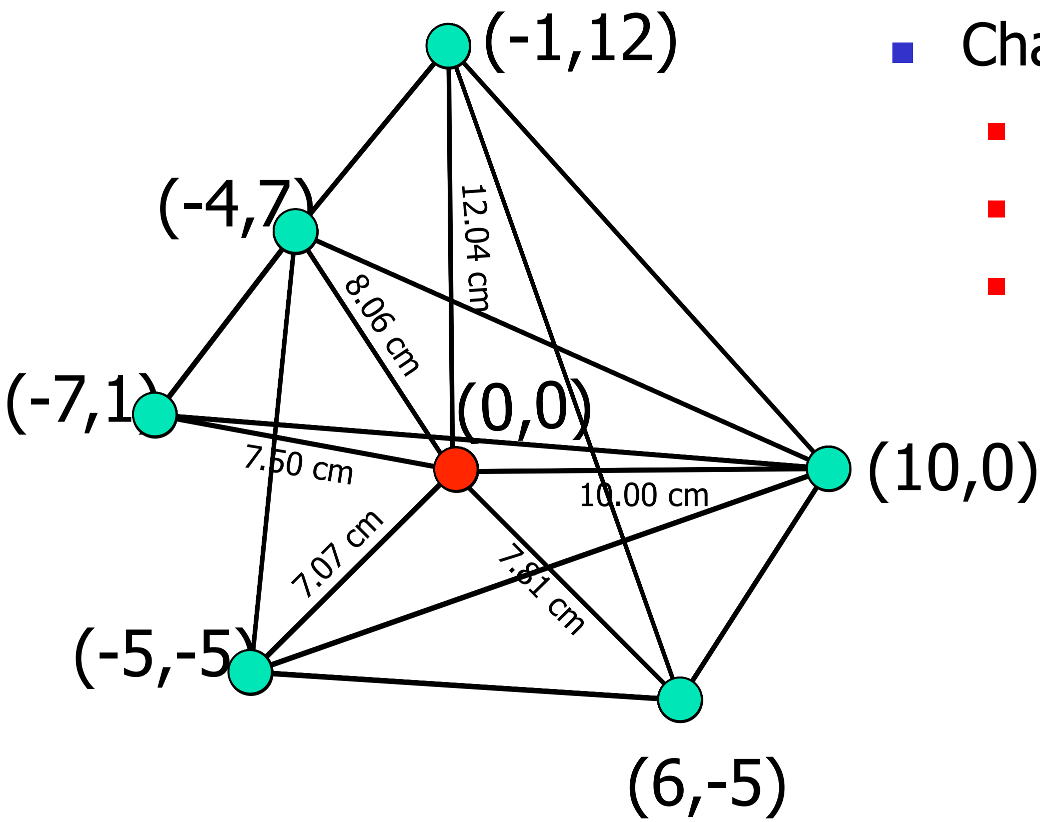
To Think About: Localization

- Below is one solution
  - If data are noiseless, is solution unique?
  - If data are noisy, is solution unique?





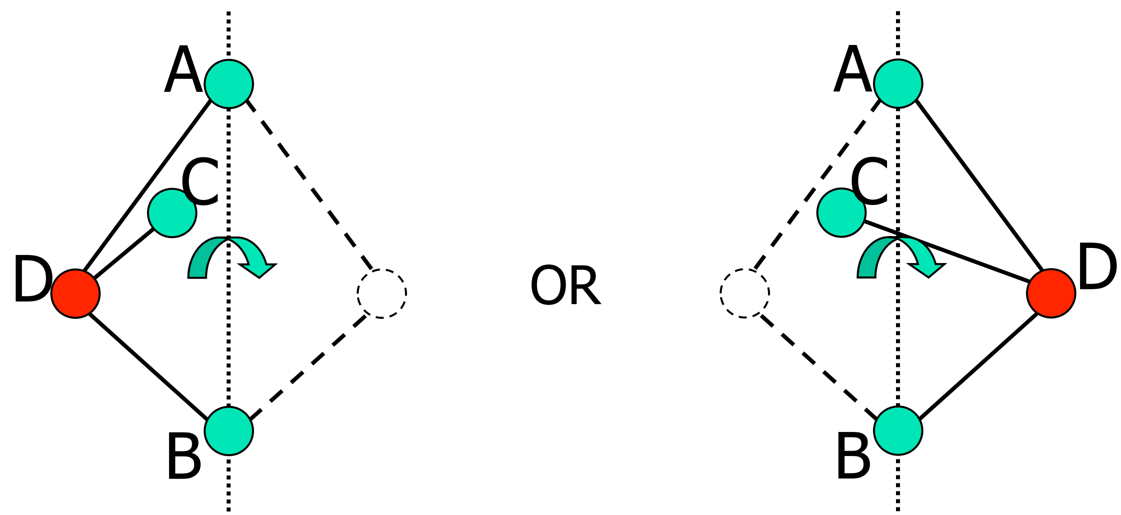
Localization With Noisy Ranges
(with D. Moore, J. Leonard, S. Teller)



- Characteristics:
  - Robust against noise
  - No beacons
  - Handles mobility

Complications of Noise

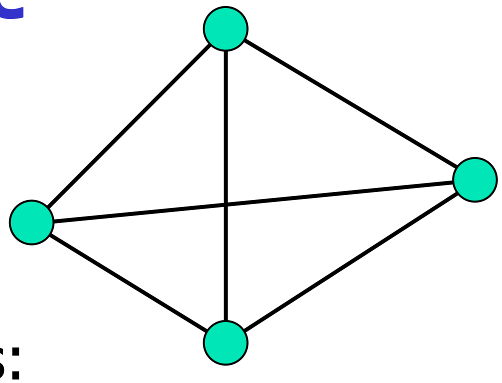
- Small measurement errors due to noise lead to large localization errors
- Example: **flip ambiguity** from noise



- Small error in CD leads to large position error of D

The Robust Quadrilateral

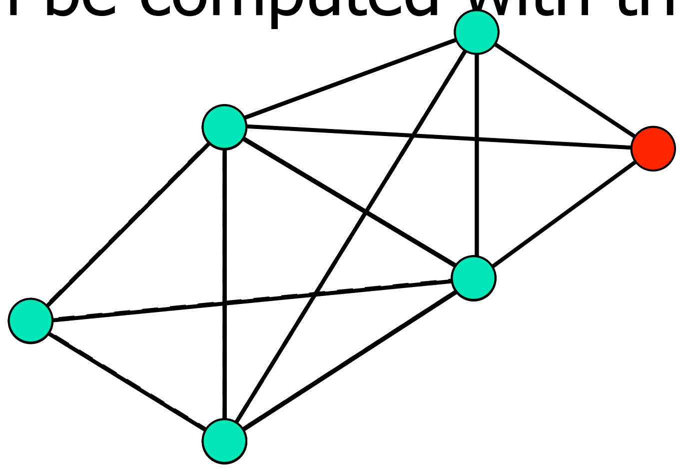
- Consider this graph:



- Robustness characteristics:
  - Rigid (no continuous deformations)
  - No discontinuous flex ambiguities (by Laman's Theorem)
  - We probabilistically constrain it to minimize the likelihood of a flip ambiguity
- We call it a *robust quadrilateral*
- A graph constructed from overlapping robust quads will itself possess the robustness characteristics

Trilateration w/ Robust Quads

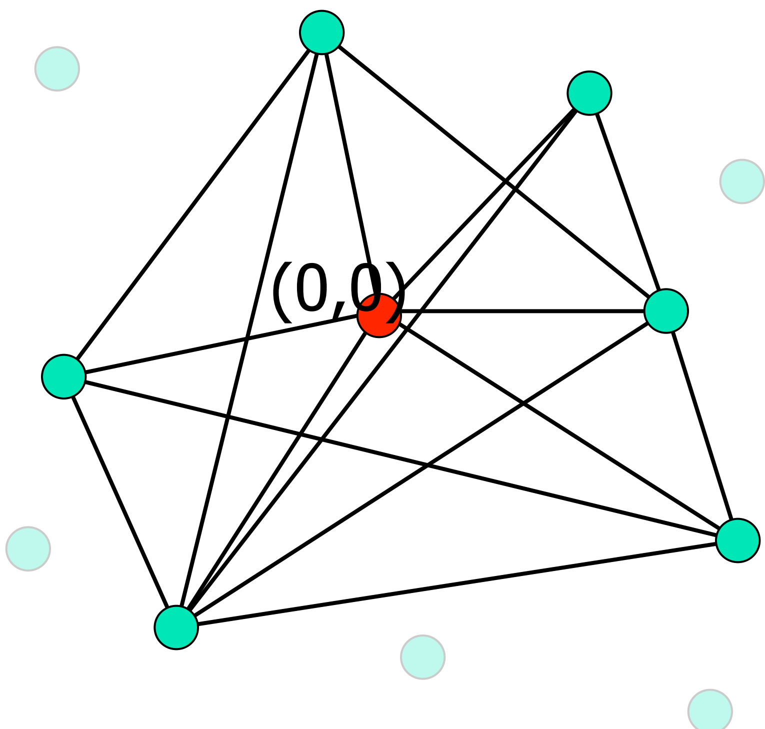
- If three nodes of a quad have known position, fourth can be computed with trilateration



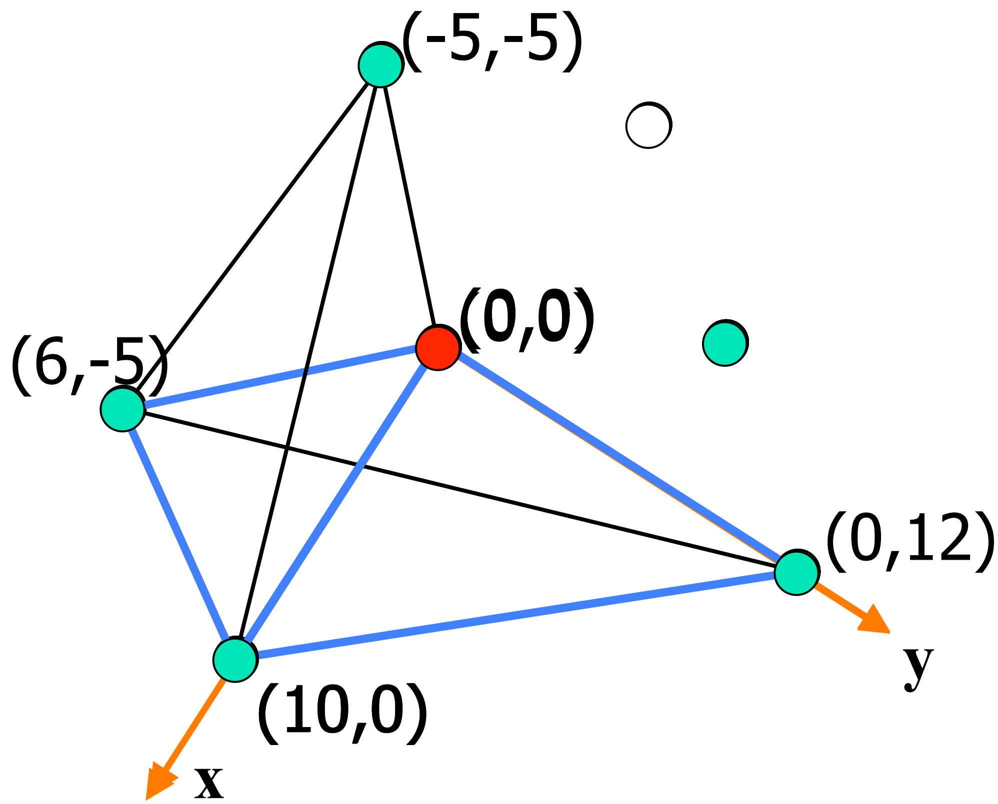
- Quads can be “chained” in this manner

Our algorithm

Starting cluster w/ distance measurements

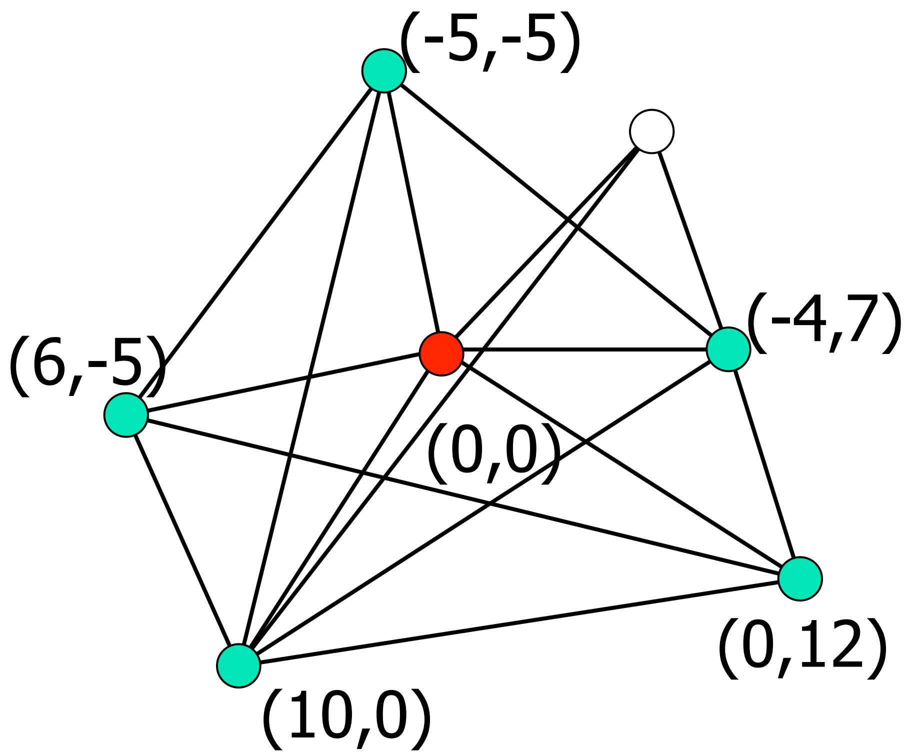


Choose two neighboring nodes for initial robust triangle



Our algorithm (cont.)

Cluster localization complete

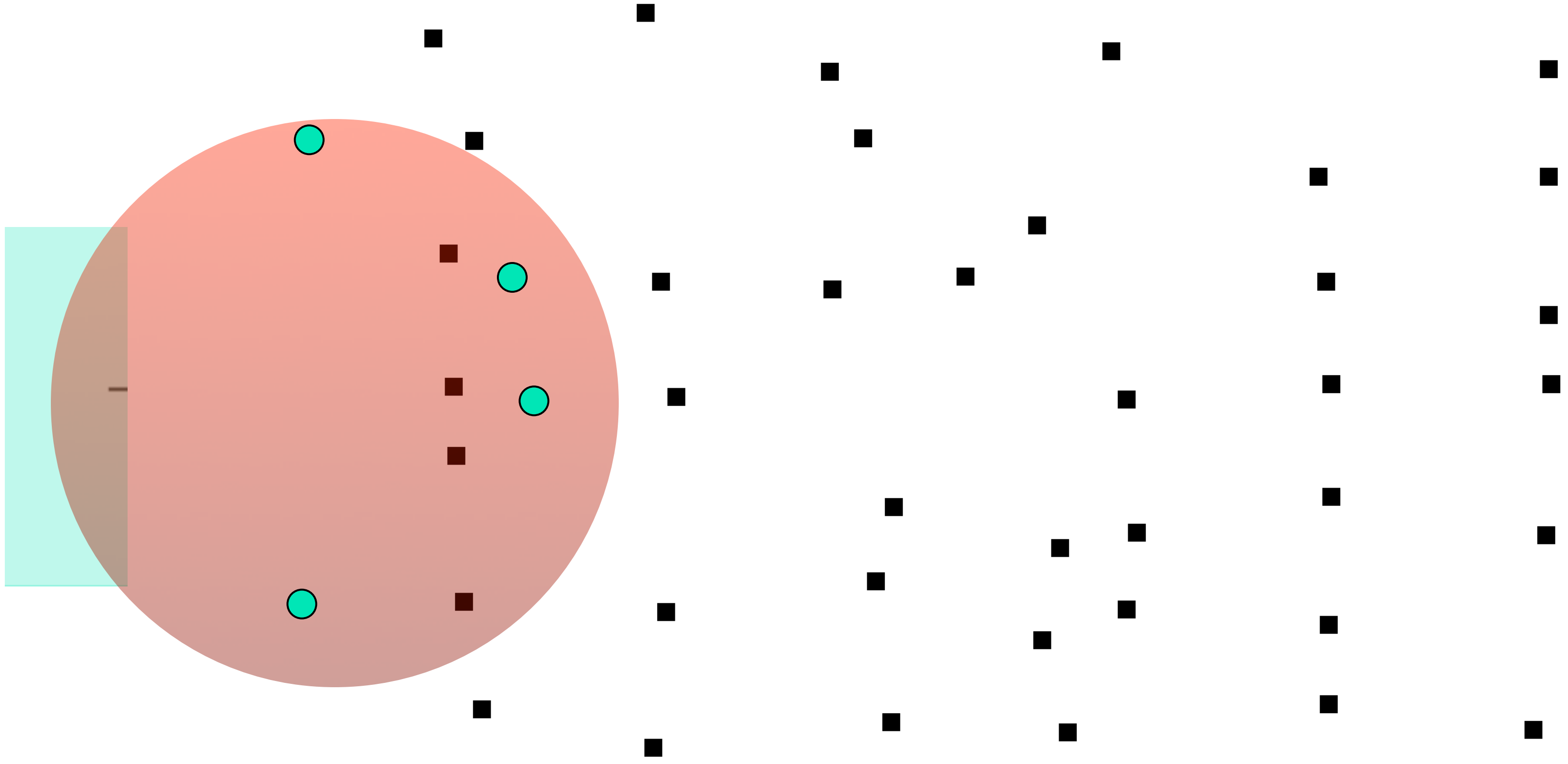


2-D Beacons from 1-D Ranging

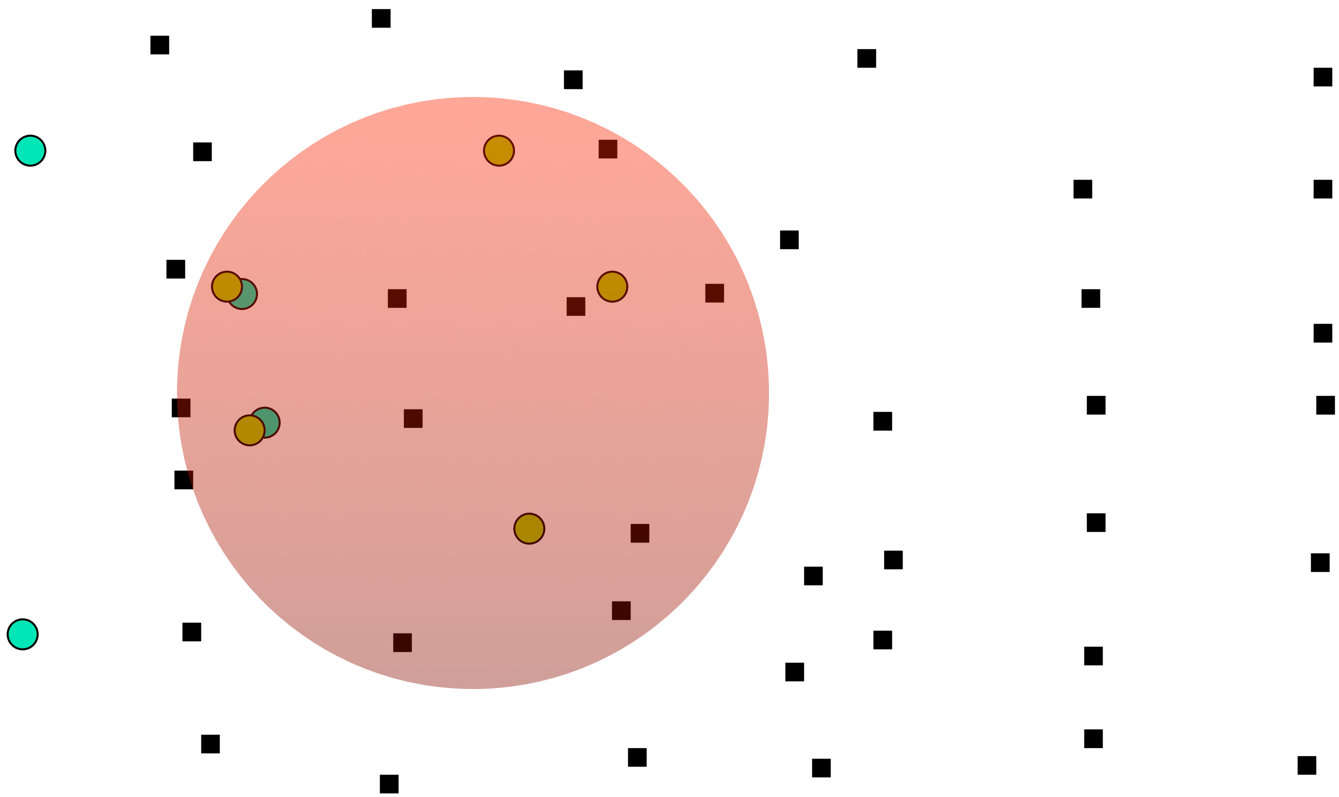
- Demo: "Cricket" RF / acoustic beacons
  - Pairwise ranging from TDoA of pulse pair, ranging σ_r ~3cm
  - End-to-end beacon localization with position error σ_{xy} ~5cm

Moore, Leonard, Rus, Teller, "Robust distributed network localization with noisy range measurements" (Sensys 2004)

Localizing the “left-over” nodes



Robot broadcasts locations



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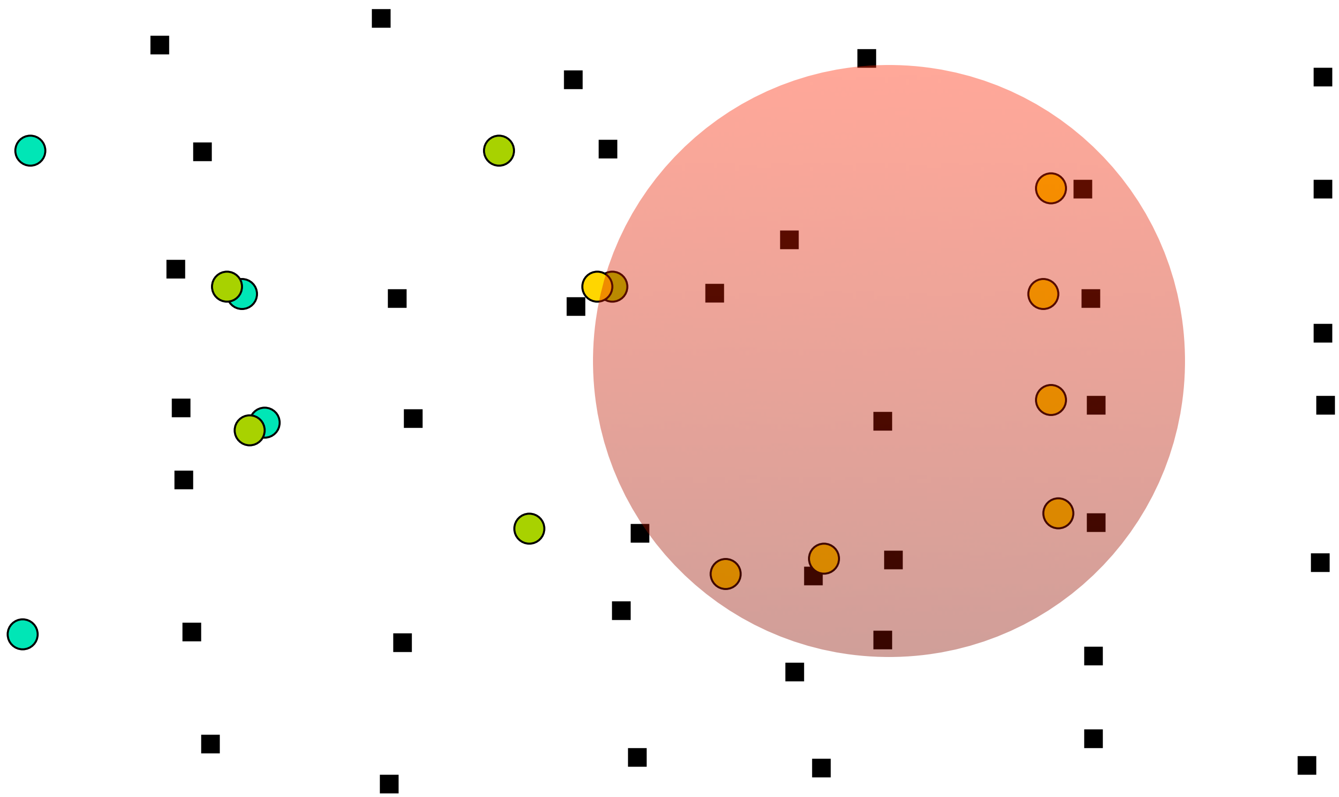

Robot broadcasts locations