Configuration Space for Motion Planning

RSS Lecture 10

Monday, 8 March 2010

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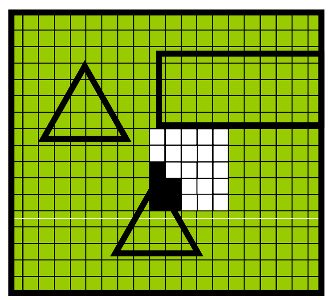
Siegwart & Nourbahksh S 6.2

(Thanks to Nancy Amato, Rod Brooks, Vijay Kumar, and Daniela Rus for some of the figures)

Last Time



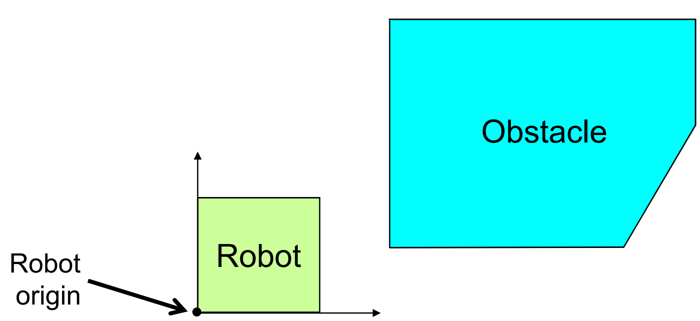
- Planning for point robots
  - Visibility graph method
  - Intermittent obstacle contact
- Ad hoc method of handling non-point robots
  - Represent robot as a (2-DOF) disk
  - Discretize Cartesian space, conservatively (Some feasible paths not identified by search)
- Today: “configuration space” methods
  - Reason directly in space with dimension = #DOFs
  - Transform, solve problem, transform back

Today

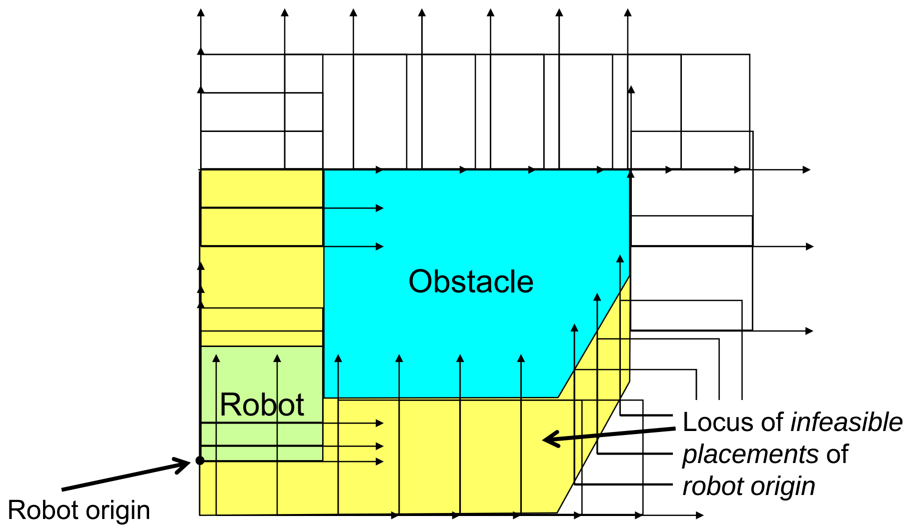
- Configuration space
  - Intuition
- Preliminaries
  - Minkowski sums
  - Convexity, convex hulls
- Configuration space
  - Definition
  - Construction
- Rigid (low-DOF) motion
  - Deterministic methods
- Articulated (high-DOF) motion
  - Randomized methods

Intuition

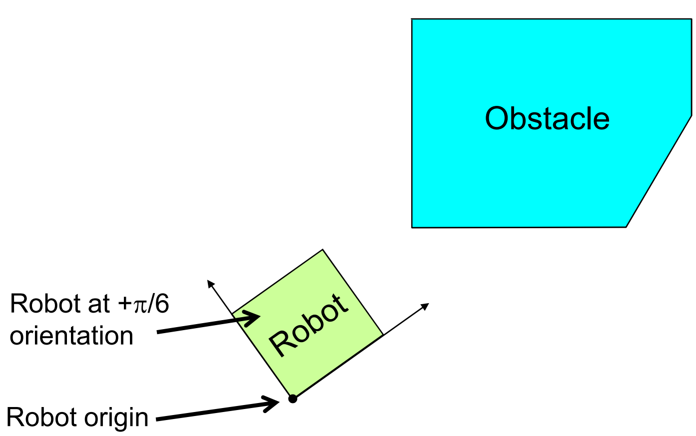
- Suppose robot can move only by translating in 2D
- How can it move in the presence of an *obstacle*?
  - How to describe *infeasible placements* of robot origin?



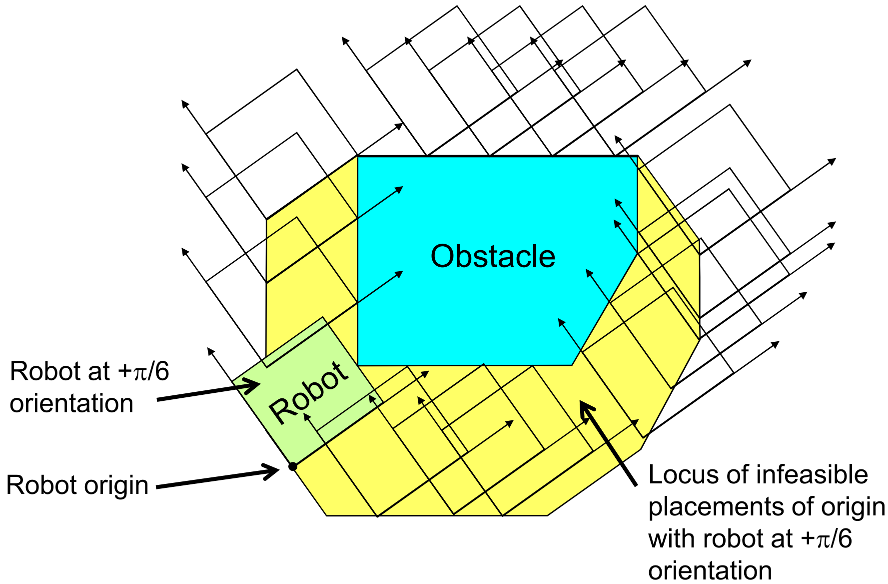
Infeasibility Under Translation



What if Robot can also Rotate?

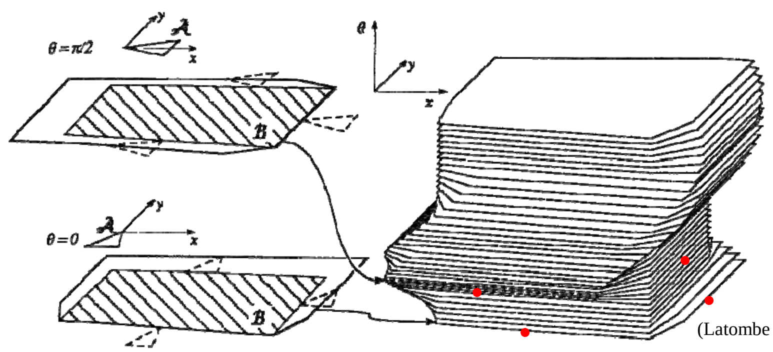


Infeasibility under 3-DOF Motion



Configuration Space

For a robot with *k* total motion DOFs, C-space is a coordinate system with *one dimension per DOF*

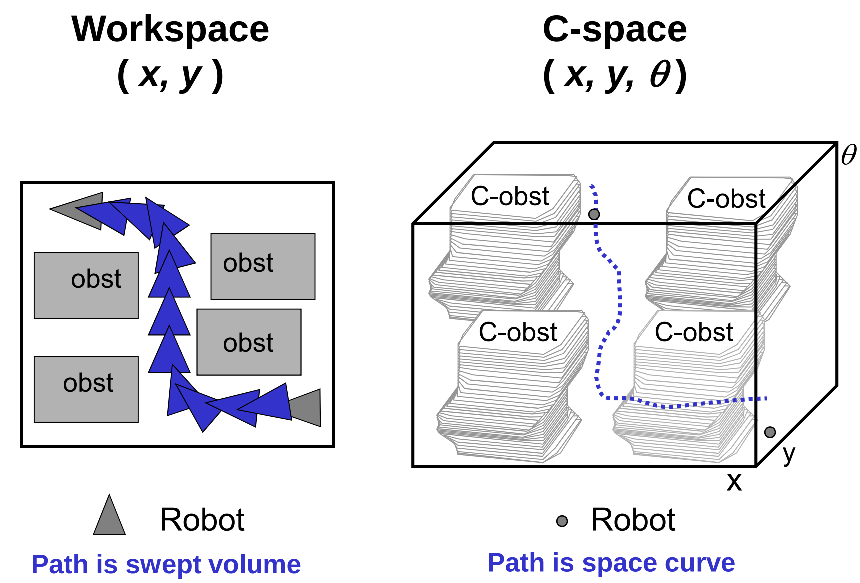


(Latombe 1991)

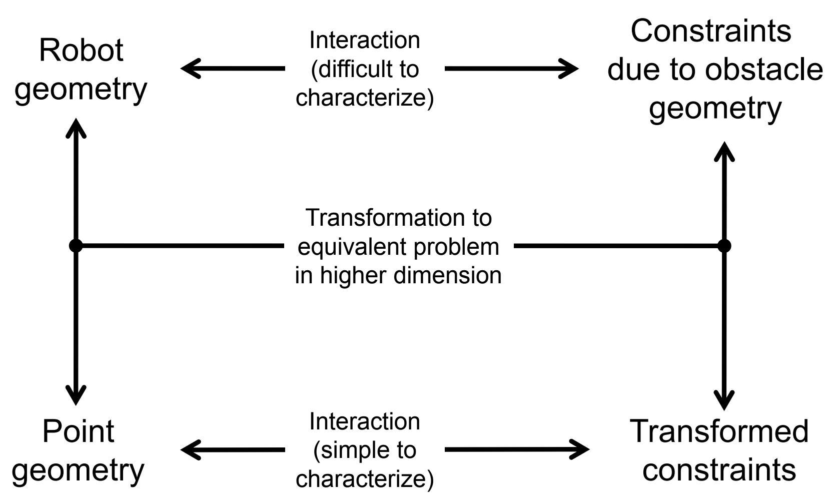
In C-space, a robot pose is simply

… and a workspace obstacle is a

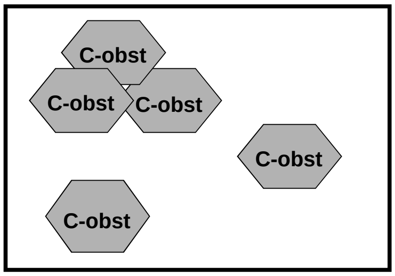
Motion Planning Transformation



Configuration Space Idea



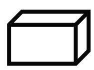
C-space Summary, Examples



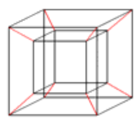
- Define space with one dimension per robot motion (or pose) DOF
- Map robot to a point in this space
- C-space = all robot configurations
- C-obstacle = locus of infeasible configurations due to obstacle

Some example configuration spaces:

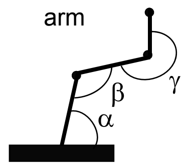
Translation + rotation in 2D



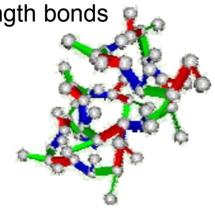
Translation + rotation in 3D



3-link arm

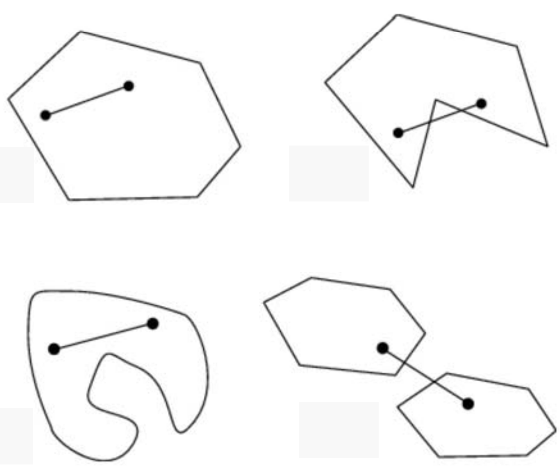


Molecule with *n* fixed-length bonds



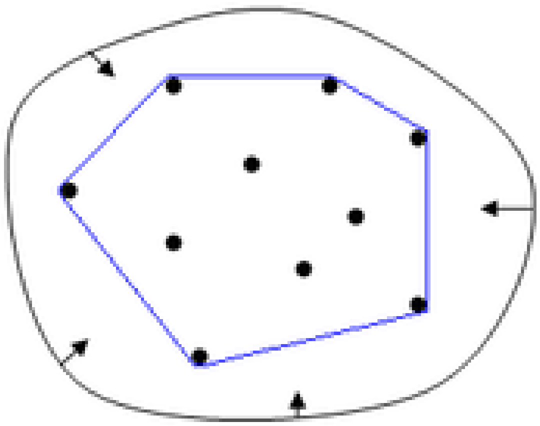
Convexity

- A set S is *convex* if and only if every line segment connecting two points in S is contained within S
- Which of these are convex?

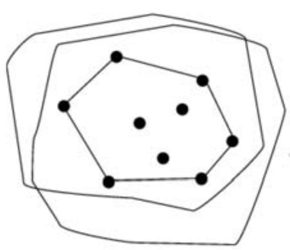


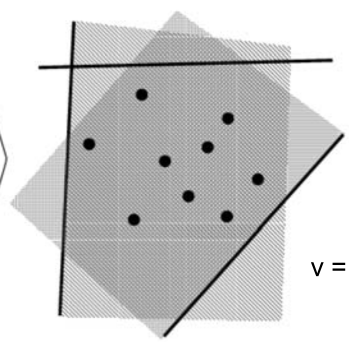
Convex Hull of a Set of Points

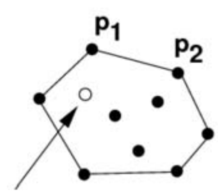
- Intuition: shrink wrap or rubber band around points



Convex Hull: Formal Definitions





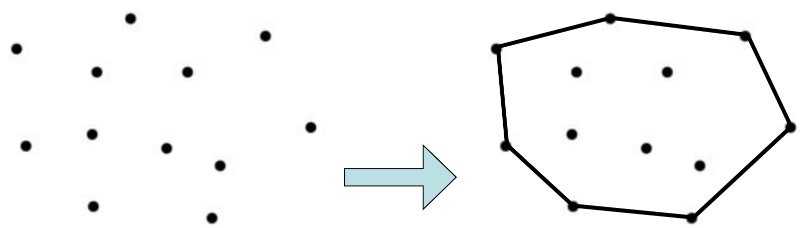


$v = \sum c_i \cdot p_i,\ c_i \geq 0,\ \sum c_i = 1$

- Which of these are constructive / algorithmic?

Computing 2D Convex Hull

- Input: set S of N points (x_i, y_i) in 2D
- Output: polygonal boundary of convex hull of S

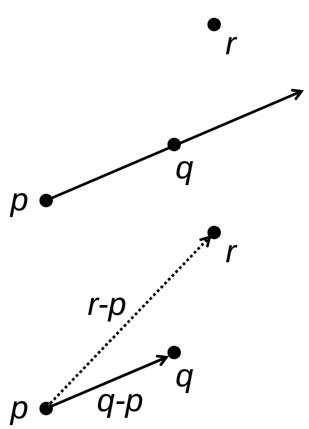


S Convex(S)

- How can Convex(S) be computed (efficiently)?

The Leftof Predicate

- Input: three points *p, q, r*
- Function Leftof (*p, q, r*) // argument order matters
- Output: 1 iff *r* is left of *directed line* $\overrightarrow{pq}$, otherwise -1



How to implement Leftof()?

1. Compute sign of determinant

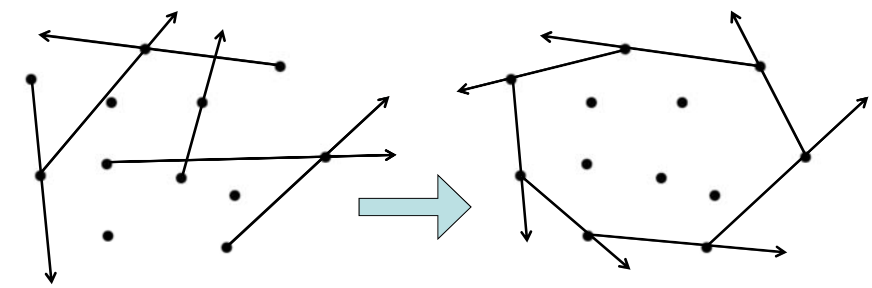
$$\begin{vmatrix} 1 & r_x & r_y \\ 1 & p_x & p_y \\ 1 & q_x & q_y \end{vmatrix}$$

2. Equivalently, find sign of *z* component of

Brute Force Solution

Identify point pairs that form edges of Convex(S)

I.e. for each pair p, q ∈ S, if ∀ r ∈ S – {p, q}, r lies left of the directed line $\overrightarrow{pq}$, emit

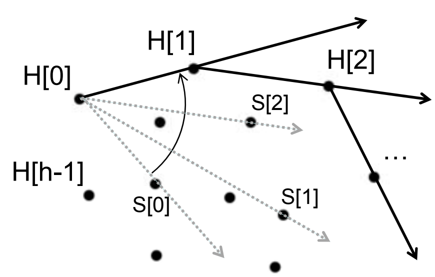


Running time for input of *n* points?

Can do better: O(n^2), O(n log n), O(nh), O(n log h) !

Jarvis March Algorithm

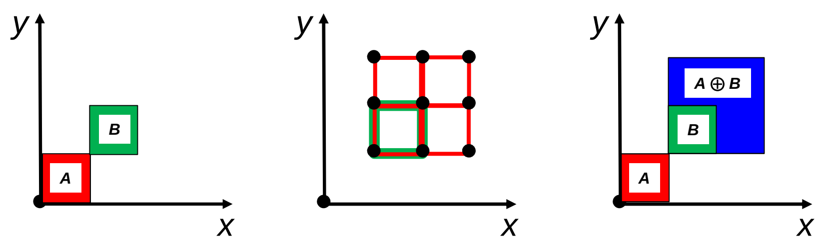
```
pivot = leftmost point in S;  i = 0   // leftmost point must be on convex hull
repeat
   H[i] = pivot           // store hull vertices in output point list H[i], 0 ≤ i < h
   endpoint = S[0]              // check candidate hull edge [pivot .. endpoint]
   for j from 1 to |S|-1
      if (Leftof (pivot, endpoint, S[j]))
         endpoint = S[j]
   pivot = endpoint; i++
until endpoint == H[0]
```



Outer loop runs times;
inner loop does work
Running time for input set of *n* points? "Output-sensitive" algorithm.

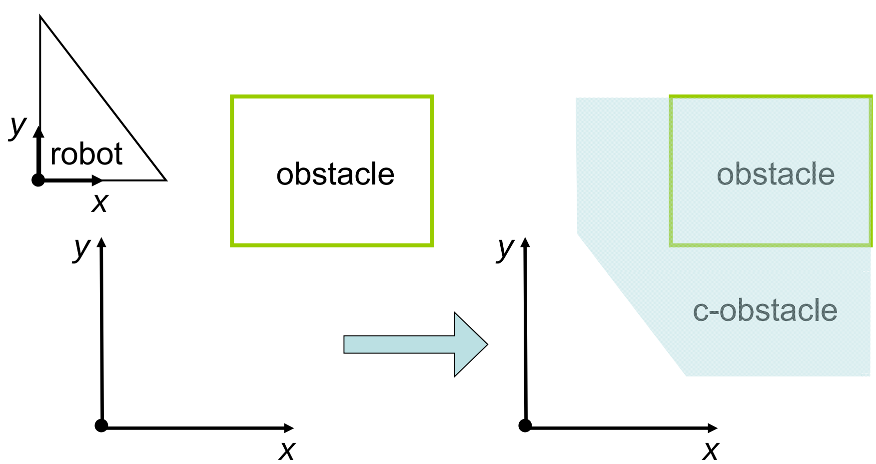
Minkowski Addition

- Given two sets $A,B \in R^d$, their *Minkowski sum*, denoted $A \oplus B$, is the set $\{ a + b \mid a \in A, b \in B \}$
  – Result of adding each element of *A* to each element of *B*
- If A & B convex, just add vertices & find convex hull:



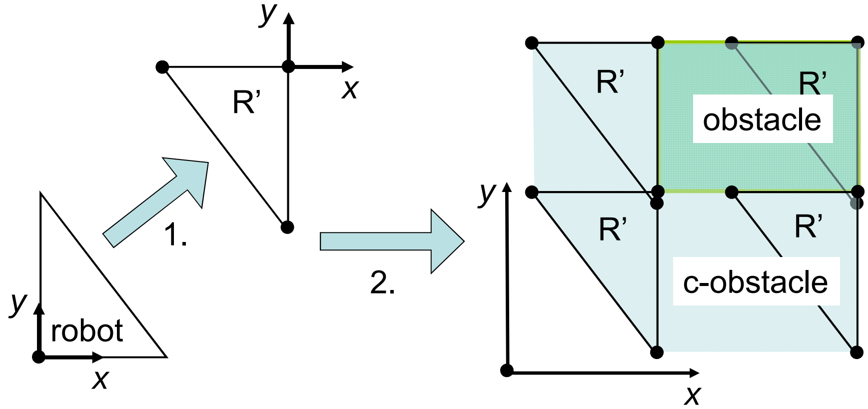
Computation of C-obstacles

- Inputs: robot polygon R and obstacle shape S
- Output: c-space obstacle c-obstacle(S, R)



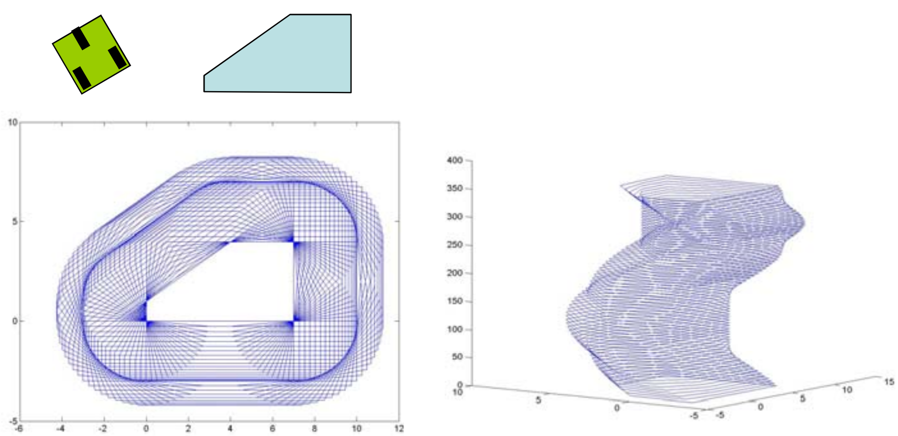
C-obstacle Computation

1. Reflect robot R about its origin to produce R’
2. Compute Minkowski sum of R’ and obstacle S



Sanity check: can robot *origin* enter c-obstacle?

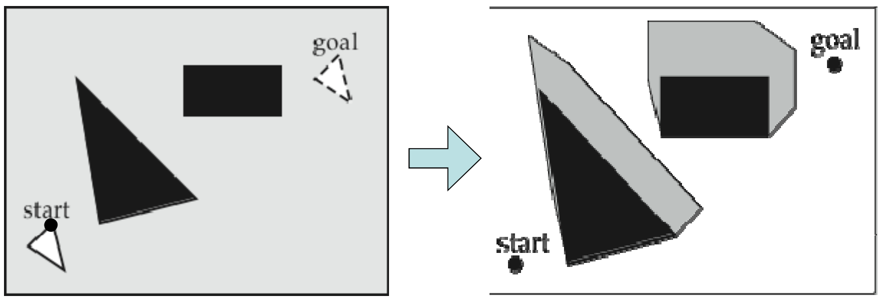
C-obstacles with Rotations



How do we compute this object?

Back to Motion Planning

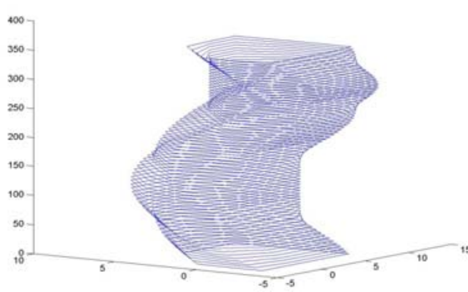
- Given robot and set of obstacles:
  - Compute C-space representation of obstacles
  - Find path from robot start pose to goal pose (point)



- Unfortunately, we have a rather serious problem:
  - We have constructed a representation of the *obstacles*
  - But we need to search a representation of the *freespace!*

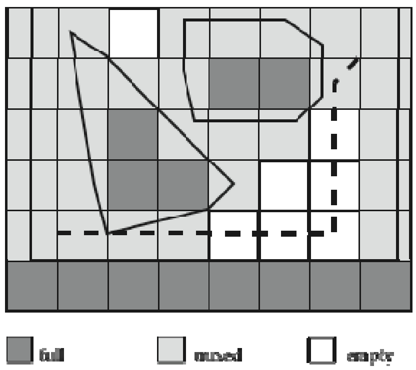
Computational Complexity

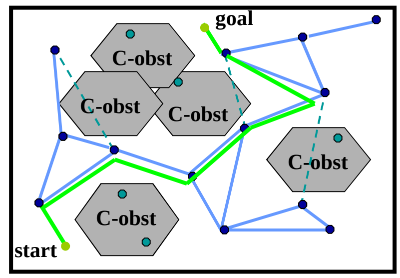
- The best deterministic motion planning algorithm known requires *exponential* time in the C-space dimension [Canny 1986]
- D goes up fast – already 6D for a rigid body in 3-space; articulation adds many more DOFs
- Simple obstacles have complex C-obstacles
- Impractical to compute explicit representation of freespace for robot with many DOFs
- What to do?



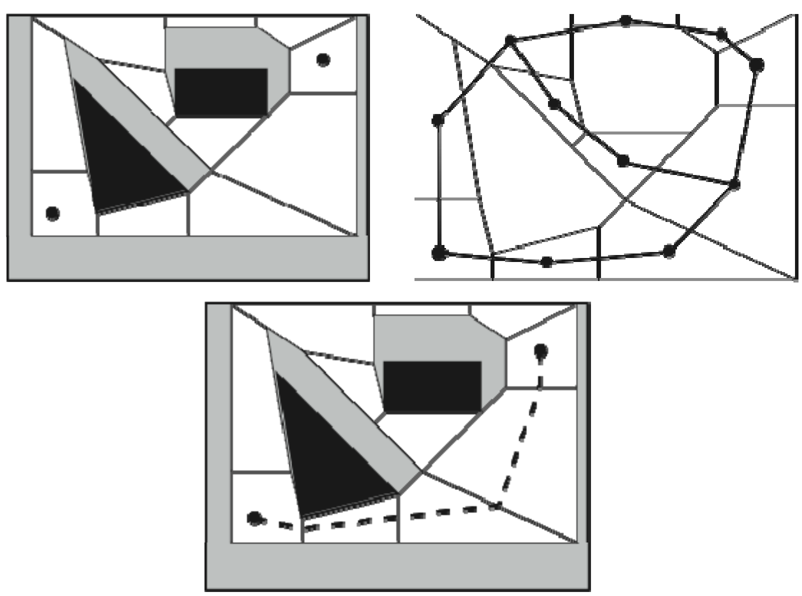
Strategies

- Approximate: use regular subdivision of freespace
- Randomize: sample and evaluate C-space poses
- Trade away completeness for gains in

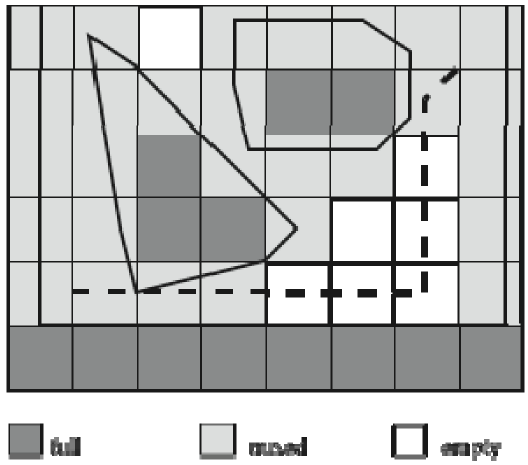




Example: Exact Decomposition



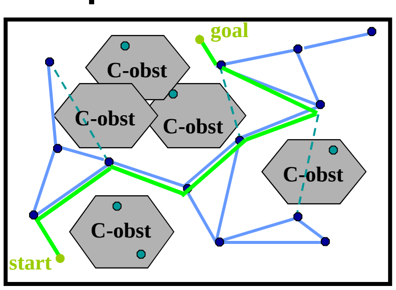
Approximate Cell Decomposition



- Advantage: recasts complex original problem as search within space of many, simpler motion plans

Probabilistic Road Maps for Motion Planning [Kavraki et al. 1996]

C-space



Roadmap Construction (Pre-processing)

1. Randomly generate robot configurations (nodes)
   - Discard invalid nodes (how?)
2. Connect pairs of nodes to form **roadmap edges**
   - Use simple, deterministic *local planner*
   - Discard invalid edges (how?)

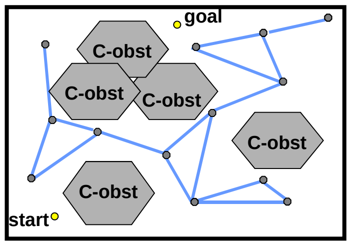
Plan Generation (Query processing)

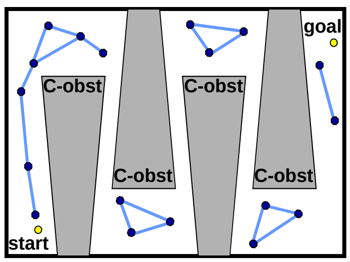
1. Link *start* and *goal* poses into roadmap
2. Find path from *start* to *goal* within roadmap
3. Generate motion plan for each edge used

Primitives Required:

1. Method for sampling C-Space points
2. Method for “validating” C-space points and edges

PRMs: Pros and Cons





Advantages

1. *Probabilistically complete*
2. Easily applied to high-dimensional C-spaces
3. Support fast queries (w/ enough preprocessing)

Many success stories in which PRMs were applied to previously intractable problems

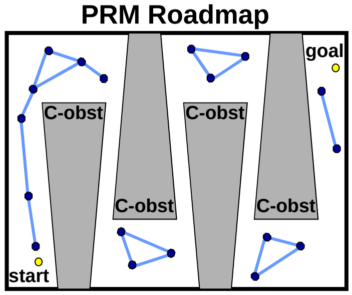
Disadvantages

PRMs don't work well for some problems:
– Unlikely to sample nodes in *narrow passages*
– Hard to connect nodes along constraint surfaces

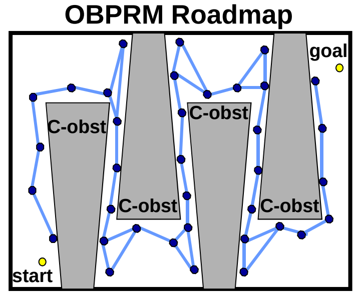
Sampling Around Obstacles: OBPRM [Amato et al. 1998]

To Navigate Narrow Passages we must sample in them
Most PRM nodes lie where planning is easy, not where it's hard

PRM Roadmap

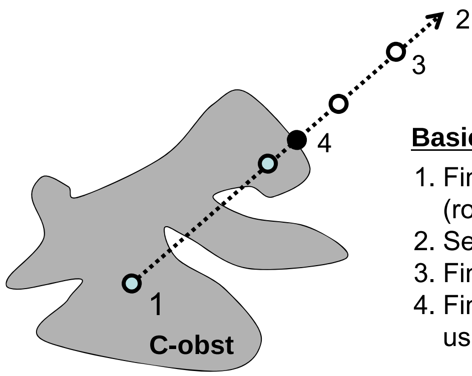


OBPRM Roadmap



Idea: Can we sample nodes near C-obstacle surfaces?
• We cannot explicitly construct the C-obstacles, but...
• We do have models of the (workspace) obstacles!

Finding Points on C-obstacles



Basic Idea (for workspace obstacle S)

1. Find a point in S's C-obstacle (robot placement colliding with S)
2. Select random direction in C-space
3. Find freespace point in that direction
4. Find boundary point between points using binary search (collision checks)

Note: we can use more sophisticated approaches to try to "cover" C-obstacle

Summary

- Introduced drastically simplifying transformation
  - Based on two useful geometric constructions
- Enables use of familiar techniques…
  - Discretization
  - Random sampling
  - Bisection
  - Graph search
- … To solve high-dimensional motion planning

- We'll use these ideas in Lab 6