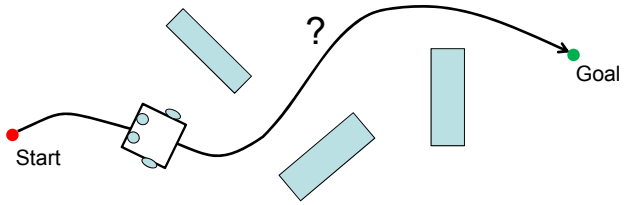
Motion Planning

RSS Lecture 9
Wednesday, 3 Mar 2010
Prof. Seth Teller

Motion Planning Intuition

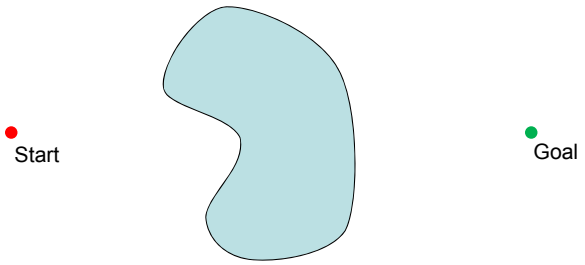
- How can robot move smoothly from start to goal?



- Are there cases in which no such motion exists?

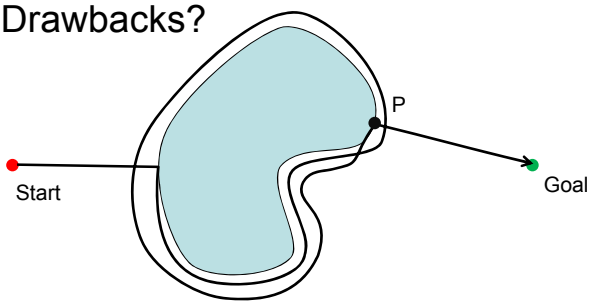
Bug Motion Planning Algorithm

- Simple algorithm based on four assumptions:
  - Perfect knowledge of direction and distance to goal
  - Ability to distinguish freespace from obstacle contact
  - Ability to move along an arbitrary obstacle boundary
  - Ability to detect whenever a location is revisited
- Which of these assumptions are strong? Weak?



Bug Motion Planning Algorithm

- Repeatedly advance toward goal
- Upon encountering an obstacle:
  - Circumnavigate it completely, then depart from point P that minimizes distance to goal
- Advantages? Drawbacks?



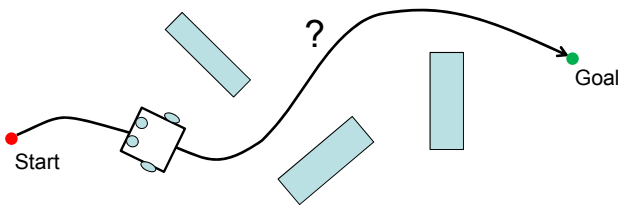
- Variants: Bug2, BugDist, BugTangent…

Complete Motion Planning

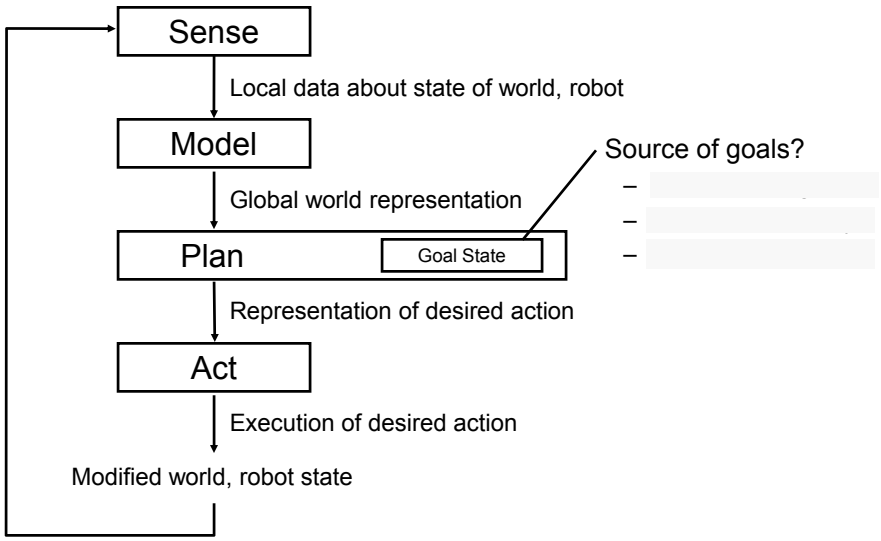
- Formal statement of motion planning problem:
  - Compute a collision-free path for a rigid or articulated moving object among static (or dynamic) obstacles
- Ideally we desire a "complete" motion planner:
  - If a solution exists, the planner is guaranteed to return it
  - Otherwise, planner indicates that no solution exists
- CMP is known to be computationally difficult
  - In general it requires exponential running time in the number of DOFs (articulation, # of obstacles etc.)
  - … Even with access to perfect, global information!

Planning Under Uncertainty

- How can robot move from starting configuration to a goal configuration despite *uncertainty:*
  - Imperfect prior knowledge
  - Imperfect perception
  - Imperfect reasoning
  - Imperfect execution
  - Imperfect prediction

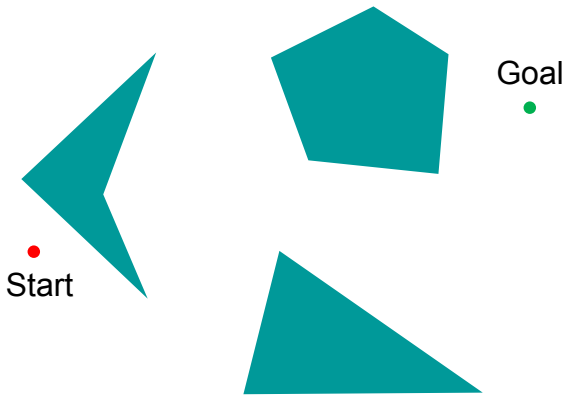


Deliberative Architecture



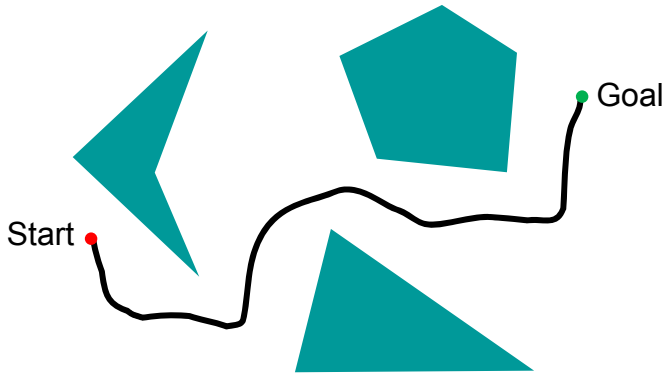
Off-Line Motion Planning

- Today, we'll make some strong assumptions:
  - Perfect map of obstacles, start, goal
  - Perfect robot localization



Motion Planning Intuition

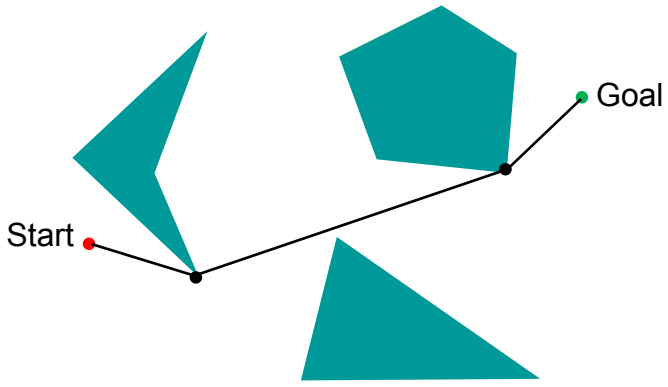
- We want robot to stay far from obstacles



… But we don't yet have a suitable representation of freespace to work with

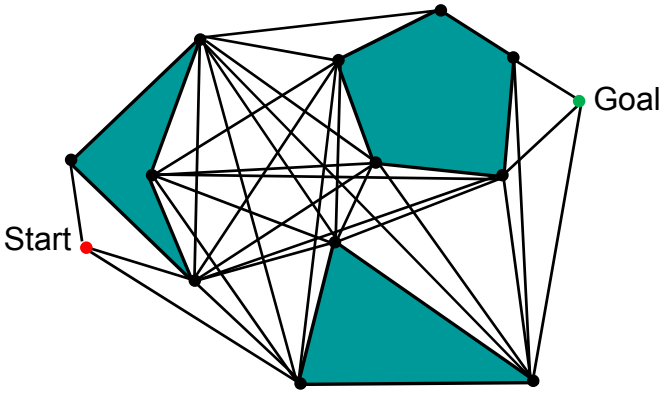
Observation

- If there exists a collision-free path from start to goal, then there exists a piecewise-linear path involving only start, goal and obstacle vertices



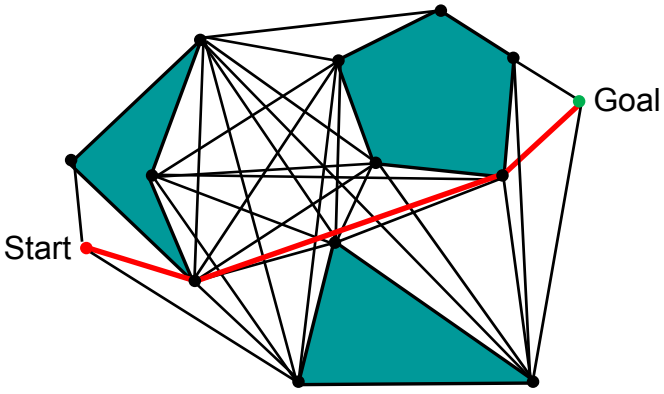
Visibility Graph Algorithm

- Construct graph G = (V, E)
  - V = {obstacle vertices} ∪ {start, goal}
  - E = edges (v_i, v_j) disjoint from obstacle interiors



Find Shortest Path in Graph G

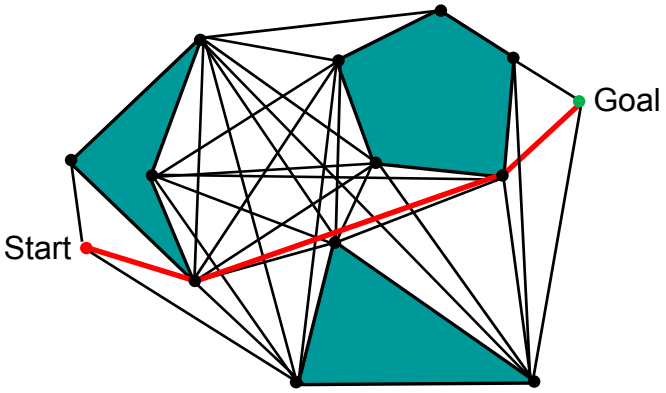
- Use rooted at start vertex



Algorithm
Single-source Shortest Path

```
1  function          (G, w, s)        // Graph G, weights w, source s
2    for each vertex v in V[G]         // Initialize d[], previous, S, and Q
3      d[v] := ∞                       // Vertex v is not yet reached
4      previous[v] := undefined        //   … so there's no path to it yet
5    d[s] := 0                         // Source reachable with zero cost
6    S := empty set                    // Set of vertices reached so far
7    Q := set of all vertices          // Set of candidate vertices
8    while Q is not an empty set       // While unreached vertices
9      u := vtx v in Q with            // O(n) search or Fibonacci heap
10     S := S union {u}                // Vertex u reached
11     for each edge (u, v)            // For each neighbor v of u
12       if                            // If lower-cost path to v exists via u
13         d[v] := d[u] + w(u,v)       //   … update cost to v
14         previous[v] := u            //   … and update path record
```

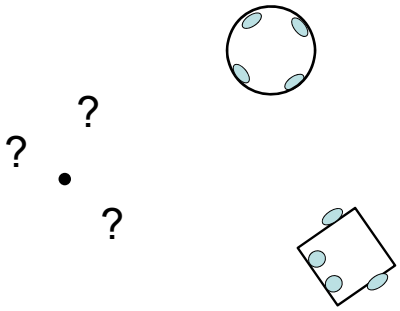
Application of Shortest-Path



- What do we use as edge weights?
- Memory usage?
- Running time?
- Can we optimize by omitting reflex vertices?
- What major assumption have we made about the robot?

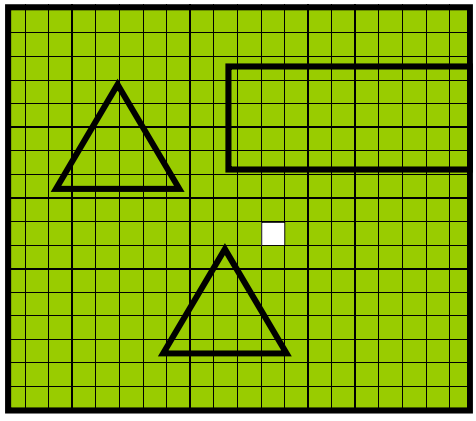
A *Point* Robot?

- Can't fit much robot into a zero-area point …
  - Today we'll address robot extent via discretization
  - Next time we'll see a much more elegant method



Discretizing Polygonal Obstacles

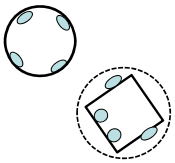
- How should we discretize freespace into a grid?
  - Is this just like rendering polygons in graphics?

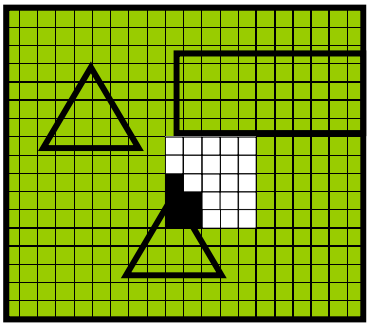


  - To avoid collisions, we must account for !

Discretizing Polygonal Obstacles

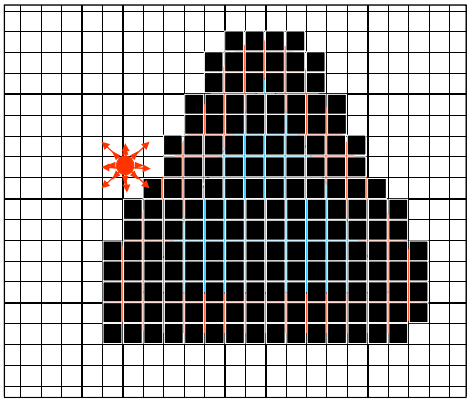
- For today, assume robot is a disk with radius R
  - Then for planning purposes, robot has only 2 DOFs (why?)
- Then a grid square represents freespace if:
  - It does not overlap with any obstacle
  - It lies further than R from all obstacle edges
- Algorithm:
  - Pick a grid square that is known to lie in freespace
  - Do breadth-first search (or "flood-fill") from that start square
  - As each square is visited by the search, compute the minimum distance d to any obstacle edge
  - Label square "free" if d > R; otherwise label square "occupied"
  - Once BFS is complete, label any unlabelled squares as "occupied"





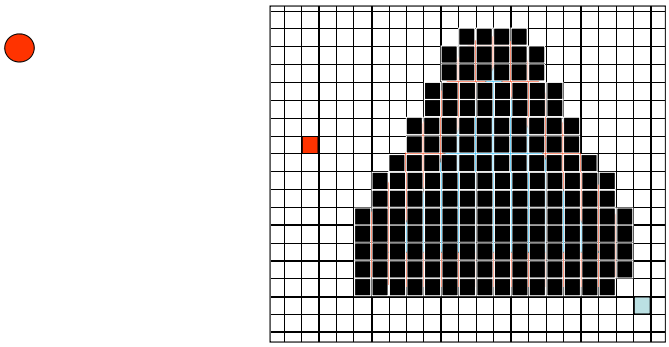
Example of a Discrete State Space

- Cartesian space
- Configuration space

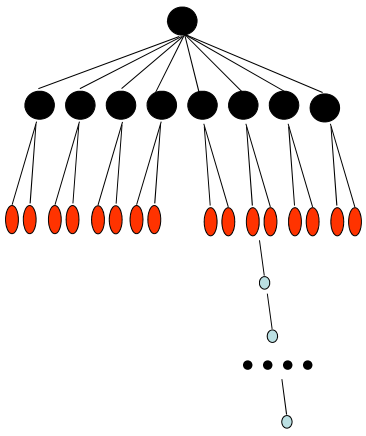


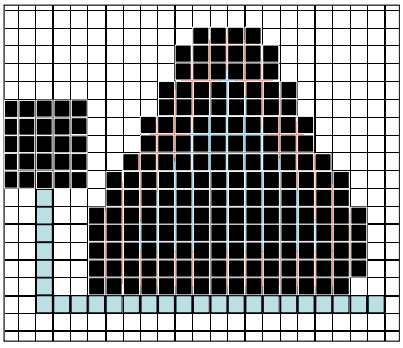
- Actions take robot from one state to another
- Objective is to find a path from the start state to the goal state

Planning as Tree Search



Planning as Tree Search

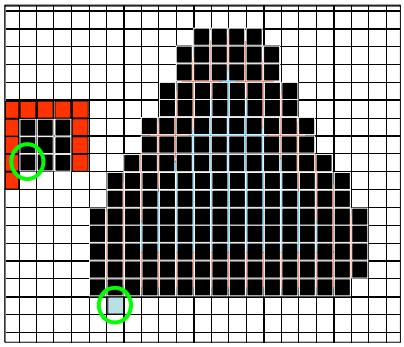




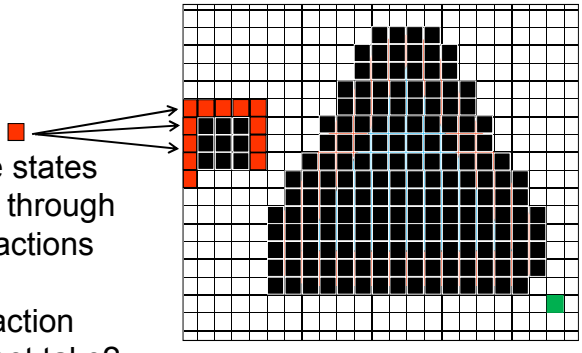
… How can such searching be made *effective* and *efficient*?

Move Generation

- Which state-action pair to consider next?
- Shallowest next
  - Aka: Breadth-first search
  - Guarantees shortest path
  - But: storage-intensive
- Deepest next
  - Aka: Depth-first search
  - Can use minimal storage
  - But: no optimality guarantee



Informed Search – A*



Candidate states reachable through available actions

… which action should robot take?

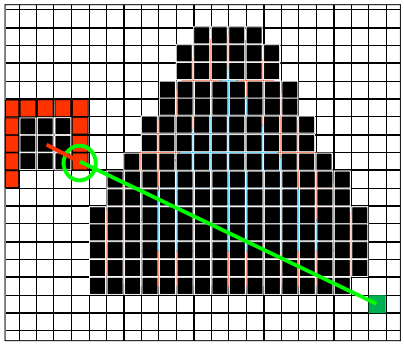
Informed Search – A*

- Use domain knowledge to bias the search
- Favor actions that might get closer to the goal
- Each state gets assigned an approximate cost

$f(x) = c(x) + h(x)$

Cost incurred to here from the start state

Estimated cost from here to the goal, aka the "heuristic" cost



- For example:
  - c(x) = 3, h(x) = ||x-goal|| = sqrt(8^2+18^2) = 19.7, so f(x)=22.7

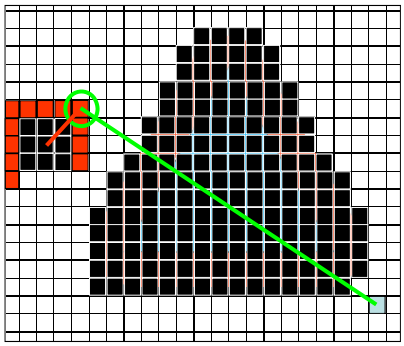
Informed Search – A*

- Each state gets assigned an approximate cost

$f(x) = c(x) + h(x)$

Cost incurred to here from the start state

Estimated cost from here to the goal, aka the "heuristic" cost



- Choose the state with the
- Cost for another example candidate action is higher:
  - c(x) = 4, h(x) = ||x-goal|| = sqrt(11^2+18^2) = 21.1, so f(x)=25.1

How to Construct Heuristics

- The more closely h(x) approximates the true cost to the goal, h*(x), the more efficient the search will be* …

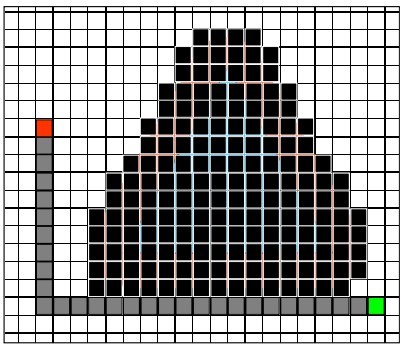
BUT:

- In order for A* to find the optimal path, it must be the case that
- Why? Suppose this was not the case. Then the search would
- Such an h is called an “admissible” heuristic

*There is an interesting design tradeoff involved here – what is it?

A Problem with Plans

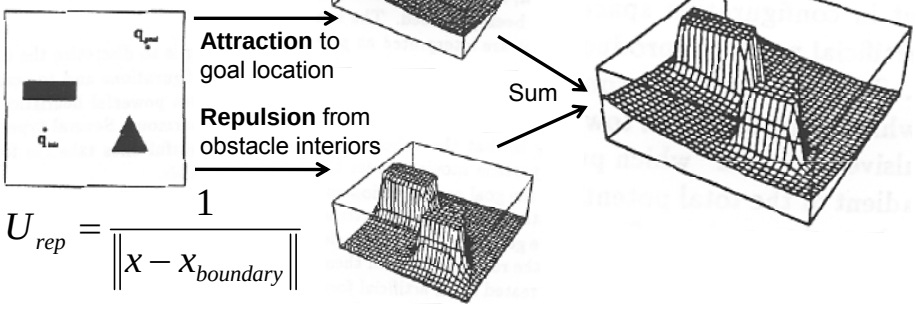
- We have a plan that gets us from the start to the goal
- But… what happens if we *depart* from the plan?
  - We can *replan*, or:
  - We can keep a *policy*



Potential Field Method

- Real-time collision avoidance method [Khatib 1986]
- Construct scalar potential field throughout freespace

$$U_{att} = \frac{1}{2}\left\|x - x_{goal}\right\|^2$$

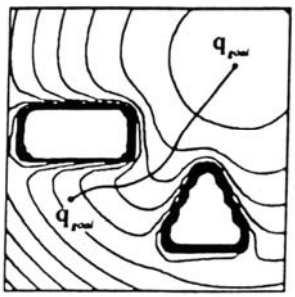


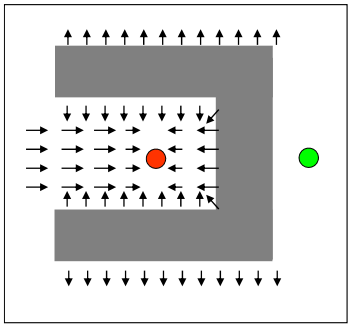
$$U_{rep} = \frac{1}{\left\|x - x_{boundary}\right\|}$$

- Robot moves along [redacted] of potential field

Ideal Potential Field

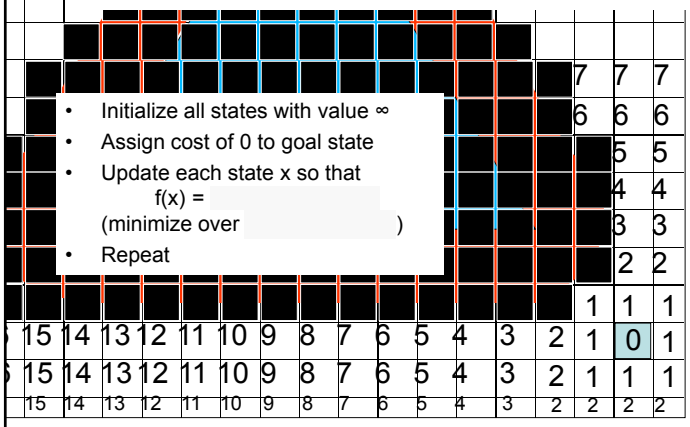
- We want to construct the potential field so that it:
  - Is nearly infinite close to obstacles
  - Has a global minimum at the goal (so no local minima)
  - Is smooth everywhere
  - Does algebraic method achieve this? [redacted]





If only life were so easy…

Numerical Potential Field

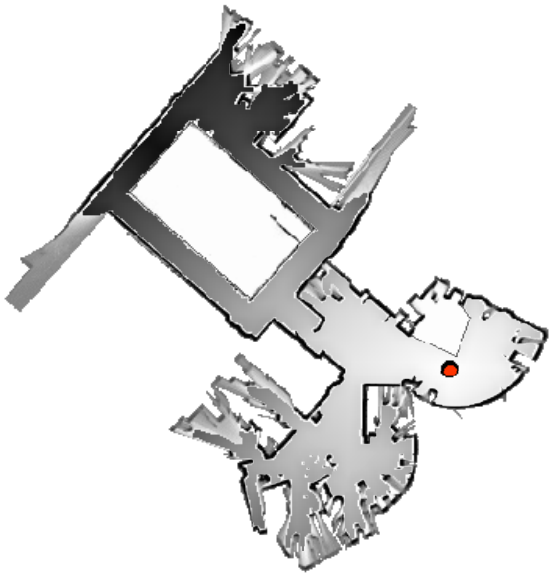


- Initialize all states with value ∞
- Assign cost of 0 to goal state
- Update each state x so that
  f(x) =
  (minimize over)
- Repeat

Numbers shown are for an obstacle-induced cost of ∞, and a goal-induced cost of 1 unit per grid cell (can also make it costly to approach obstacles)

Post-plan: for each state

Example Output Value Function



Completeness Questions

- Recall our definition of complete MP
  - Is the visibility graph algorithm complete?
  - Is the potential field algorithm complete?

Recap: Design Decisions

- How is your map described? This will have an impact on the state space for your planner
  - Is it a list of polygons?
  - Is it a grid map?
- What are you trying to optimize?
  - The fastest path (time)?
  - The shortest path (wear and tear)?
  - The lowest-energy path (battery usage)?
- What kind of search should you use?
  - Can you formulate a reasonably good heuristic?
  - If so, then maybe A* is a good idea
- Physical intuition can yield useful algorithms
  - Potential field method