Localization

RSS Lecture
Monday, March 9, 2009
Prof. Teller
Text: Siegwart and Nourbakhsh Ch. 5
Dudek and Jenkin Ch. 7

Navigation Overview

- Where am I?
  - Localization (Today)
  - Assumes accurate map, but imperfect sensing
- Where have I been?
  - Mapping (Wednesday)
  - Assumes effective localization
- How can I get there from here?
  - Planning (Next M & W)
  - Assumes perfect map, sensing, and actuation

Thought experiment

- Does it make sense to localize in a void (an environment containing absolutely nothing)?
  ... not very interesting; We conclude that there has to be some kind of “stuff” in environment
- What if the environment is isotropic (space, fog, water, desert, jungle etc.)?
  ... again, not very interesting for robot to move or perform tasks within such an environment

We conclude that environment must contain features that can be sensed (distinguished) by bot

Localization Problem Statement

- Given some representation of the environment, to localize, robot must, through sensing, determine its pose with respect to the specified representation

- Defined with respect to some frame or feature set that is external to robot:
  - Global coordinate frame
    • E.g., GPS (Earth) coordinates
  - Local coordinate frame
    • Ceiling or floor tiles
    • Mission starting pose
  - Environment features
    • E.g., nearby walls, corners, markings
Basic Localization

- **Open-loop pose estimation:**
  - Maintain pose estimate based on expected results of motion commands (no sensing)

- **Dead reckoning:**
  - Use proprioception (odometry, inertial) to estimate pose w.r.t. initial coordinate frame
  - Multiple error sources:
    - Wheel slip, gear backlash
    - Noise (e.g. from encoders)
    - Sensor, processor quantization errors
  - Pose error accumulates with time and motion
  - Typically ~ a few percent of distance traveled

Dead Reckoning Error

- Two hours of slow, rolling motion through MIT main campus corridors at third-floor level
- High-precision inertial sensors exist... do they solve problem?

Landmark Attributes

- Is landmark passive or active?
  - Must sensor emit energy to sense landmark?
- Is landmark natural or artificial?
  - If placed in env’t, how are locations chosen?
- Which sensor(s) can detect it?
  - Vision, sonar, radio, tactile, chemical, ...
- What are landmark’s geometric properties?
  - Plane, line, segment, point, diffuse source, ...
- What is discriminability of landmark?
  - (Will discuss this in detail in a minute)

Landmark Types

<table>
<thead>
<tr>
<th>Passive</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural</td>
<td></td>
</tr>
<tr>
<td>Wall corner</td>
<td>Sun, North star</td>
</tr>
<tr>
<td>Texture patch</td>
<td>Magnetic dipole</td>
</tr>
<tr>
<td>River bend</td>
<td>Pressure gradient</td>
</tr>
<tr>
<td>Earth’s surface</td>
<td>Mineral vent</td>
</tr>
<tr>
<td>Artificial</td>
<td></td>
</tr>
<tr>
<td>Surveyor’s mark</td>
<td>Chemical marker</td>
</tr>
<tr>
<td>Retro-reflector</td>
<td>Radio beacon</td>
</tr>
<tr>
<td>Lighthouse (day)</td>
<td>Lighthouse (night)</td>
</tr>
<tr>
<td>Trail blaze</td>
<td>LORAN</td>
</tr>
<tr>
<td>Buoy, channel marker</td>
<td>GPS</td>
</tr>
</tbody>
</table>
Robot Landmark/Sensor Types

- Range to surface patch, corner
  - Sonar return
- Bearing (absolute, relative, differential)
  - Compass; vision (calibrated camera)
- Range to point
  - RSS, TOF from RF/acoustic beacon
  - Cricket (TDoA acoustic, RF)
- Range and (body-relative) bearing to object
  - Radar return
  - Laser range scanner return
  - Vision (stereo camera rig)
- Distance to sea surface, floor
  - Pressure (depth), bathymetry (depth, altitude)

Discriminability Challenges

- Landmark Detection
  - Is landmark distinguishable from background?
- Landmark Measurement, Data Fusion
  - Sensor gives a noisy, quantized measurement of landmark geometry (bearing and/or range)
  - How accurately can one measurement localize landmark?
  - How can multiple corrupted measurements be combined into one accurate landmark estimate?
- Landmark Identification
  - To which element of representation (i.e., map) does the detected and measured landmark correspond?
  - To which previously-observed landmark (if any) does currently observed landmark correspond?
  - Also known as the “data association” or “feature correspondence” or “matching” problem

Localization Degrees of Freedom

- Model robot/vehicle as a single rigid body
- Aerial, orbital, underwater navigation
  - 6 DOFs: three position + three orientation
- Terrestrial operation (rolling, walking)
  - 3 DOFs: two position + one orientation
  - Used for planar, mildly non-planar terrain
- Underwater surveying (high C. O. B.)
  - 4 DOFs: three position + one orientation

Localization Examples

- Two dimensions
  - Ideal sensors
  - From measured ranges (distances)
  - From measured bearings (directions)
- One dimension
  - Real sensor (noisy measurements)
  - From range and odometry
  - Filtering, outlier rejection
- Two dimensions
  - Mobility with RF/acoustic beacons

WHOI AUV, Hanu Singh (Aug. 2004)
Triangulation
- Natural geometry for 2D localization
  - Simplest framework combining range, bearing
  - Used by Egyptians, Romans for engineering
  - Features for which only some geometric attributes can be sensed directly are called "partially observable" features

Triangulation from ranges
- Robot at unknown position $P$ measures distances $d_1, d_2$ to known landmarks $L_1, L_2$
- What are possible solutions for $P$?
- Robot must lie on circles of radius $d_1, d_2$ centered at $L_1, L_2$ respectively
- Two solutions in general, $P$ and $P'$
- How to select the correct solution?
Disambiguating solutions
• *A priori* information (richer map)

Disambiguating solutions
• Continuity (i.e., spatiotemporal information)

Disambiguating solutions
• Additional landmarks (redundancy)

Triangulation from ranges
• Are we done yet, i.e., is pose fully determined?
• No: absolute heading is *not determined*

• How to get heading?
  - Motion (difference of positions inferred across time)
  - Extent (using two ranges measured over ship baseline)
Triangulation from bearings

- *Body-relative* bearings to two landmarks
  - Bearings measured relative to “straight ahead”

Robot observes:
- $L_1$ at bearing $\theta_1$
- $L_2$ at bearing $\theta_2$

... are two bearings enough for unique localization?

"differential bearing" $\alpha = \theta_2 - \theta_1$

Triangulation from two bearings

- Robot somewhere on circular arc shown
  - Can it be anywhere on circle? (No; ordering constraint)

Measurement Uncertainty

- Ranges, bearings are typically *imprecise*
- Range case (estimated ranges $\sim d_1$, $\sim d_2$)

Locus of likely positions

Triangulation from bearings

- Measure bearing to third landmark
  - Yields robot position and orientation
  - Also called robot *pose* (in this case, 3 DoFs)
Measurement Uncertainty

- Two-bearing case (estimated bearings \( \sim \theta_1, \sim \theta_2 \))
- What is locus of recovered vehicle poses?
- Solve in closed form? Is there an alternative?

Landmark, sensor geometry

- Consider off-axis and near-axis bearing measurements to two known landmarks (simplification: assume absolute heading is known)

Dilution of Precision

- General phenomenon that sensor, landmark, and motion geometry can degrade solution quality, even for a fixed set of observed landmarks

  - Geometric DOP = GDOP
    - Also Vertical DOP, Horizontal DOP etc.

  - How to take GDOP into account?
    - If sufficiently many landmarks are available, select those with minimal GDOP
    - Decouple pose, solve separately, recombine
Sensing uncertainty
- Time series of round-trip-time to one acoustic beacon for an underwater autonomous vehicle

Gaussian noise model
- Measurement returns a corrupted value

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( \frac{(x-\mu)^2}{2\sigma^2} \right) \]

- Gaussian or "normal" distribution with std. dev. \( \sigma \), variance \( \sigma^2 \)

Outliers
- Many measurements are outliers; their frequency is not well-modeled by a Gaussian distribution

Filtering
- Consider one-dimensional localization:
  - Robot measures range \( r(i) \) at \( i \)th time step
  - Ranges corrupted by Gaussian noise, outliers

- Filter measurements; combine over time
  - Deal with each measurement as it arrives
  - Recursive or on-line filtering (contrast batch)
Filtering with no outliers

- Suppose robot does not move
  - What should our filtering strategy be?
  - Call $x(t)$ our estimate of $x$ after $t$ time steps

\[
\text{Call } x(t) \text{ our estimate of } x \text{ after } t \text{ time steps}
\]

- Take mean (arithmetic average)
  - $x(i) = (r(1) + r(2) + \ldots + r(i)) / i$ (batch)
  - $x(i) = [x(i-1) \times (i-1) / i] + [r(i) / i]$ (recursive / on-line)
  - ... if no outliers, no change over time, filter is optimal
- Computational complexity of each update?

Dealing with outliers

- Suppose a fraction of $r(i)$ are wildly wrong
  - Classify $r(i)$ as inliers or outliers
  - How to do this?

-\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{outliers_diagram.png}
  \caption{Noisy range measurement $r(i)$}
  \end{figure}\n
Modeling measurement noise

- Estimate sample variance as well as mean

\[
\text{Estimate sample variance as well as mean}
\]

- Reject unlikely samples (e.g., $p < 1\%$)
  - Filter only inliers, by averaging as before
- ... But where does variance come from?
  - Determine it \textit{a priori} (e.g. from bench tests)
  - Or, estimate it \textit{on-line}, in addition to mean
    - Chicken-and-egg problem (could be unlucky)
    - If "outliers" become frequent, what can you do?

Estimating variance

- Define $\sigma^2(i)$ as variance after $i$ steps
- Batch computation:
  - As before, $x(i)$ is the mean after $i$ steps
  - Then variance $\sigma^2(i)$ is $[\Sigma(r(i)-x(i))^2] / i$
- Recursive (on-line) computation:
  - Estimate $x(i)$ recursively as before
  - Define $\sigma^2(1) = 0$; then for $i > 1$:

\[
\sigma^2(i) = \frac{(i-1)}{i} \times \sigma^2(i-1) + \frac{1}{(i-1)} \times (r(i) - x(i))^2
\]
Robustness, Validation

- Additional measurements can be used
- Increase robustness to noise:
  - Average measurements as shown earlier
  - Require more than minimum # of landmarks
  - Drawbacks? Takes more time, or restricts space in which method works. These are fundamental tradeoffs in localization
- Enable validation w.r.t. gross error:
  - Decompose into subsets, solve independently; compare solutions
  - Predict additional landmarks from observed

Localization challenges

- Partial observability
- Measurement noise
  - Amplified by GDOP
- Outlier measurements
- ... Are those all we have to worry about?

  Light at bearing $b_1$
  Light at bearing $b_2$
  Light at bearing $b_3$

Data association problem

- General problem: determining how an observation corresponds to a map feature, or to a previously observed feature (also called correspondence problem)
- How to tackle?
  - Initialization and continuity
  - Identify distinguishing features among landmarks
  - Combinatorial testing / cross-validation
    - RANSAC, Random Sampling and Consensus, 1981

Localization: Summary

- Localization: from a map and its sensors, robot must determine its pose with respect to map
- Challenging problem in general, due to:
  - Partial observability
  - Data association
  - Noise & GDOP
  - Outliers
- Strategies for robust localization
  - Geometric decoupling
  - Landmark selection
  - Initialization, continuity, combinatorial search
  - Filtering
  - On-line variance estimation, outlier rejection