

# 6.141/16.405J: Robotics systems and science Lecture 2: Motor Control

Lecture Notes Prepared by Daniela  
Rus  
EECS/MIT  
Spring 2009

<http://courses.csail.mit.edu/6.141/>  
Challenge: Build a Shelter on Mars

## This week we will see

---

- Why is robot control hard
- Basic Control: Bang-Bang, P, D, I, PID
- DC Motors
- Stepper motors
- PWM control
- Encoders

## Today:

---

- The role of *control*
- Open-loop (feed-forward) control
- Closed-loop (feedback) control
- Basic controllers:
  - P
  - PD
  - PID

## The Role of Control

---

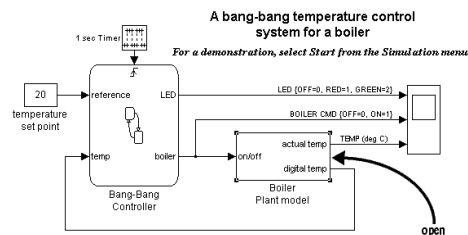
- Many tasks in robotics are defined by *achievement* goals
  - Go to the end of the maze
  - Push that box over here
- Other tasks in robotics are defined by *maintenance* goals:
  - Drive at 0.5m/s
  - Balance on one leg

## The Role of Control

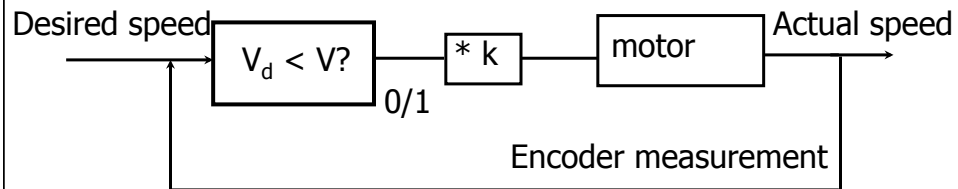
- *Control theory* is generally used for low-level maintenance goals
- General notions:
  - output = Controller(input)
  - output is control signal to actuator (e.g., motor voltage/current)
  - input is either goal state or goal state error (e.g., desired motor velocity)
- Controller is stateless

## Bang-bang control

- Discrete on/off
- Example: furnace:  
goal temp = 70  
when temp < 70  
BANG! Heat;  
when temp > 70  
BANG! Stop the heat

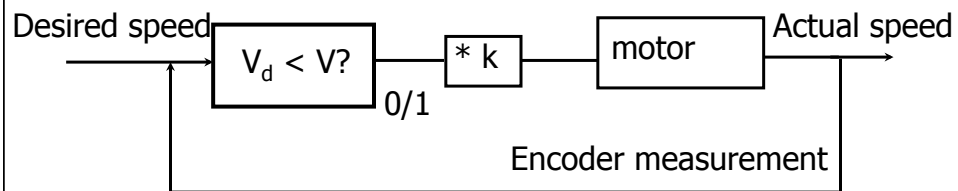


## Bang-bang control



$$O(t) =$$
$$O(t) =$$

## Bang-bang control

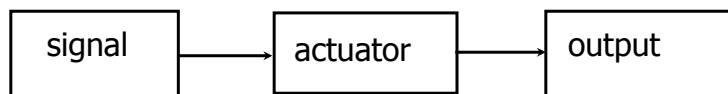


$$O(t) = k \text{ if } v(t) < V_d$$
$$O(t) = 0 \text{ otherwise}$$

## Motor Control: Open Loop

---

- Give robot task with no concern for the environment
- Applications: ???
- Open loop: signal to action
- Not checked if correct action was taken
- Example: go forward for 15 secs, then turn left for 10 secs. Issues?



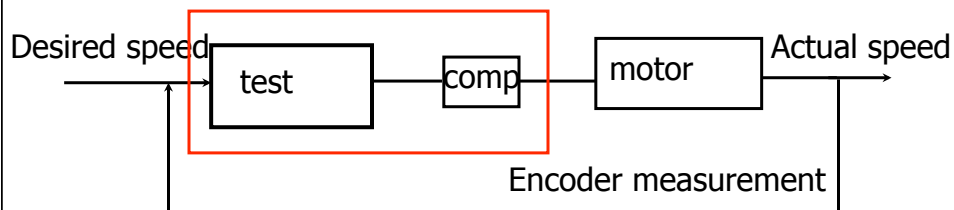
## Open loop (feed-forward) control

---

- Open loop controller:
  - $\text{output} = \text{FF}(\text{goal})$
- E.g.: motor speed controller (linear):
  - $V = k * s$
  - V is applied voltage on motor
  - s is speed
  - k is gain term (from calibration)
- Weakness:
  - Varying load on motor => motor may not maintain goal speed

## Motor Control: closed loop

- A way of getting a robot to achieve and maintain a goal state by constantly comparing current state with goal state.
- Use sensor for feedback



## Feedback Control

- Feedback controller:
  - $\text{output} = \text{FB}(\text{error})$
  - $\text{error} = \text{goal state} - \text{measured state}$
  - controller attempts to minimize error
- Feedback control requires sensors:
  - Binary (at goal/not at goal)
  - Direction (less than/greater than)
  - Magnitude (very bad, bad, good)

## Example: Wall Following

---

- How would you use feedback control to implement a wall-following behavior in a robot?
- What sensors would you use, and would they provide magnitude and direction of the error?
- What will this robot's behavior look like?

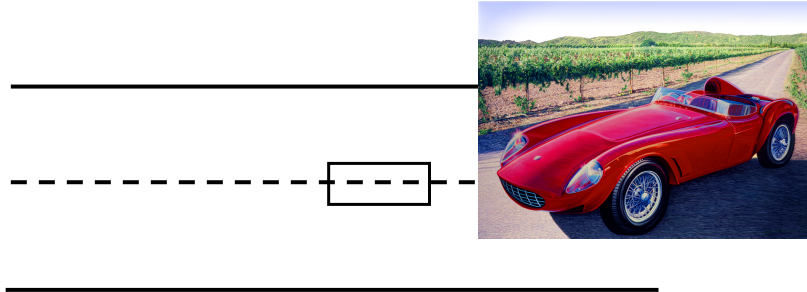
## Motor Control: PID

---

- *Control theory is the science that studies the behavior of control systems*
- $CurrentState - DesiredState = Error$
- Three main types of simple linear controllers:
  - P: proportional control
  - PD: proportional derivative control
  - PID: proportional integral derivative control
- All use direction and magnitude of error

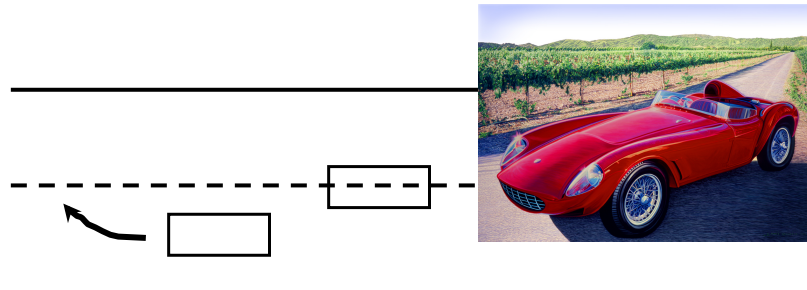
## Example: driving

- Steer a car in the center of a lane



## Example: driving

- Steer a car in the center of it lane

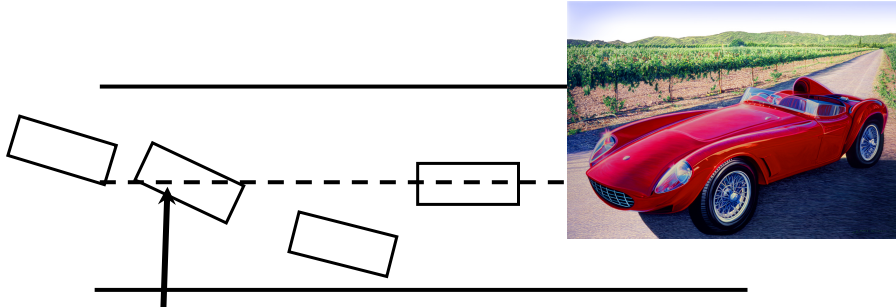


Observed error: speed of lateral movement



## Example: driving

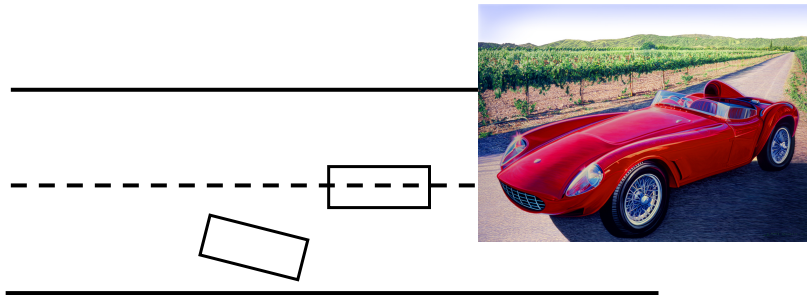
- Steer a car in the center of it lane



Error is zero but how is the car pointed?  
What will this do to the car?  
P controller is happy on line independent of orientation!

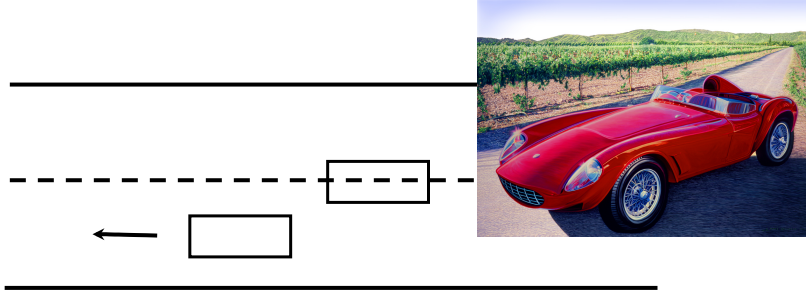
## What if respond $\sim$ rate of change ?

- Steer a car in the center of it lane



## Example: driving

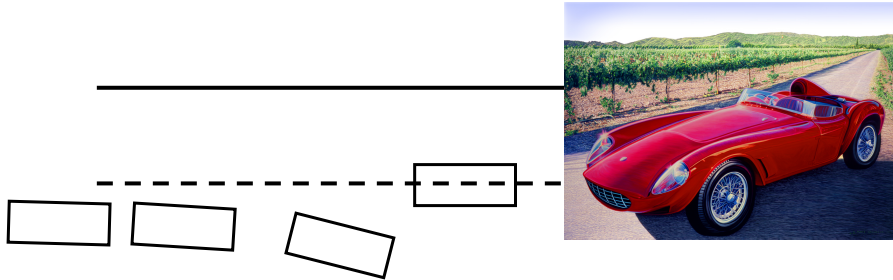
- Steer a car in the center of it lane



What is the observed rate of error?  
Other error?

## What if respond $\sim$ rate of change ?

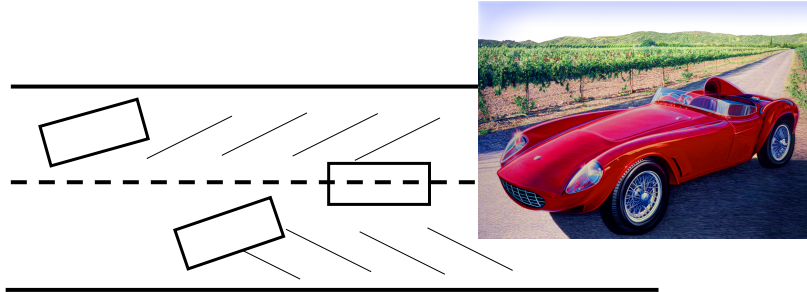
- Steer a car in the center of it lane



D controller is Happy on any parallel line!

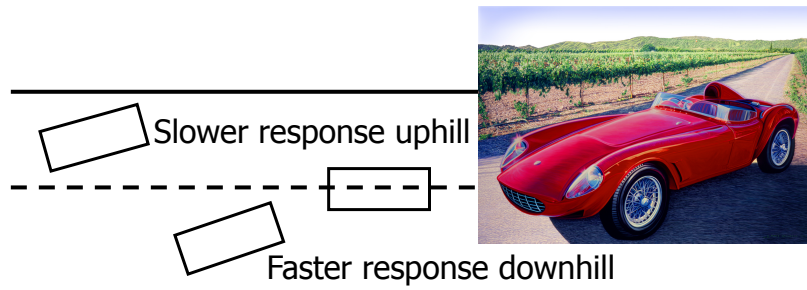
## What if Road Sloped ?

- Steer a car in the center of its lane



## What if Road Sloped ?

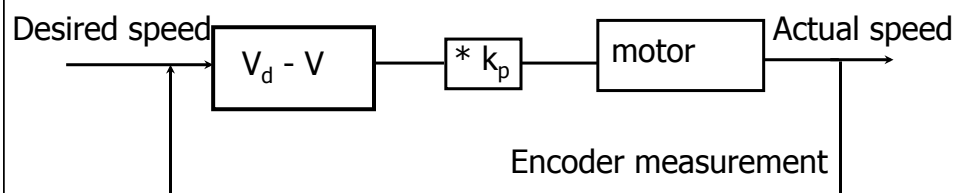
- Steer a car in the center of it lane



Gravity contributes a steady-state error

## Proportional Control

- Act in proportion to the error
- A proportional controller has an output  $o$  proportional to its input  $i$ :  
$$o = K_p * (V_d - v) = K_p * i = K_p * \text{err}(t)$$
- $K_p$  is a proportionality constant (gain)



## Gains

- How do we decide how much to turn, or how fast to go?
- Parameters ( $K_p$ ) are called gains
- Determining the right gains is difficult
- It can be done
  - analytically
  - empirically
  - trial and error

## Damping

---

- Damping is the process of systematically decreasing oscillations
- A system is *properly damped* if it does not oscillate with increasing magnitude, i.e., if its oscillations are either avoided, or decrease to the desired set point within a reasonable time period

## Integral Control

---

- Act in proportion to the *accumulated* error
- An integral controller has an output  $o$  proportional to the integral of its input  $i$ :
$$o = K_i * \int i(t) dt$$
- $K_i$  is a proportionality constant
- Integral control is useful for eliminating steady-state errors

## Derivative Control

---

- Act in proportion to the *rate of change* of the error
- A derivative controller has an output  $o$  proportional to the derivative of its input  $i$ :

$$o = K_d * di/dt = K_d * derr(t)/dt$$

- $K_d$  is a proportionality constant

## PD Control

---

- PD control combines P and D control:

$$o = K_p * i + K_d * di/dt$$

- P component minimizes error
- D component provides damping
- Gains  $K_p$  and  $K_d$  must be tuned together

## PID control

---

- PID control combines P and D control:
  - $o = K_p * i + K_i * \int i(t) dt + K_d * di/dt$
  - $o = K_p * err + K_i * \int err(t) dt + K_d * derr/dt$
- P component minimizes instantaneous error
- I component minimizes cumulative error
- D component provides damping
- Gains must be tuned together

## Ziegler-Nichols Tuning Method

---

Exploration: set the plant under P control and start increasing the gain until loop oscillates  
 Note critical gain  $K_C$  and oscillation period  $T_C$

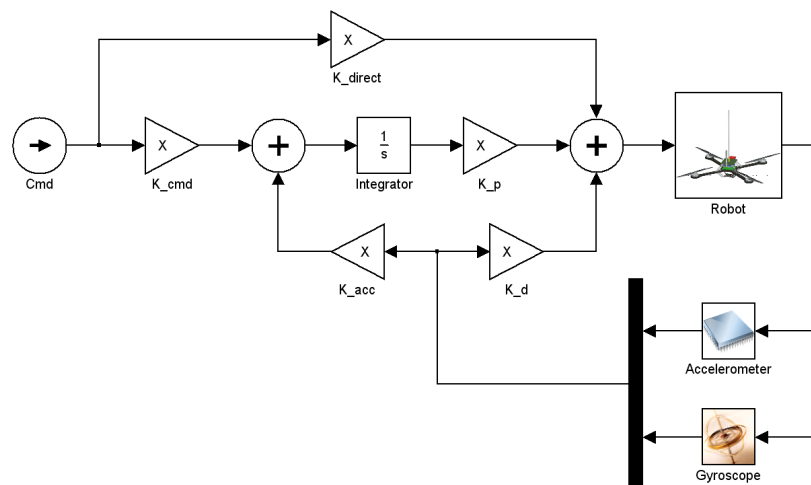
	$K_p$	$K_I$	$K_D$
P	$0.5K_C$		
PI	$0.45K_C$	$1.2K_p/T_C$	
PID	$0.5K_C$	$2K_p/T_C$	$K_p T_C/8$

Z & N developed rule using Monte Carlo method  
 Rule useful in the absence of models

## Implementation Issues

- How do we approximate  $K_i * \int \text{err}(t) dt$  to implement an I controller?
- How do we approximate  $K_d * d\text{err}/dt$  to implement a D controller?

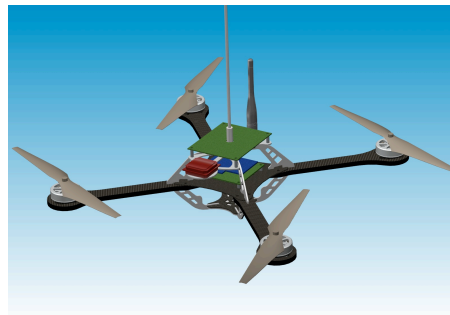
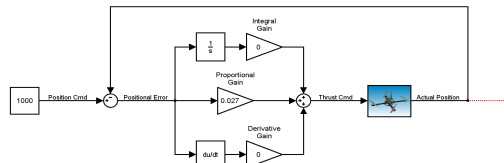
## Quad-Rotor Controller





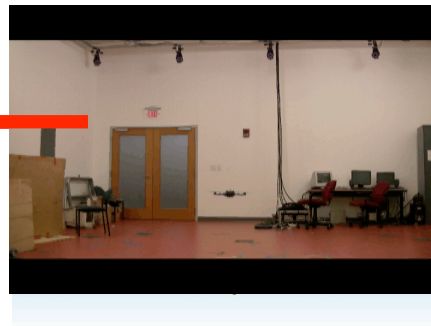
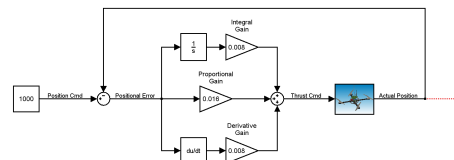
## Example UFO Control Using Ziegler-Nichols Method

- Integral and Derivative gains are set to zero
- Proportional gain is increased until system oscillates in response to a step input
- This is known as the critical gain  $K_C$  and the system oscillates with a period  $P_C$



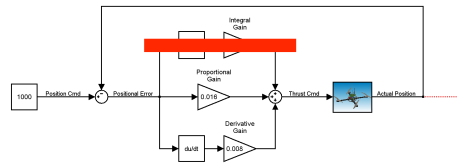
## Calculating PID parameters

- $K_p = 0.6 * K_C$
- $K_I = 2 * K_p / T_C$
- $K_D = 0.125 * K_p * T_C$
- $T_C = 4$
- The Ziegler-Nichols Method is a guideline for experimentally obtaining 25% overshoot from a step response.



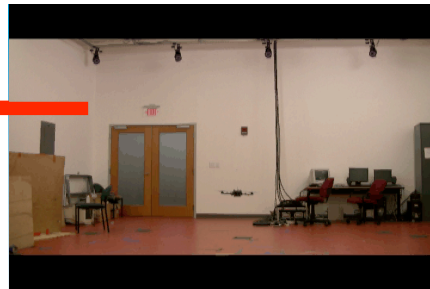
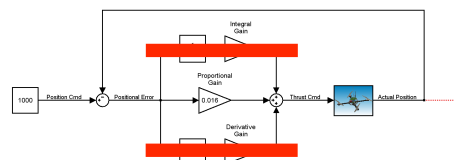
## What does $K_I$ do?

- For this system,  $K_I$  is crucial in eliminating steady state error primarily caused by gravity.
- The PD system overshoot and damping are still reasonable.



## What does $K_D$ do?

- Reasonable  $K_D$  helps minimize overshoot and settling time.
- Too much  $K_D$  leads to system instability.
- P system has unacceptable overshoot, settling time, and steady state error. However, it is stable.



## Control summary

---

	Control type	Feedback	Pro/Con
Bang-bang	discreet	environment	Simple/ Discreet
Open loop	Control law	no	Simple/may be unrepeatable
Closed loop	P, I, D	yes	Continuous/ Tune Gains