6.141:
Robotics systems and science
Lecture 12: Implementing Motion Planning

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Spring 2009
Based on Slides by Nick Roy
Reading: Chapter 3, and Craig: Robotics

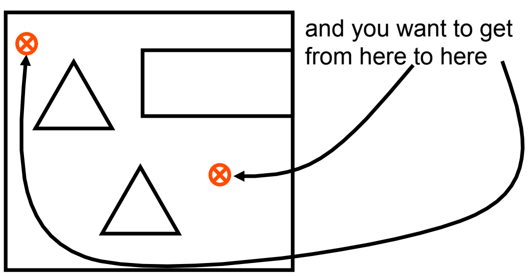
http://courses.csail.mit.edu/6.141/
Challenge: Build a Shelter on Mars

Today's Objectives

- Planning and search
  - Search methods
- Plans vs. Policies
  - Numerical potential fields

Let's Recap

Your mapping software gives you a good map....



Planning as Search

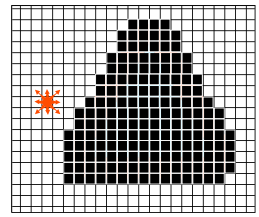
- Find a sequence/set of actions that can transform an ***initial state*** of the world to a ***goal state***
- Planning Involves ***Search*** Through a ***Search Space***
  - How to represent the search space?
  - How to conduct the search?
  - How to evaluate the solutions?

Motion Planning as Search

- Find a sequence of poses that connects the ***initial pose*** of the robot to the ***goal pose***
- State space is configuration space
  - To perform search, we discretize the space
  - This is a big issue in planning: how do we discretize the search space? Is it a graph? Is it a grid?
- Actions connect pairs of states
  - Assume a P-D controller
  - If the controller can get you from one pose to the other, then that action connects those states
  - In this course, we assume pairs of mutually visible states are connected

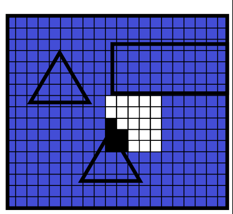
Setting up the State Space

- Real space
- Configuration space
- State space
- Actions get you from one state to another



Finding the free part of c-space using a grid?

- A grid square is in the c-space if it is:
  - not inside an obstacle
  - further than the radius of the robot from all obstacle edges
- Algorithm:
  - Pick a grid square you know is in free space
  - Do breadth-first search (or "flood-fill") from that start square
  - As each square is visited by the search, compute the distance to all obstacle edges
  - label as "free" if the distance is greater than the radius of the robot or "occupied" if the distance is less
  - Once breadth-first search is done, also label all unlabelled squares as "occupied"



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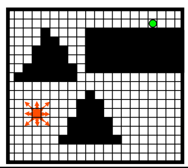
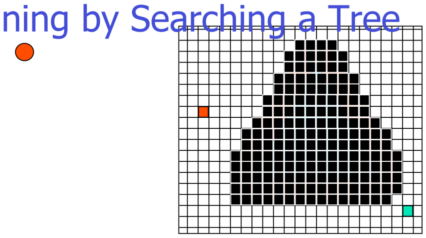
Planning as Tree Search

- Perform tree-based search (need c-space, cost)
  - Construct the root of the tree as the start state, and give it value 0
  - While there are unexpanded leaves in the tree
    - Find the leaf x with the lowest value
    - For each action, create a new child leaf of x
    - Set the value of each child as:
      g(x) = g(parent(x))+c(parent(x),x)
      where c(x, y) is the cost of moving from x to y (distance, in this case)



Planning by Searching a Tree



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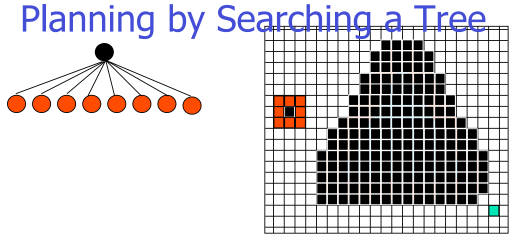

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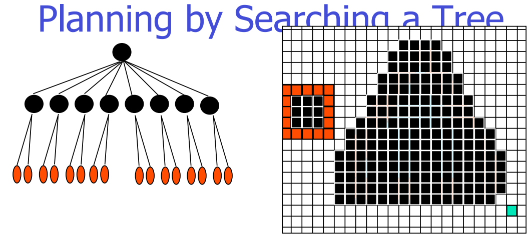
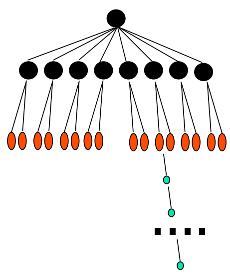
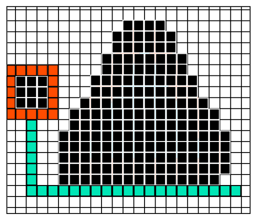


Planning by Searching a Tree





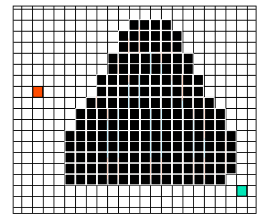
Simple Search Algorithm

Let Q be a list of partial paths,

Let S be the start node and

Let G be the Goal node.

1. Initialize Q with partial path (S) as only entry; set Visited = {}
2. If Q is empty, fail. Else, pick some partial path N from Q
3. If head(N) = G, return N (goal reached!)
4. Else
   a) Remove N from Q
   b) Find all children of head(N) not in Visited and create all the one-step extensions of N to each child.
   c) Add to Q all the extended paths;
   d) Add children of head(N) to Visited
   e) Go to step 2.



	Q	Visited
1	(3, 11)	
2		
3		
4		

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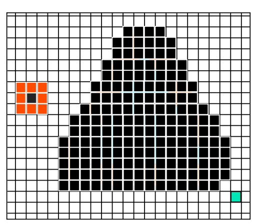

	Q	Visited
1	(3, 11)	
2	(2, 11) (2, 10), (3, 10), ….	(3, 11)
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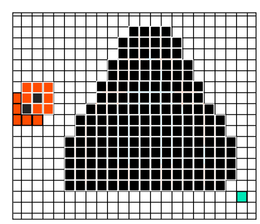


	Q	Visited
1	(3, 11)	
2	(2, 10) (3, 10), (4, 10), ….	(3, 11)
3	(1, 9), (2, 9), (3, 9), ….	(3, 11), (2, 10)
4		

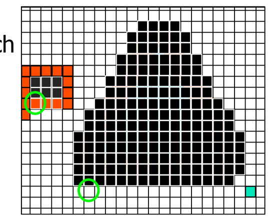
Simple Search Algorithm

```
public class Search {
  static Path search(State start, State goal) {
    Queue q = new Queue();
    HashSet<State> visited = new HashSet<State>();
    q.add(new Path(start));
    while (!q.empty()) {
      Path partialPath = q.pop();
      State head = partialPath.head();
      if (head.matches(goal))
        return partialPath;                       // Goal reached!
      for (int i = 0; i < head.numNeighbours(); i++) {
        if (visited.contains(head.neighbour(i))
          continue;
        // Create a new path to a node we haven't seen before
        // by adding the neighbour of the head to the current path
        Path extension = new Path(head.neighbour(i), partialPath));
        visited.add(head.neighbour(i));
        q.push(extension);
      }
    } // End of while (q.empty());
    return null; // No path found
  }
}
```

Careful: the HashSet object to get O(1) tests on whether we have seen this state before, but we may have to override the Object.hashCode method for our State class.

Move Generation

- How to determine the lowest-cost child to consider next?
- Shallowest next
  - aka: Breadth-first search
  - Guaranteed shortest
  - Storage intensive
- Deepest next
  - aka: Depth-first search
  - Can be storage cheap
  - No optimality guarantees
- Cheapest next
  - aka: Uniform-cost search
  - Breadth-first search is the same if the cost == depth



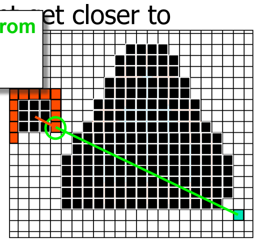
Informed Search – A*

- Use domain knowledge to bias the search
- Favour act... et closer to ...
- ... value

$f(x)=g(x)+h(x)$

Cost incurred from the start

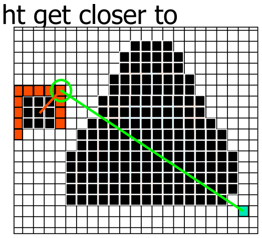
Estimated cost from here to the goal: "heuristic" cost



- For example
  - g(x) = 3
  - h(x) = ||x-g|| $=\sqrt{8^2+18^2}$ =19.7
  - f(x) =22.7

Informed Search – A*

- Use domain knowledge to bias the search
- Favour actions that might get closer to the goal
- Each state gets a value $f(x)=g(x)+h(x)$



- For example
  - g(x) = 4
  - h(x) = ||x-g|| $=\sqrt{11^2+18^2}$ =21.1
  - f(x) =25.1

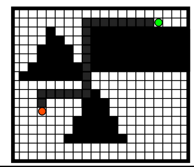
How to choose heuristics

- The closer h(x) is to the true cost to the goal, h*(x), the more efficient your search BUT
  - h(x) ≤ h*(x) to guarantee that A* finds the lowest-cost path
  - In this case, h is an “admissible” heuristic

Once the search is done, and we have found the goal

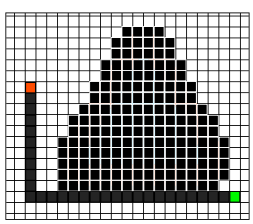
- We have a tree that contains a path from the start (root) to the goal (some leaf)
- Follow the parent pointers in the tree and trace back from the goal to the root, keeping track of which states you pass through
- This set of states constitutes your plan

- To execute the plan, use your PD controller to face the first state in the plan, and then drive to it
- Once at the state, face and drive to the next state



A problem with plans

- We have a plan that gets us from the start to the goal
- What happens if we take an action that causes us to leave the plan?



1) It's a problem with planners! We should use behaviours!
2) We can replan
3) We can keep a cached conditional plan
4) We can keep a policy

A Reactive Motion Planner

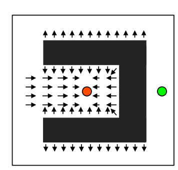
- The potential of each obstacle generates a repulsive force

$$U_{rep} = \frac{1}{\|x - x_c\|}$$

and the potential of the goal generates an attractive force

$$U_{att} = \frac{1}{2}\|x - x_{goal}\|^2$$

- Easy and fast to compute
- Susceptible to local minima



Numerical Potential Functions

- We can compute the "true" potential at each point x by integrating the forces along the desired path from the goal to x

$$V(x) = \min_{\pi} \int_{\pi} -\nabla U_{att}(\pi(t)) - \nabla U_{rep}(\pi(t)) dt$$

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$$V(x) = \min_{x \to x_{goal}} \sum_{x' \in x \to x_{goal}} \left(-\nabla U_{att}(x') - \nabla U_{rep}(x') \right) \delta x'$$

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Potential Field

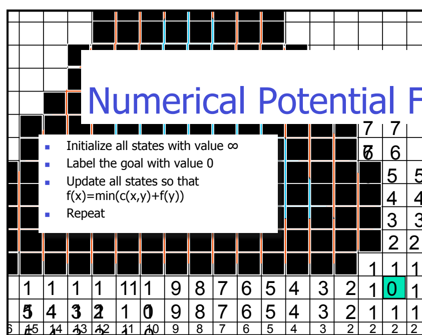
- Let's write this recursively:

$$V(x) = -\left(\nabla U_{att}(x) + \nabla U_{rep}(x)\right)\delta x + \min_{x' \in a(x)} V(x')$$

$$= C(x) + \min_{x' \in a(x)} V(x')$$

$C(x) = F(x) = \nabla U_{att}(x) - \nabla U_{rep}(x)$

Numerical Potential Field

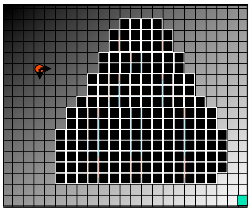


- Initialize all states with value ∞
- Label the goal with value 0
- Update all states so that f(x)=min(c(x,y)+f(y))
- Repeat

- The numbers shown are for an obstacle-induced cost of 0, and a goal-induced cost of 1 unit per grid cell

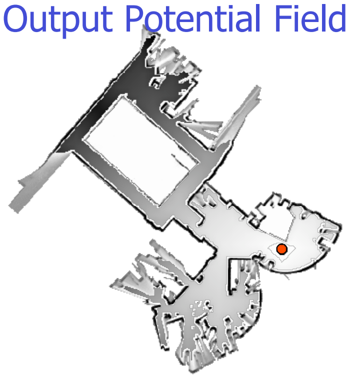
Uniform Cost Regression

- Initialize all states with value ∞
- Label the goal with value 0
- Update all states so that f(x)=min(c(x,y)+f(y))
- Repeat
- aka: Dijkstra's algorithm



- After planning, for each state, just look at the neighbours and move to the cheapest one, i.e., just roll down hill

The Output Potential Field



Progression vs. Regression

- **Progression (forward-chaining):**
  - Choose action whose preconditions are satisfied
  - Continue until goal state is reached
- **Regression (backward-chaining):**
  - Choose action that has an effect that matches an unachieved subgoal
  - Add unachieved preconditions to set of subgoals
  - Continue until set of unachieved subgoals is empty
- Progression: + Simple algorithm ("forward simulation")
  - Often large branching factor
- Regression: + Focused on achieving goals
  - Need to reason about actions
  - ***Regression is incomplete, in general, for functional effects***

Data Structures Prof. Roy has Known and Loved

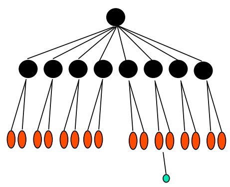
- Priority Heap
  - Look it up in Cormen, Leiserson, Rivest and Stein. (MIT gives you free access to the online edition.)
  - Add nodes in O(log n) time, remove the lowest (or highest) priority node in O(log n) time.
- If your heap consists of `State` data structures of the form:

```
public class State {
        int id;
        double coordinates[2];
        int neighbours[];
        double priority;
}
```

then you can implement any search algorithm by changing the priority scheme
  - Uniform-first search if
    - current priority = parent state's priority + action cost from parent to current state
    - choose lowest-priority state
  - Depth-first search if
    - current priority = parent state's priority + 1
    - choose highest-priority state
  - A* search if
    - current priority = parent state's priority + action cost from parent to current state+ heuristic cost from current state to goal
    - choose lowest-priority state

More on Data Structures

- While there are unexpanded leaves in the tree
  - Find the leaf x with the lowest value
  - For each action, create a new child leaf of x
  - Set the value of each child
- Let's say that each tree node is given by



```
public class State {
          int id;
          double coordinates[2];
          int neighbours[];
          double priority;
          State parent;
          State [] children;
}
```

- Then as you create new children, you store them in the children array inside the parent State
- The tree structure, however, does **not** automatically tell you the lowest (or highest) priority child
- Therefore, as you add each child to the parent state in the tree, also add the child to a sorted set (e.g., java.util.TreeSet) that has the methods add() and first() that will let you add items and retrieve the lowest (highest) items in O(log n) time. (NB: If using TreeSet, you would need to make sure your State class implements the comparable interface.)

Design Choices

- How is your map described? This may have an impact on the state space for your planner
  - Is it a grid map?
  - Is it a list of polygons?
  - The critical choice for motion planning is state space
  - The other choices tend to affect computational performance, not robot performance
- What kind of controller do you have?
  - Do you just have controllers on distance and orientation?
  - Do you have behaviours that will let you do things like follow walls?
- What do you care about?
  - The shortest path?
  - The fastest path?
- What kind of search to use?
  - Do you have a good heuristic?
  - If so, then maybe A* is a good idea.

Summary

- Planning as search
- The design decisions in setting up a planner
- Different forms of search
- A* and what an admissible heuristic is
- What a policy is, why it's different from a plan, and when you might want one
- When to use each