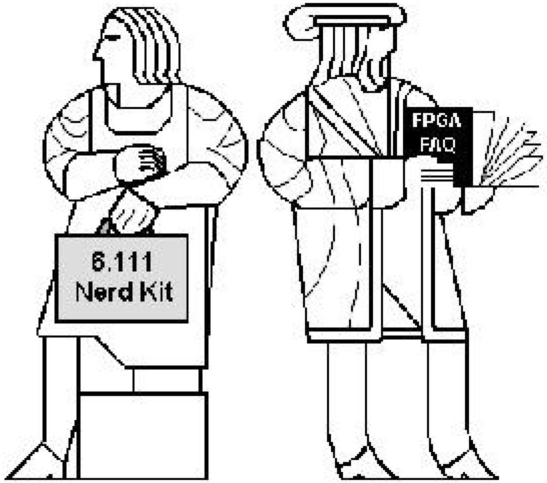
6.111 Lecture 16

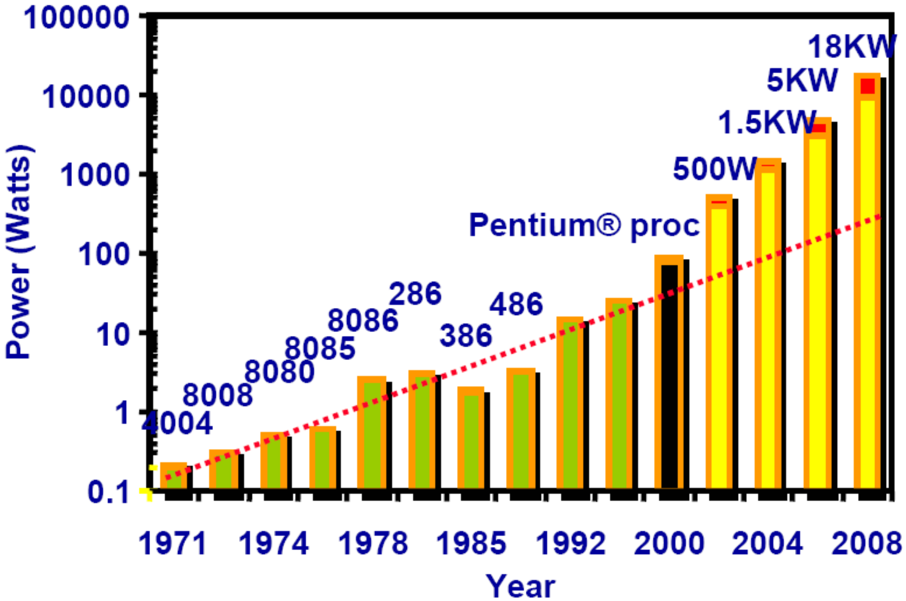
Today: Power Dissipation, Reversible Computing, and Quantum Computation

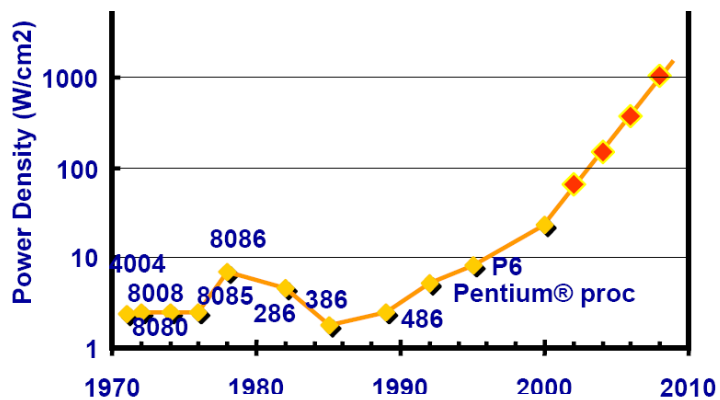
1. Power dissipation: CMOS
2. Physics of computation
3. Reversible computation
4. Fault tolerance
5. Quantum computation



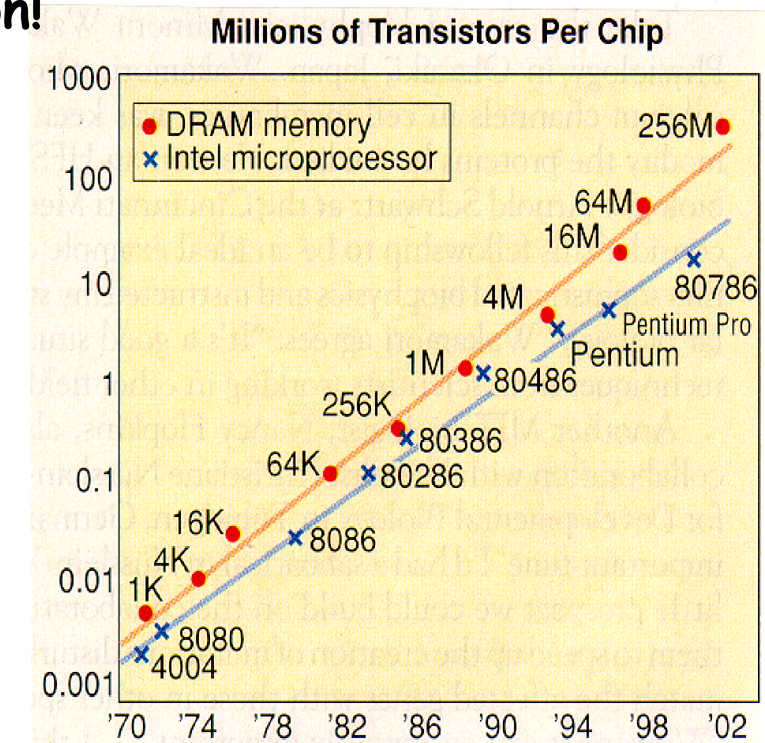
1. Power dissipation in CMOS

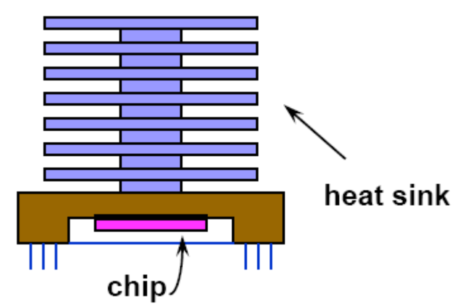
- Moore's law: # of transistors/chip doubles every ~18 months
- Exponential rise in power dissipation!





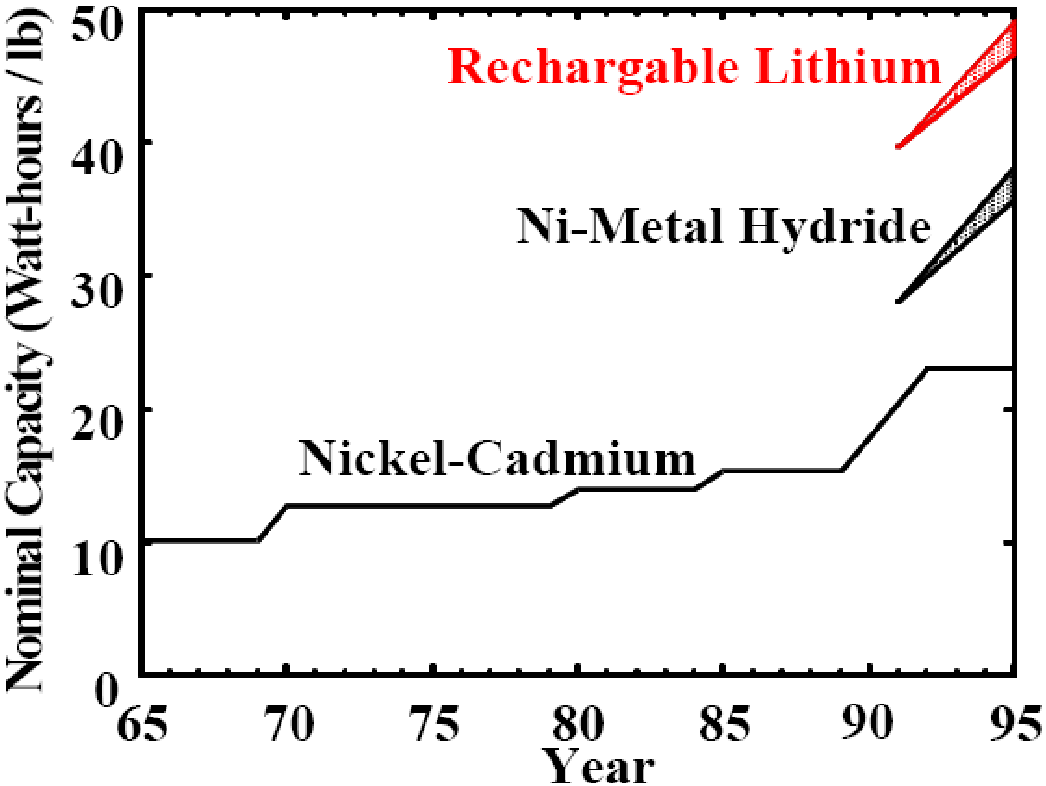
Courtesy Intel (S. Borkar)



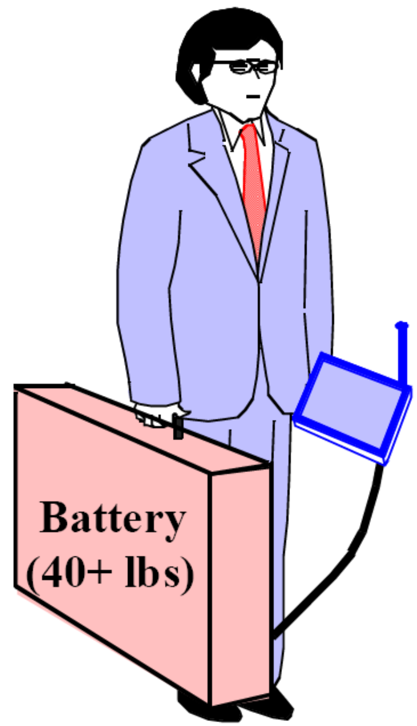


Problem: Energy Capacity

- No Moore's law for batteries!

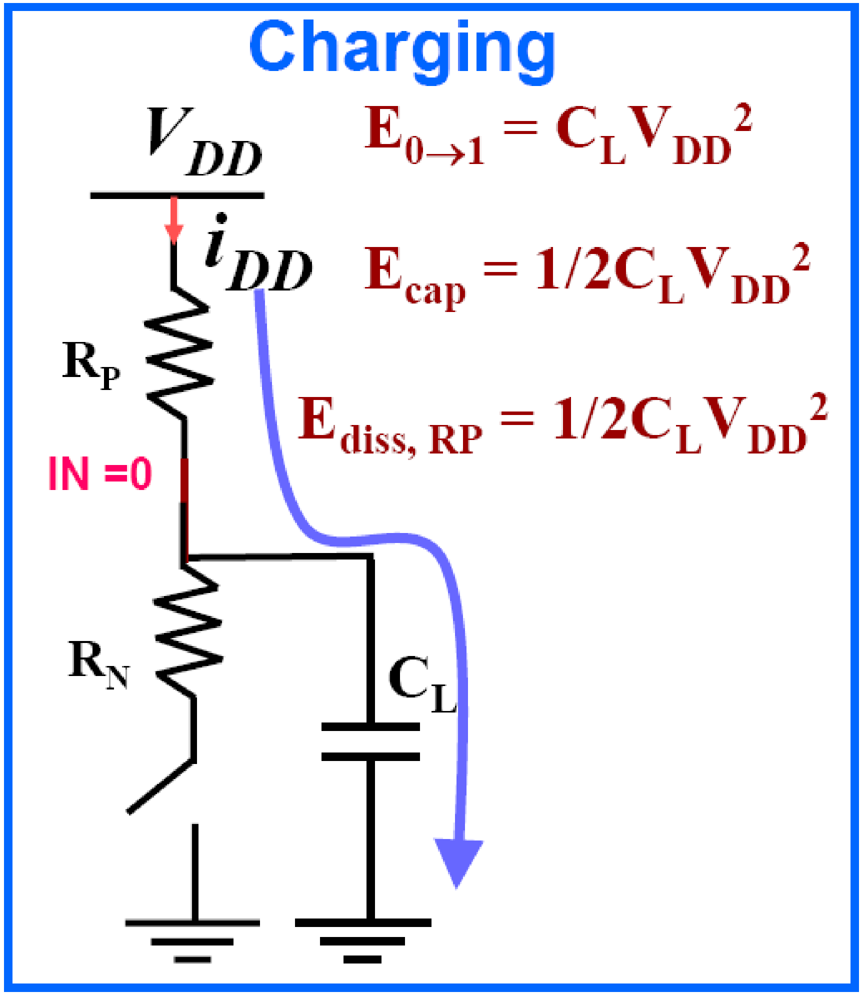


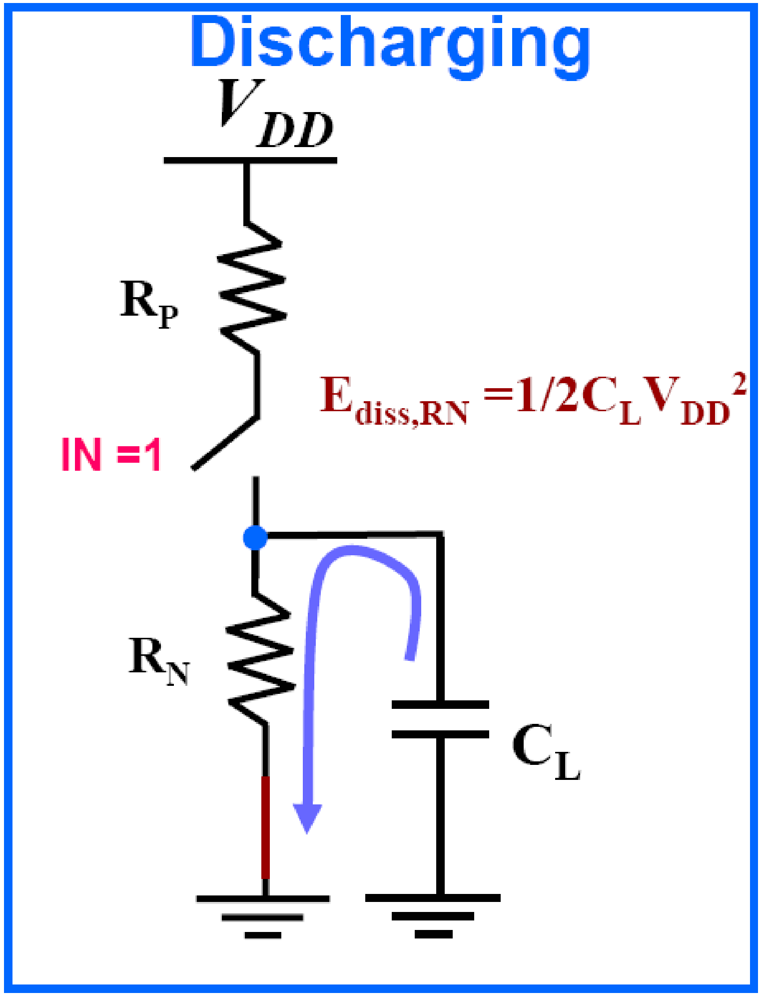
(from Jon Eager, Gates Inc. , S. Watanabe, Sony Inc.)



- Today: Understand where power goes, fundamental limits on power consumption in computation, and frontiers in new approaches to computation

Dynamic Energy Dissipation

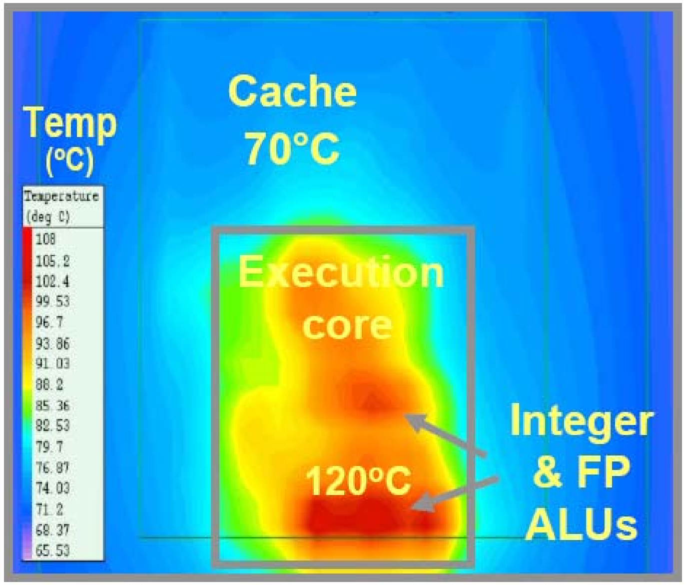




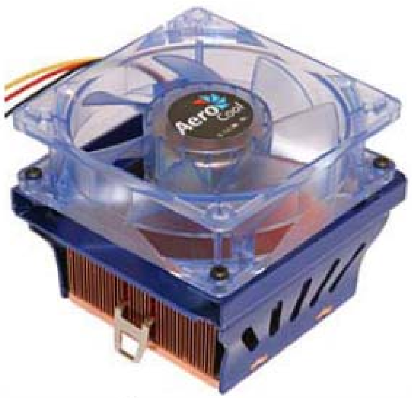
$$P = C_L V_{DD}^2 f_{clk}$$

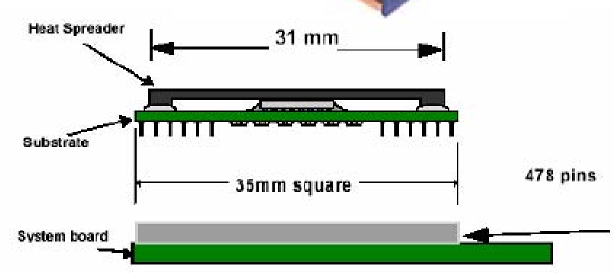
Intel Pentium 4 Thermal Guidelines

- Pentium 4 @ 3.06 GHz dissipates 81.8W!
- Maximum T_C = 69 °C
- R_{CA} < 0.23 °C/W for 50 C ambient
- Typical chips dissipate 0.5-1W (cheap packages without forced air cooling)



Courtesy of Intel (Ram Krishnamurthy)





Processor and Core Frequency	Thermal Design Power[1,2] (W)
Processors with VID=1.500V	
2 GHz	52.4
2.20 GHz	55.1
2.26 GHz	56.0
2.40 GHz	57.8
2.50 GHz	59.3
2.53 GHz	59.3
Processors with VID=1.525V	
2 GHz	54.3
2.20 GHz	57.1
2.26 GHz	58.0
2.40 GHz	59.8
2.50 GHz	61.0
2.53 GHz	61.5
2.60 GHz	62.6
2.66 GHz	66.1
2.80 GHz	68.4
Processors with multiple VIDs	
2 GHz	54.3
2.20 GHz	57.1
2.26 GHz	58.0
2.40 GHz	59.8
2.50 GHz	61.0
2.53 GHz	61.5
2.60 GHz	62.6
2.66 GHz	66.1
2.80 GHz	68.4
3.06 GHz	81.8

2. The Physics of Computation

- Q: What is the minimum energy dissipation necessary for computation?
- A: $k_B T \log 2$ per operation – J. von Neumann 1949

R. Landauer, "Irreversibility and heat generation in the computing process," IBM Journal of Research and Development, vol. 5, pp. 183-191, 1961.

J. von Neumann, *Theory of Self-Reproducing Automata*, Univ. of Illinois Press, 1966.

- AND gate:

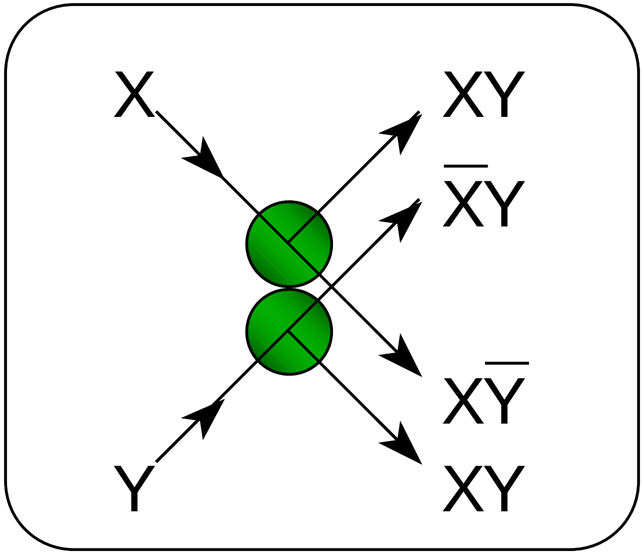
This is irreversible!

In	Out
00	0
01	0
10	0
11	1

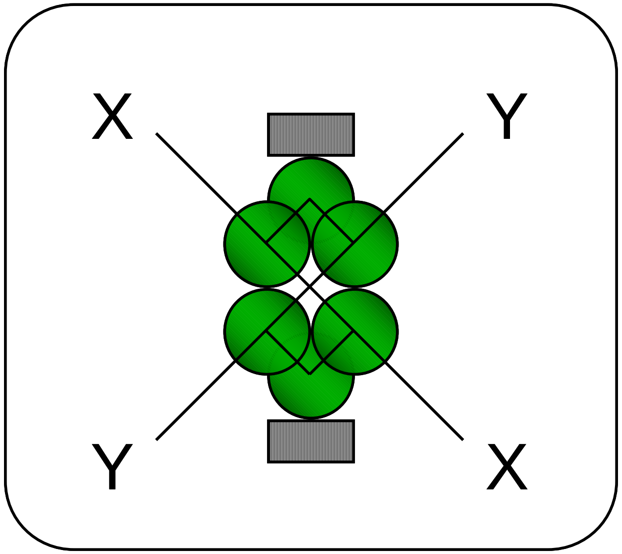
***Landauer's Principle*: Energy Dissipation comes from irreversibility**

Billiard ball model of computation

Billiard ball collisions may be used to build logic gates



Interaction Gate

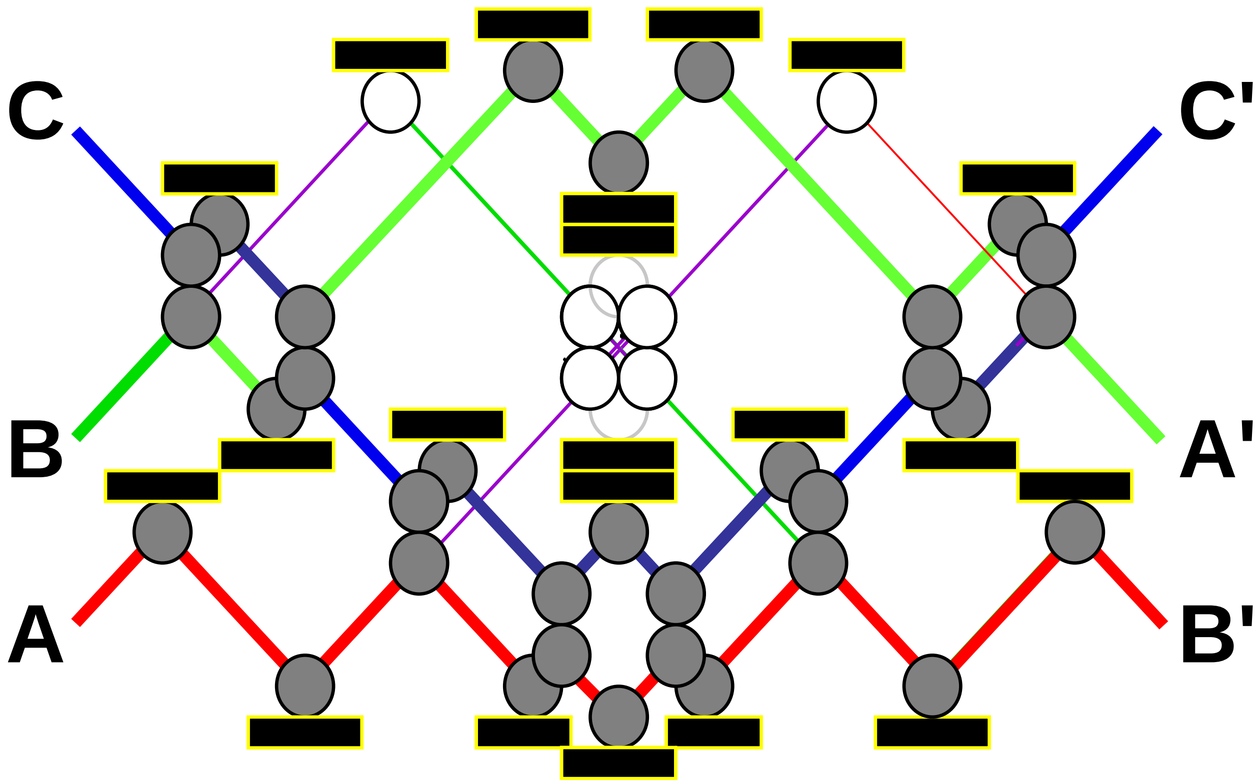


Crossover

- Billiard balls obey classical Newtonian equations of motion, and thus follow perfectly reversible trajectories.

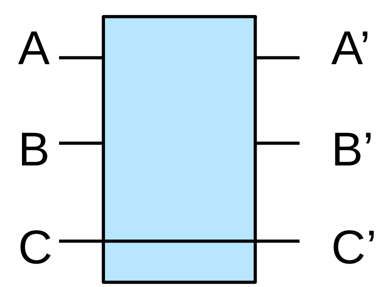
(Feynman, Optics News V11 p11, 1985)
(Bennett, IBM J. Research and Dev, V6, p525, 1973)

The Fredkin Gate



IN	OUT
A B C	*A' B' C'*
0 0 0	0 0 0
0 0 1	0 0 1
0 1 0	0 1 0
1 0 0	1 0 0
0 1 1	1 0 1
1 0 1	0 1 1
1 1 0	1 1 0
1 1 1	1 1 1

- Without C, A'=A and B'=B
- When C is present, A'=B and B'=A



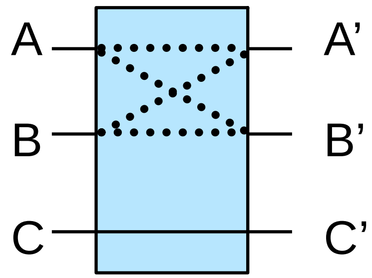
(Ressler, A. L., MIT EECS MS Thesis, Jan. 1981, "The Design of a Conservative Logic Computer and a Graphical Editor Simulator")

3. Reversible Computation

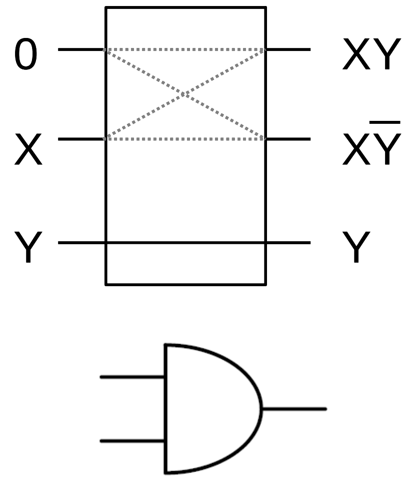
- Build an entire computer from Fredkin gates (vs AND/NOT)

IN	OUT
A B C	*A'B'C'*
0 0 0	0 0 0
0 0 1	0 0 1
0 1 0	0 1 0
1 0 0	1 0 0
0 1 1	1 0 1
1 0 1	0 1 1
1 1 0	1 1 0
1 1 1	1 1 1

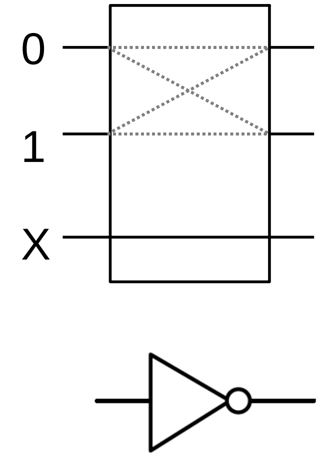
Schematic symbol



"AND" function



"NOT" function

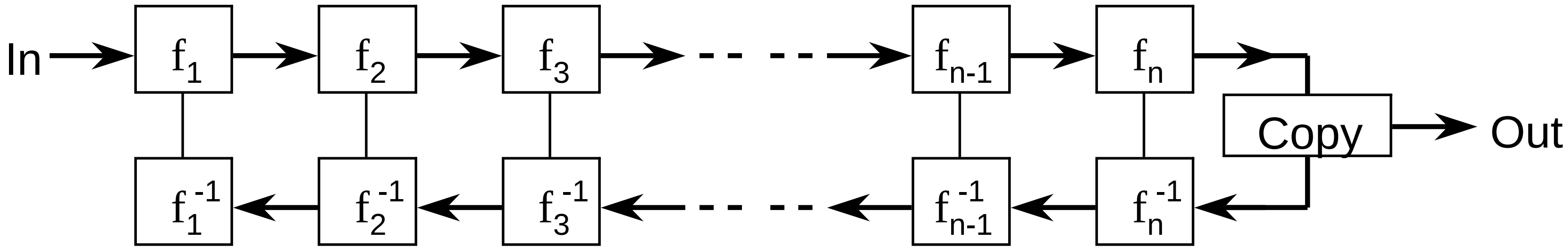


- The Fredkin gate is computationally universal
  - This reversible logic gate is a fundamental building block for a reversible computer
  - Its function can be understood as a simple "controlled exchange-bypass switch"

(Fredkin and Toffoli, Int. J. of Theor. Phys., V21 p219, 1982, "Conservative Logic")

Uncomputing

- All functions can be computed reversibly
  - Reconstruct and erase intermediate results (garbage) by using inverse functions:



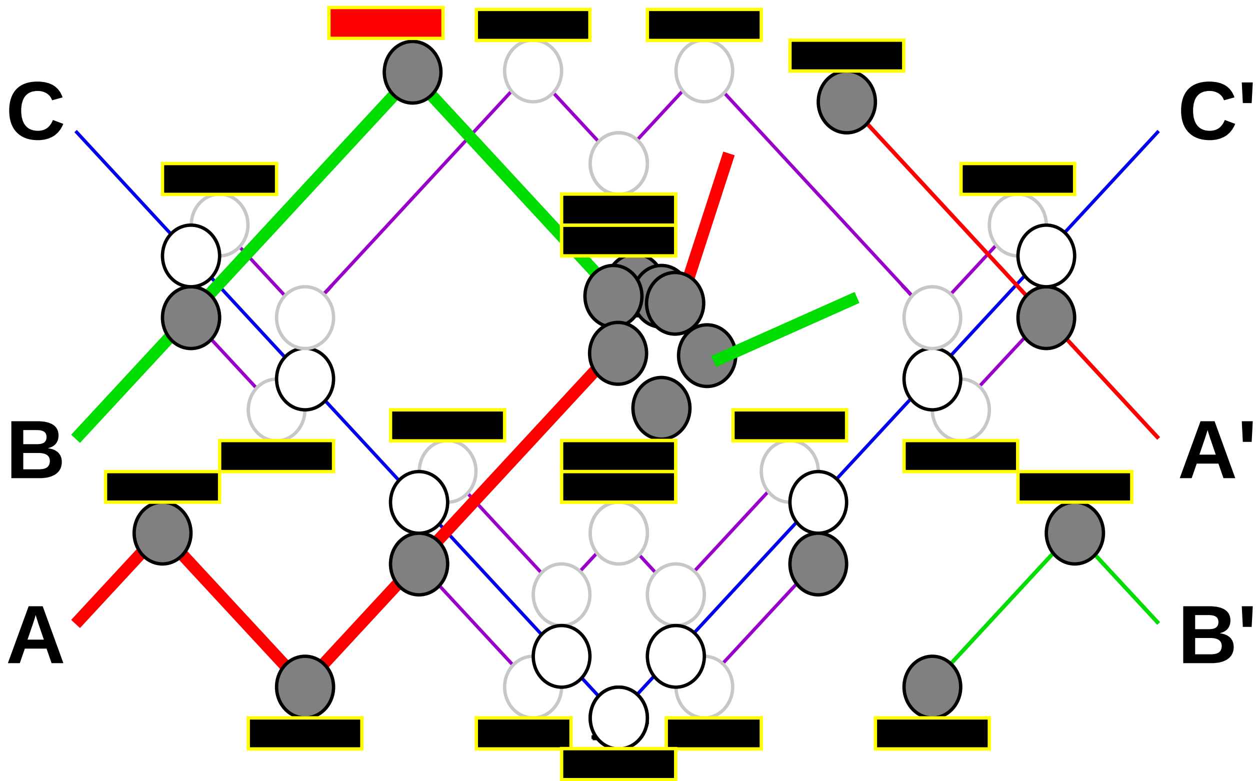
$$(x,y) \rightarrow (x, y \oplus f(x))$$

- All computation can be done, in principle, at zero cost in energy!

- Q: When is energy dissipation necessary?

Irreversibility => Energy Dissipation

- Consider an error, eg the ball going off trajectory or timing



- Correct by expending energy to monitor ball trajectories, and actively correcting errors: erasure of errors costs energy

Energy dissipation is needed only to provide stability & correct errors!

4. Fault Tolerant Computation

- Instead of constructing perfect digital systems, we can choose a different strategy to achieve reliability.

Reliable computers can be constructed from faulty components

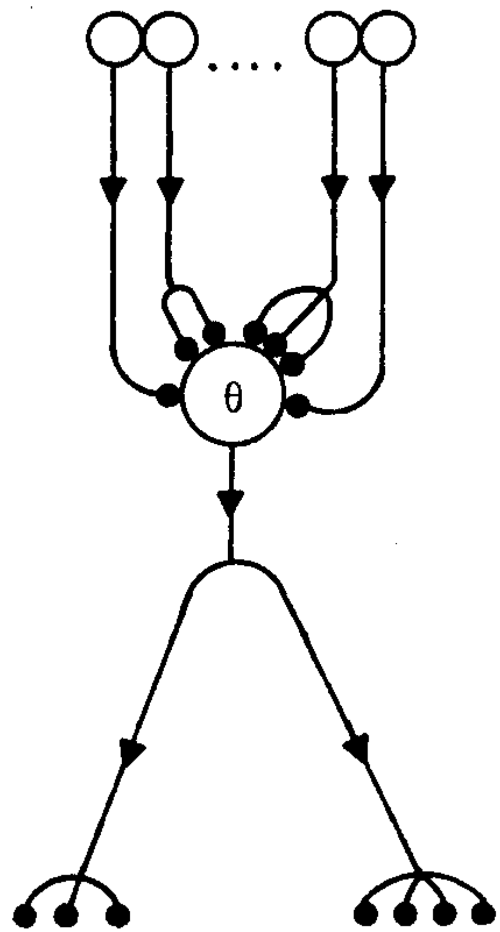
- A circuit containing N (error-free) gates can be simulated with probability of error at most ε, using N $\log(N/\varepsilon)$ faulty gates, which fail with probability p, so long as $p<p_{th}$.   von Neumann (1956)

Fault-Tolerant Circuits

Reliable Computation
in the Presence of Noise

S. Winograd and *J. D. Cowan*

The M.I.T. Press
Massachusetts Institute of Technology
Cambridge, Massachusetts



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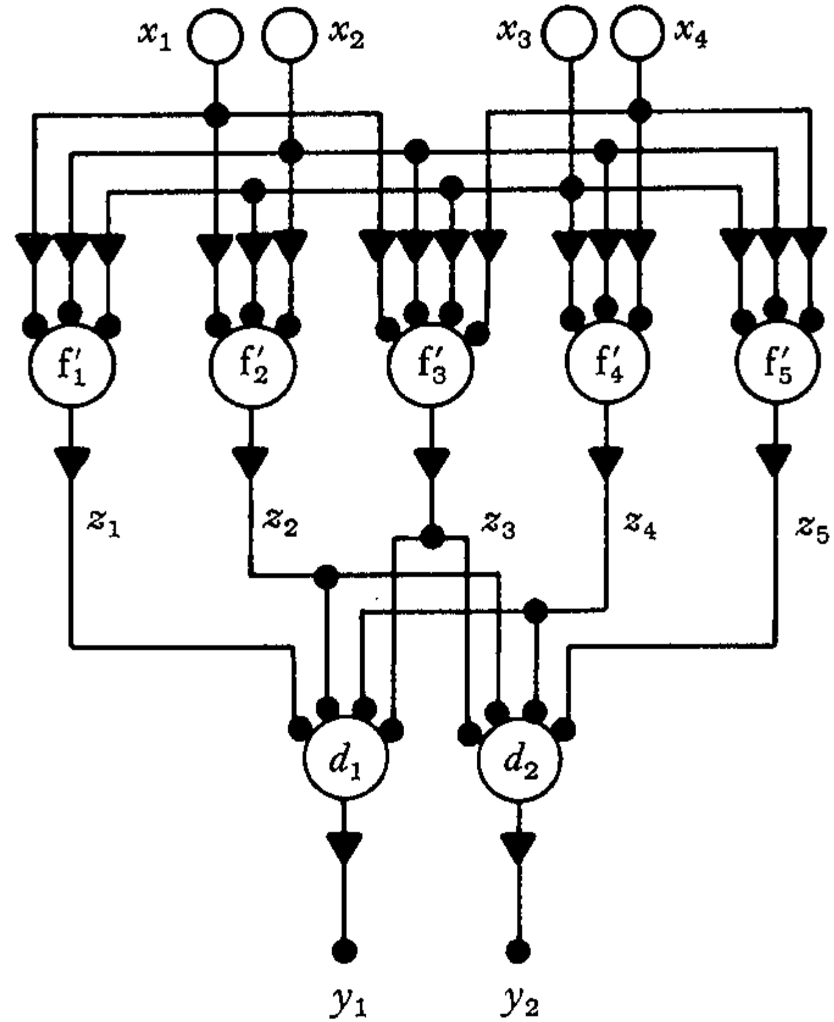
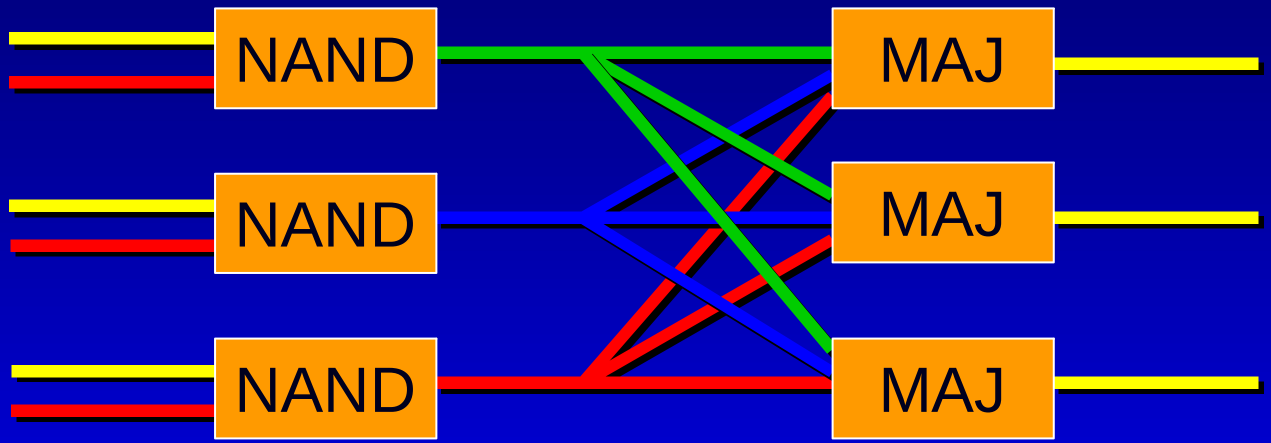


Ex: Fault Tolerant NAND

- Fails with probability p

Ex: Fault Tolerant NAND

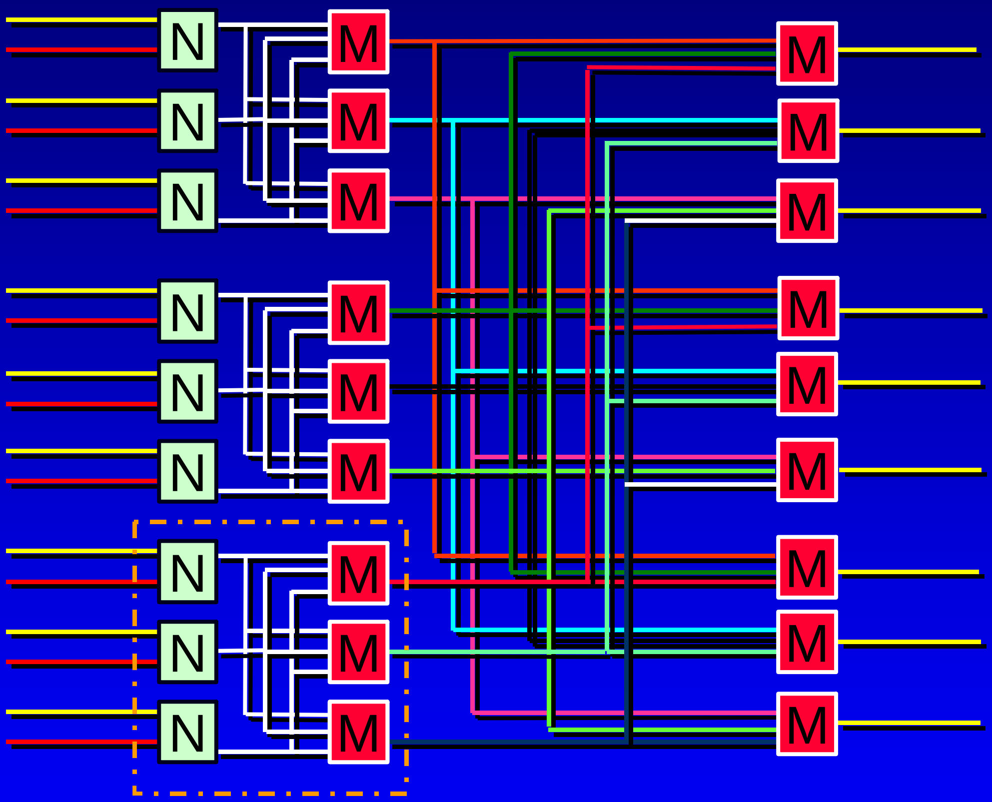


- Encode data: 0 $\rightarrow$ 000, 1 $\rightarrow$ 111
- Assume each gate fails with probability p
- Circuit fails only if 2 gates fail (6 possibilities)

$$p_{fail} \leq 6p^2 \quad \Rightarrow \quad p \leq \frac{1}{6}$$

Ex: Fault Tolerant NAND

- Recursive construction: $0 \rightarrow 000000000$, $1 \rightarrow 111111111$
- Circuit fails only if 2 modules fail



$$p_{fail} \leq 6(6p^2)^2$$

$$p_{fail} \leq \frac{(cp)^{2^k}}{c}$$

Fault Tolerance: Threshold

- Use k recursive levels of error correction:

Circuit failure

Gate failure

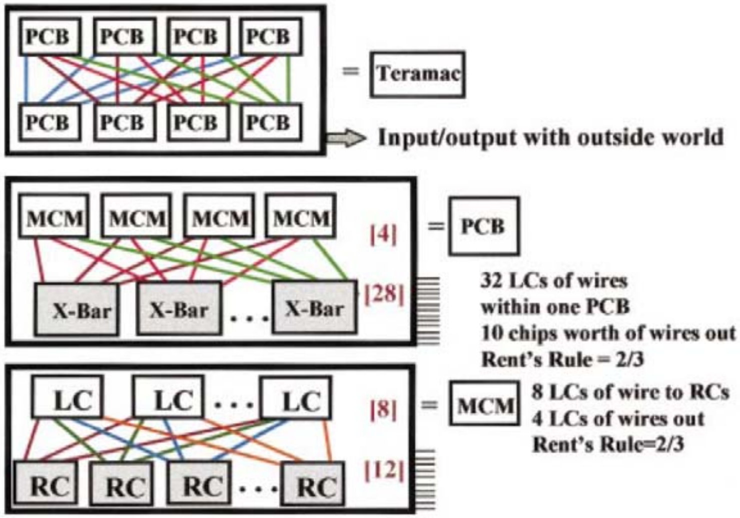
$$\frac{p_{fail}}{p_{th}} = \left(\frac{p_0}{p_{th}}\right)^{2^k}$$

Threshold

- Error reduction is exponential in resources!

Teramac: "Defect tolerant" FPGA Array

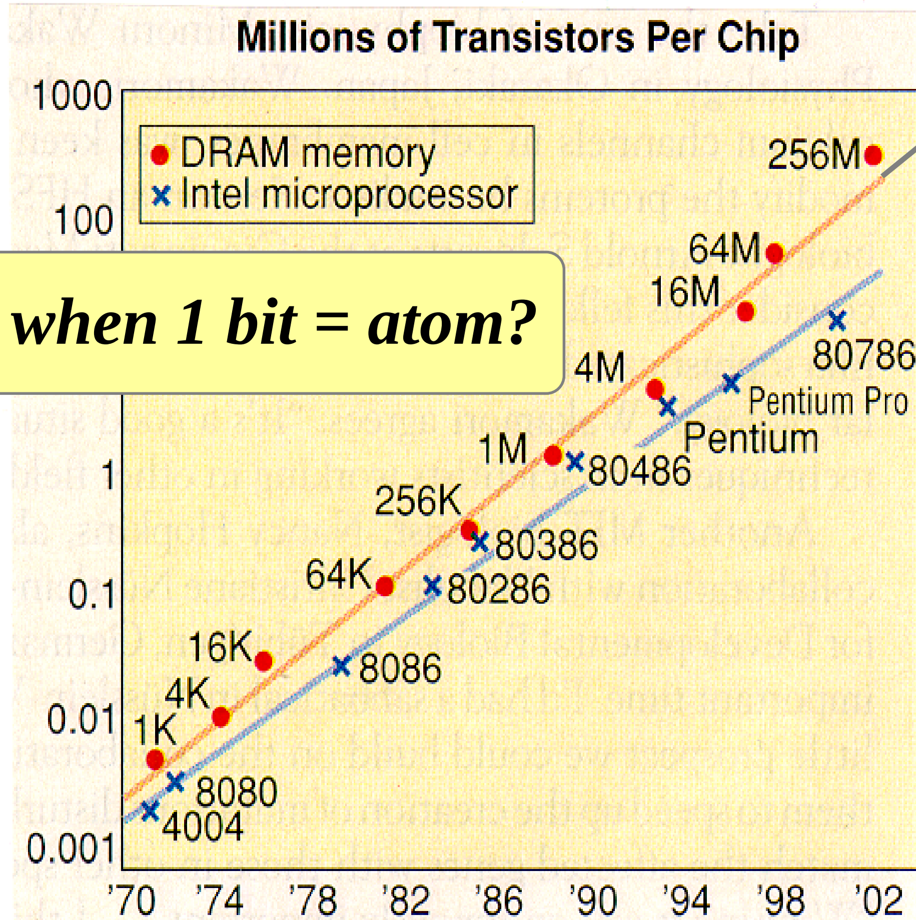
- Modern digital electronics vs fundamental limits:
  - Intel 4004: 500 additions/Joule
  - Modern microprocessors: 3×10^6 additions/Joule
  - Von Neumann-Landauer limit of $k_B T$ log 2: 10^{18} add./Joule
- Teramac: an experimental digital system using faulty devices:



- 864 FPGA's
- 217 defect-free, 647 uncertified
- 10% of FPGA logic cells defective
- 10% of interconnects unreliable
- System made reliable through redundancy
- Static, not dynamic faults

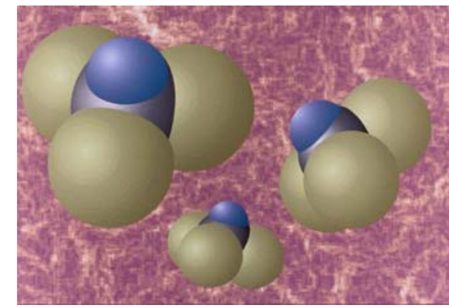
(Heath, Kuekes, Snider, Williams, Science v280, p1716, 1998)

The Quantum Limit



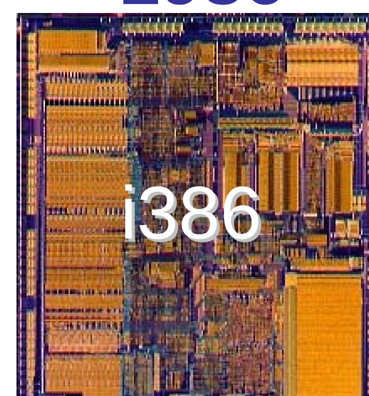
What happens when 1 bit = atom?

2020



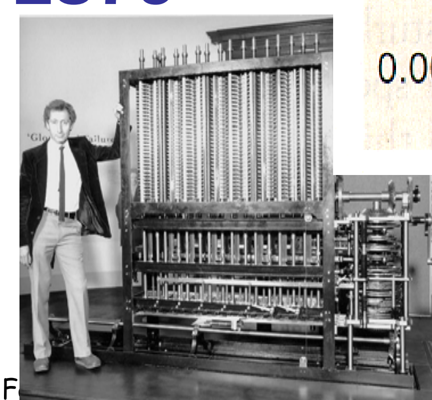
1 nanometer

1986



1 micrometer

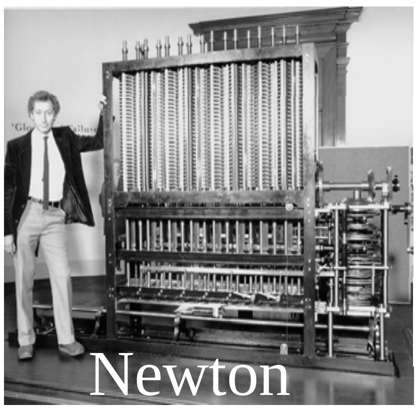
1879

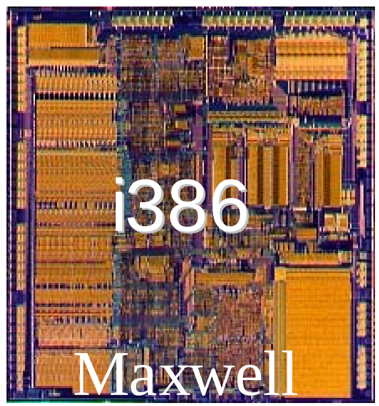


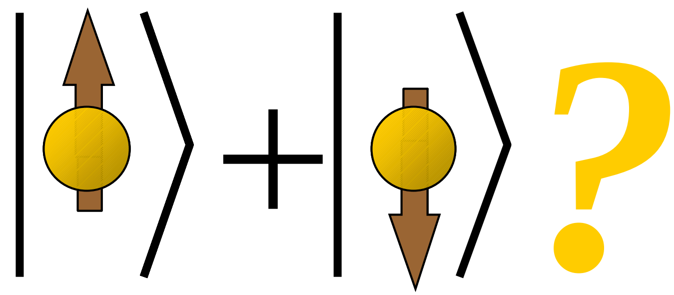
1 inch

Quantum Computation

- Fundamental Motivations
  - Quantum physics provides new physical resources
  - Computer science = new mathematical tools for physics





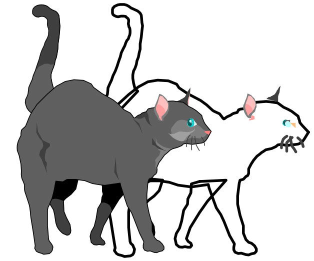


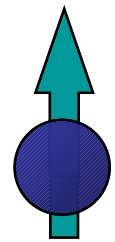
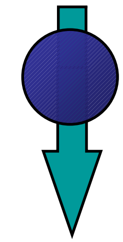
Schrödinger

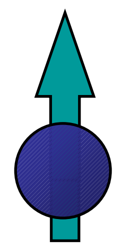
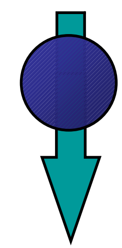
- Status
  - ✓ Factoring: Exponential speedup **Shor '94**
  - ✓ 7-qubit QC demonstrated **IBM / MIT 2001**
  - ✗ Need: new algorithms?

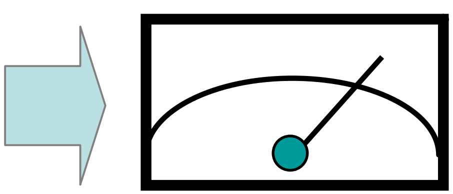
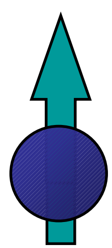
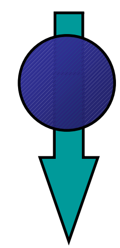
System	*qubits*
Trapped Ion	1-4
Superconding JJ	1-2
Quantum dots	0-1
NMR	7

Quantum Bits



- **Classical states:** **1 =**  **0 =** 

- **Arbitrary superposition:** C_1  **+** C_0 

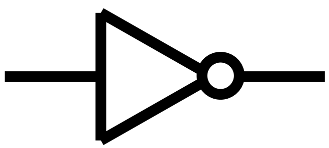
 =  w.p. $\left|C_1\right|^2$ **<u>OR</u>**  w.p. $\left|C_0\right|^2$

Classical vs. Quantum Circuits

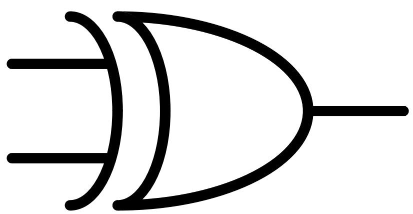
- **bit: 0 or 1**
- **Boolean logic:**

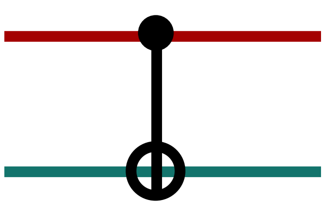
- **qubit:** $a|0\rangle + b|1\rangle$
- **Unitary transform:**

NOT



XOR





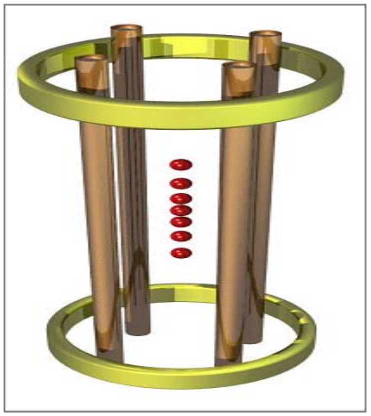
In	Out
00	00
01	01
10	11
11	10

Hadamard **N/A**

In	Out
0	0+1
1	0 - 1

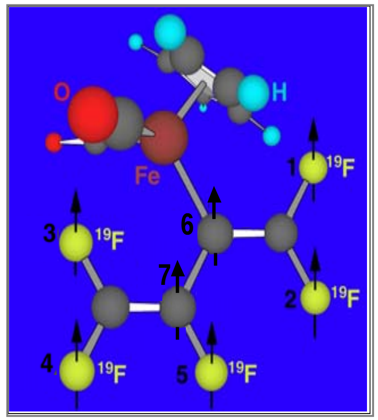
Quantum Computers Today

- Atoms



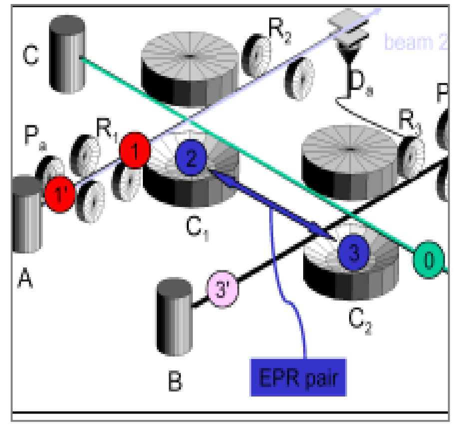
(Blatt / Wineland)

- NMR



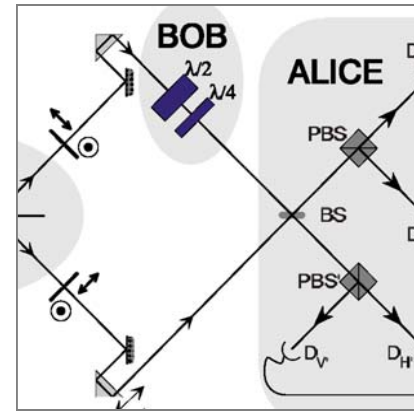
(Vandersypen et al)

- Cavity QED



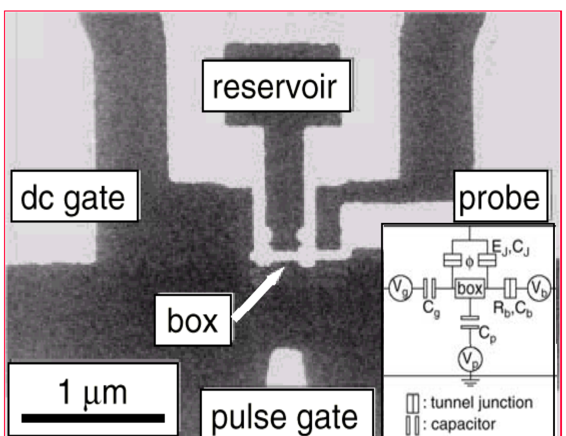
(Brune / Haroche)

- Optics



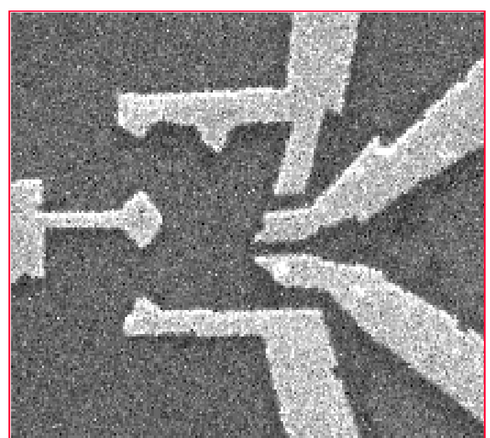
(Zeilinger)

- Superconductor



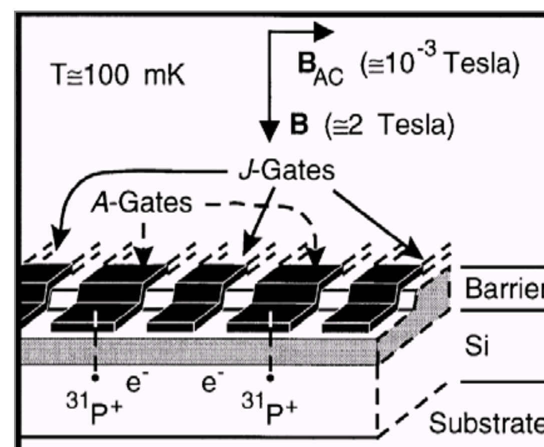
(Nakamura)

- Quantum Dots



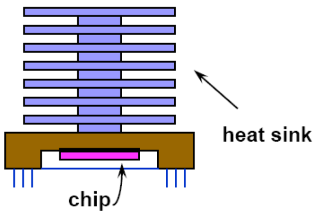
(Marcus / Tarucha)

- ^{31}P in Silicon

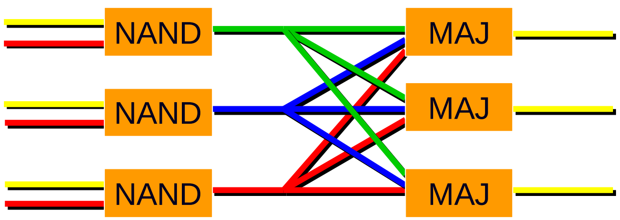


(Kane)

Summary



- **Power dissipation in CMOS:**
  - Power = $C_L V_{dd}^2 f_{clk}$
  - Fundamental limit:)



- **Fundamental Limits:**
  - $k_B T$ log 2 Joules/op (for irreversible logic)
  - Power dissipation only required to correct errors!
- **Reliable computers:**
  - Can be constructed from faulty components, using recursive error correction
- **Future digital system technologies:**
  - Bio: fault tolerant through redundancy
  - Quantum: new computing resource primitives