Lecture 10: March 15, 2001

Today: 2-3 Trees

- Successor
- Insert
- Delete

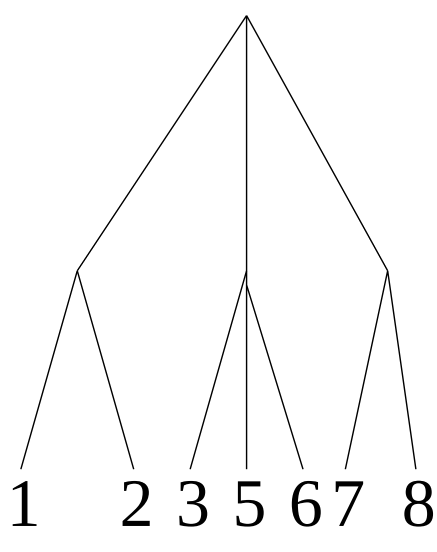
in $O(logn)$ time.

What is a 2-3 tree ?

- elements only in the leaves, the internal nodes hold maximum of their subtrees
- elements in increasing order from left to right
- every internal nodes has 2 or 3 children
- all leaves have equal depth

More rigorous structure than BST.

An example



What is the depth of a 2-3 tree ?

- a 2-3-tree with depth h contains (as a subgraph) a complete binary tree of depth $h \Rightarrow$ has $\geq 2^h$ leaves
- therefore, any 2-3-tree of depth $\geq \log(n+1)$ has $\geq n+1$ leaves
- ...so, any such tree with $\leq n$ leaves has depth $< \log(n+1)$

Depth is $O(\log n)$ - can perform operations efficiently.

How to find a successor ?

To find a successor for k:

- start from the root
- find the first child with max $\geq k$ (if it does not exists, successor undefined since k is larger than all elements)
- search the child's subtree

The general philosophy as for BST's. Time $O(\log n)$.

How to insert ?

To insert a node x:

- find a successor y of $key[x]$ (if does not exist, find a predecessor and perform symmetric operations)
- attach x to the parent p of y and hope p has still ≤ 3 children
- if the hope fails (i.e., p has 4 children now):
  - create a new node p' and attach 2 children of p to it (so now p has only two children)
  - insert (recursively) p' to p's parent; if p is an orphan, create its parent first

Does not violate the "equal depth" constraint
$\Rightarrow$ still 2-3-tree!

Insertion example