**Problem 1:** These are the key concepts from lecture this week:

1. Mapping Reducibility - pages 189-194 (make sure you understand Theorems 5.16, 5.17, 5.22, and 5.23)
2. Rice’s Theorem

**Problem 2:** Give yourself the following test, then check your answers on the back of the handout. Classify each of the following problems as either

- (D) decidable,
- (R) recognizable but not decidable,
- (C) co-recognizable but not decidable, or
- (N) neither recognizable nor co-recognizable,

and indicate which undecidable examples follow from Rice’s Theorem.

1. $E_{Q_{NFA}}$, the Equivalence problem for NFA’s.
2. $\{\langle M \rangle | M$ is a Turing Machine that runs for at least $n$ steps when started with a blank input tape, where $n$ is the length of the string $\langle M \rangle \}$.
3. $\{\langle M \rangle | M$ is a Turing Machine that accepts at least two inputs\}.
4. $E_{Q_{TM}}$

**Problem 3:** (Mapping Reducibility) Answer the following True or False:

6. $A_{TM}$ is mapping reducible to $E_{TM}$.
7. $A_{TM} \leq_m 0^*1^*$.

**Problem 4:** (Applications of Rice’s Theorem and Mapping Reducibility)

1. Let $L_1 = \{ \langle M \rangle | M$ accepts 01 in a perfect number of steps $\}$. Show that $L$ is undecidable. Does Rice’s Theorem apply?

   **Answer:** Rice doesn’t apply. Show that $\mathcal{A}01 \leq_m L$.

2. Let $L_2 = \{ \langle M \rangle | L(M)$ is recognized by a TM having an even number of states $\}$. Show that $L_2$ is decidable.

   **Answer:** Even though we have a language property, notice that any language has the property, so Rice’s Thm doesn’t apply.
3. Let $L_3 = \{ < M > \mid L(M) \text{ is not regular} \}$. Show that $L_3$ is undecidable.

   Answer: Rice’s Thm applies.

**Problem 5:** (Rice’s Theorem and Mapping Reducibility)
Consider the problem of testing whether a Turing machine $M$ accepts any binary string with an odd number of zeros.

1. Formulate this problem as a language; call it $ODDZ$.
2. Show that $ODDZ$ is undecidable.

   Answer: Use Rice’s Theorem, show hypotheses are satisfied.


   Answer: Yes, by running the TM in parallel (i.e., using the dove-tailing technique from class) on all inputs strings with an odd number of zeros until it accepts.


   Answer: no, undecidable, but recognizable

**Problem 2 Solutions:**

1. D; recall the $EQ_{DFA}$ algorithm from textbook.
2. D; just simulate $M$ for up to $|\langle M \rangle|$ steps.
3. R; Undecidable by Rice’s Theorem; Recognizable by running the TM in parallel (i.e., using the dove-tailing technique from class) on all input strings until it accepts two strings.
4. N; Undecidable by Rice’s Theorem; Neither recognizable nor co-recognizable from textbook.

**Problem 3 Solutions:**

1. False; $A_{TM}$ is recognizable, $E_{TM}$ is not. See Corollary 5.17.
2. False; $0^*1^*$ is decidable, $A_{TM}$ is not. See Theorem 5.16.