Reading: Sipser, Sections 1.3 and 1.4

Problem 1: Taken from Sipser 1.18. Give regular expressions generating the following languages. In all cases the alphabet is \{0, 1\}.
1. \(L_1 = \{w | w \text{ contains at least three 1s}\}\).
2. \(L_2 = \{w | w \text{ has length at least 3 and its third symbol is 0}\}\).
3. \(L_3 = \{w | w \text{ doesn’t contain the substring } 110\}\).
4. \(L_4 = \{w | \text{ every odd position of } w \text{ is a 1}\}\).
5. \(L_5 = \{w | w \text{ contains at least two 0s and at most one 1}\}\).

Problem 2: Sipser 1.19. Use the procedure described in Lemma 1.55 to convert the following regular expressions to nondeterministic finite automata.
1. \((0 \cup 1)^*000(0 \cup 1)^*\)
2. \(((00)^*(11)) \cup 01)^*\)
3. \(\emptyset^*\)

Problem 3: Sipser 1.21. Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.

\[
\begin{array}{c}
\text{(a)}
\end{array}
\]

Problem 4: Use the pumping lemma to show that the following languages are not regular.
1. \(A_1 = \{w w w | w \in \{0, 1\}^*\}\).
2. \(A_2 = \{w \in \{0, 1\}^* | \text{ the number of 0s in } w \text{ is a perfect square }\}\).
**Problem 5:** Based on Sipser 1.30. Describe the error in the following “proof” that $0^*1^*$ is not a regular language. (An error must exist because $0^*1^*$ is regular.)

The proof is by contradiction. Assume that $0^*1^*$ is regular. Let $p$ be the number of states in a DFA recognizing $0^*1^*$. Choose $s$ to be the string $0^p1^p$. You know that $s$ is a member of $0^*1^*$, but Example 1.73 (in the text) shows that $s$ cannot be pumped. Thus you have a contradiction. So $0^*1^*$ is not regular.

**Problem 6:** Sipser 1.47

Let $\Sigma = \{1, \#\}$ and let

$$Y = \{w \mid w = x_1 \# x_2 \# \ldots \# x_k \text{ for } k \geq 0, \text{ each } x_i \in 1^*, \text{ and } x_i \neq x_j \text{ for } i \neq j\}.$$ 

Prove that $Y$ is not regular.