Readings: Sections 7A, 7.5

Problem 1: Let $A$ and $B$ be nontrivial languages over an alphabet $\Sigma$ (that is, not equal to $\emptyset$ or $\Sigma^*$). State whether each of the following is KNOWN TO BE TRUE, KNOWN TO BE FALSE, or UNKNOWN. Explain carefully why. For example if you claim that a reduction exists, then you should actually define the reduction.

1. If $A \leq_P B$, then $\overline{A} \leq_P \overline{B}$.
2. If $B \in P$ and $A$ is nontrivial (not equal to $\emptyset$ or $\text{Sigma}^*$), then $A \cap B \leq_P A$.
3. If $B \in P$ and $A$ is nontrivial, then $A \cup B \leq_P A$.
4. If $A \cap B$ is NP-complete, $A \in NP$ and $B \in P$, then $A$ must be NP-complete.
5. If $A \cup B$ is NP-complete, $A \in NP$ and $B \in P$, then $A$ must be NP-complete.
6. If $A$ is NP-complete, $\overline{A} \in NP$ and $B \in NP$, then $\overline{B}$ must be in $NP$.
7. If $A$ is NP-complete, $\overline{A} \in NP$ and $B \in NP$, then $\overline{B}$ must be in $P$.

Problem 2: For each of the following pairs of sets $A$ and $B$, show that $A \leq_P B$.

1. $A = SAT$, and $B = SAT-UNSAT = \{\langle \phi \rangle \mid \phi$ is a satisfiable Boolean formula that is not a tautology (that is, it has at least one non-satisfying assignment)\};
2. $A = SAT$, and $B = TRIPLE - SAT = \{\langle \phi \rangle \mid \phi$ is a satisfiable Boolean formula that has at least three distinct satisfying assignments }.
3. $A = VC$, the Vertex Cover problem, and $B = HALF-VC$, defined as $\{\langle G \rangle \mid G$ is an undirected graph with an even number of vertices, of which some half form a vertex cover }.