6.045J/18.400J: Automata, Computability and Complexity	Prof. Nancy Lynch

Practice Quiz 2

March 21, 2007 *Elena Grigorescu*

Please write your name in the upper corner of each page.

INFORMATION ABOUT QUIZ 2:
Quiz 2 will be closed-book, closed-notes. However, you are allowed to bring one sheet of paper with review notes.

Problem	Score
1	
2	
3	
4	
5	
6	
Total	

Name: ____

Problem 1: **True or False (20 points)** Full credit will be given for correct answers. If you include justification for your answers, you may obtain partial credit for incorrect answers.

1. True or False: There exists a Turing machine that enumerates a set S of (encodings of) decider Turing machines, such that S includes Turing machines that decide infinitely many different decidable languages.

2. True or False: There exists a Turing machine that enumerates a set S of (encodings of) decider Turing machines, such that S includes at least one Turing machine that decides each decidable language.

3. True or False: There exists a Turing machine that enumerates an infinite set S of (encodings of) decider Turing machines, such that every machine that S outputs is "minimal". (Here "minimal" means that there is no other smaller decider Turing machine that decides the same language.)

4. True or False: Rice's Theorem immediately implies that

$$\{\langle M\rangle | M \text{ is a Turing machine and } L(M) \subseteq 0^*1^*\}$$

is undecidable.

Name: ____________________

5. True or False: If L_1 is a decidable language and L_2 is a Turing recognizable language, then $L_1 - L_2$ must be Turing-recognizable.

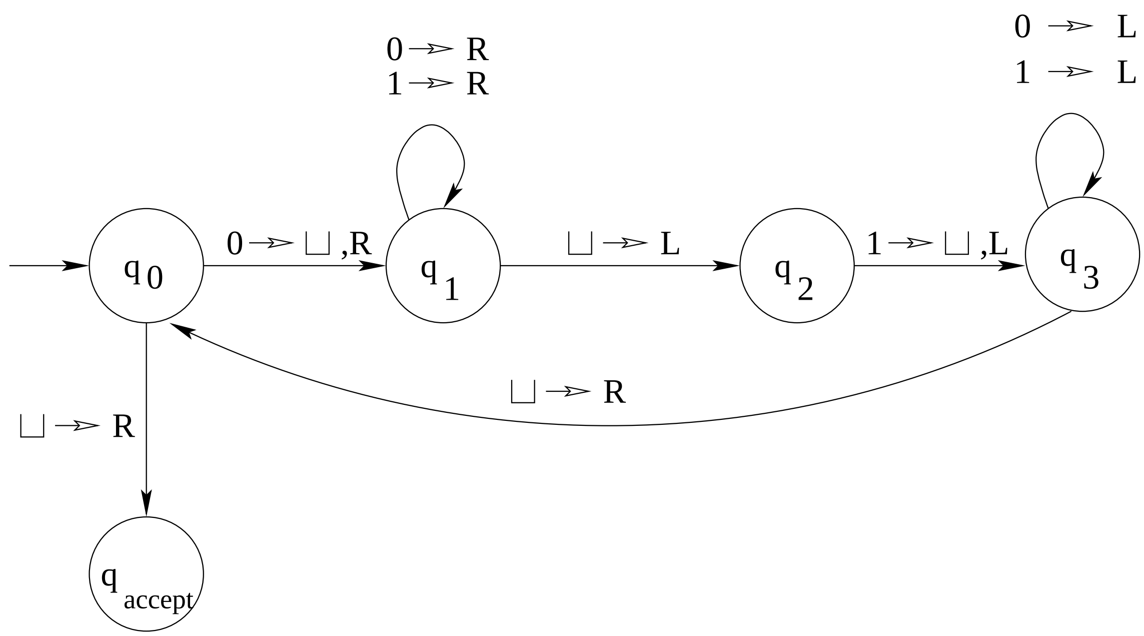
6. True or False: If L_1 is a Turing recognizable language and L_2 is a decidable language, then $L_1 L_2$ must be Turing-recognizable.

7. True or False: A three-dimensional Turing machine is like an ordinary Turing machine except that its "tape" storage consists of a three dimensional "tape", where each tape cell is a unit cube. In one step, the single tape head can move north, south, east, west, up, or down.
   The class of languages recognized by three-dimensional Turing machines is exactly the Turing-recognizable languages.

8. True or False: The class of languages recognized by three-stack machines is exactly the Turing recognizable languages.

Name: ______________________________

Problem 2: **(25 points)** Consider the following formal description of a Turing Machine M, where $Q = \{q_0, q_1, q_2, q_3, q_{accept}, q_{reject}\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \sqcup\}$. Assume that any unspecified transitions go to q_{reject}.



1. (5 points) Write out the accepting computation history of M on input 01, in the form given in class and in Sipser's book. Describe the behavior represented in this computation history in words.

Name: ______________________________

2. (5 points) What language does M recognize? (Give a precise definition.)

3. (10 points) Give the set of tiles for the *modified* Post Correspondence Problem, for this particular machine M and input 01. Indicate which is the initial tile.

   We have started things off by listing the tiles needed for completing the match from the point where an accepting state is encountered. You must define the initial tile and the tiles needed to represent all the moves.

The initial tile :

Tiles for the right moves :

Tiles for the left moves :

The alphabet tiles: $\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \sqcup \\ \sqcup \end{pmatrix}, \begin{pmatrix} \sharp \\ \sharp \end{pmatrix}, \begin{pmatrix} \sharp \\ \sqcup\sharp \end{pmatrix}$

The "clean-up" tiles : $\begin{pmatrix} 0\, q_{accept} \\ q_{accept} \end{pmatrix}, \begin{pmatrix} 1\, q_{accept} \\ q_{accept} \end{pmatrix}, \begin{pmatrix} \sqcup\, q_{accept} \\ q_{accept} \end{pmatrix}, \begin{pmatrix} q_{accept}\, 0 \\ q_{accept} \end{pmatrix},$
$\begin{pmatrix} q_{accept}\, 1 \\ q_{accept} \end{pmatrix}, \begin{pmatrix} q_{accept}\, \sqcup \\ q_{accept} \end{pmatrix}, \begin{pmatrix} q_{accept}\, \sharp\sharp \\ \sharp \end{pmatrix}$

Name: ____________________

4. (5 points) Write the accepting computation history you wrote for part (b) twice, one above the other, and mark the boundaries of the MPCP tiles involved in the match. You may skip the part involved in terminating the computation—just mark the tiles up to the first occurrence of the accept state.

Name: ____________________

Problem 3: **(10 points)** Suppose we have a Turing Machine M that, on each input string x, either halts with an output string on its tape or loops forever. Describe briefly how to constuct an enumerator Turing machine E that enumerates the outputs produced by M on all the inputs.

Name: ______________________________

Problem 4: **(20 points)** Let $EVENODD = \{\langle M \rangle |\ M$ accepts all strings of even length and does not accept any strings of odd length $\}$

1. (2 points) Does Rice's Theorem apply to $EVENODD$? Why or why not? If it does apply, then what does this imply about EVENODD?

2. (9 points) Is EVENODD Turing-recognizable? Prove your answer. You may use any results proved in class or in Sipser's book, but if you do, then cite the results explicitly.

Name: ____

3. (9 points) Is the complement of EVENODD Turing-recognizable? Again, prove your answer. Again, you may use any results proved in class or in the book, but cite them explicitly.

Name: ____

Problem 5: **(15 points)** Let L be the following language of Turing machine descriptions:
$\{\langle M\rangle$: M is a Turing machine with input alphabet $\{0, 1\}$ and M accepts every string consisting of just zeros (it may accept other strings) $\}$
Prove that L is undecidable using the Recursion Theorem. Do this by filling in the following proof outline:

Suppose for the sake of contradiction that ____

Let D be ____

Define a Turing machine R:
R: On input w do:
Obtain _________________________;

This is possible because of ____________________.

Run D on input ________________________.

If D accepts then _______________________________.

If D rejects then ________________________________.

If R accepts all strings consisting of only zeros, then ____

____

But this implies that, ____

____, which is impossible.

On the other hand, if R does not accept all strings consisting of only zeros, then ______________

____

____.

But this implies that, ____

____, which is also impossible.

Therefore, we have a contradiction, and L cannot be decidable.

Name:

Problem 6: **(10 points)** Consider a new kind of machine, a k-Queue Machine. A k-Queue Machine has the same general structure as a k-Counter Machine or a k-Stack Machine. However, it has k queues for storage instead of k counters or stacks. Initially, each queue q is empty.

It supports the following operations (Let Γ be the alphabet of queue symbols):

1. $enqueue$: takes a symbol in Γ , and adds it to the end of queue q.
2. $dequeue$: removes the symbol at the front of queue q, if q is nonempty. If q is empty, this operation does nothing.
3. $empty$: a boolean, which returns 1 if queue q is currently empty, 0 otherwise.

Briefly outline an argument that the acceptance problem for 2-Queue-Machines is undecidable.

Name:

Scratch Work