Quiz 1

Problem 1. For any arithmetic expression e, the sequence of equalities and inequalities below is a proof in our formal system for proving inequalities. (The Appendix contains all the rules of the system.) For each line, write the line numbers of its antecedents in the space to its left; if a line is an axiom, write its own line number. For example, the required antecedent numbers for lines (1) and (4) have been filled in for you.

(1)	`0 + (-0) = -0`	(1)
_____	`0 + (-0) = 0`	(2)
_____	`-0 = 0 + (-0)`	(3)
(3) (2)	`-0 = 0`	(4)
_____	`0 <= 1`	(5)
_____	`-1 <= -0`	(6)
_____	`-1 <= 0`	(7)
_____	`-1 + e <= 0 + e`	(8)
_____	`0 + e = e`	(9)
_____	`-1 + e <= e`	(10)

Problem 2. Explain why `( x <= (x * x) )` is not provable in our formal system.

Problem 3.

3(a) Here is a useful inference rule for inequalities:

$$\frac{(e \texttt{ <= } f), \quad (g \texttt{ <= } h)}{(\ (e \texttt{ + } g) \texttt{ <= } (f \texttt{ + } h)\ )} \quad (\texttt{++-<=}\text{-congruence})$$

Explain why adding the rule (**++-<=**-congruence) to our proof system will not change the set of provable inequalities.

3(b) Explain why deleting the rule ($\leq$-antisymmetry) will not change the set of provable inequalities.

Problem 4. Here is a body from our functional kernel of Scheme:

```
(define l 1)
(define m 2)
(define n 3)
(define add_n (lambda (m) (+ m n)))
(define mul_m_add5+n (lambda (n) (+ (2 3) (add_n (* m n)))))
(mul_m_add5+n ( (lambda (m) (+ m n)) 5 ))
```

4(a) Underline *all* the redexes in this body.

4(b) Write out the body that results from one rewriting at the lambda application redex followed by all possible applications of the garbage collection rule.

Problem 5. Give an example of a functional kernel Scheme body that is

5(a) an ⟨immediate value⟩ that is not terminated.

5(b) a terminated expression that is not an ⟨immediate value⟩.

5(c) a body that *cannot* reach a terminated body by repeated rewriting.

Appendix

Table 1: Standard Equational Inference Rules.

$$\frac{}{e = e}\ \text{(reflexivity)} \qquad \frac{e = f}{f = e}\ \text{(symmetry)} \qquad \frac{e = f, \quad f = g}{e = g}\ \text{(transitivity)}$$

$$\frac{e_1 = e_2, \quad f_1 = f_2}{e_1 + f_1 = e_2 + f_2}\ \text{(+-congruence)} \qquad \frac{e_1 = e_2, \quad f_1 = f_2}{e_1 * f_1 = e_2 * f_2}\ \text{(*-congruence)} \qquad \frac{e = f}{-e = -f}\ \text{(--congruence)}$$

Table 2: Equational Axioms for Arithmetic

$(e + f) + g$	$=$	$e + (f + g)$	(associativity of $+$)
$(e * f) * g$	$=$	$e * (f * g)$	(associativity of $*$)
$e + f$	$=$	$f + e$	(commutativity of $+$)
$e * f$	$=$	$f * e$	(commutativity of $*$)
$0 + e$	$=$	e	(identity for $+$)
$1 * e$	$=$	e	(identity for $*$)
$e + (-e)$	$=$	0	(inverse for $+$)
$e * (f + g)$	$=$	$(e * f) + (e * g)$	(distributivity)

Table 3: Inference Rules for Inequalities.

$$\frac{e = f}{e \le f}\ \text{(}\le\text{-reflexivity)} \qquad \frac{e \le f, \quad f \le e}{f = e}\ \text{(}\le\text{-antisymmetry)} \qquad \frac{e \le f, \quad f \le g}{e \le g}\ \text{(}\le\text{-transitivity)}$$

$$\frac{e \le f}{e + g \le f + g}\ \text{(+-}\le\text{-congruence)} \qquad \frac{e \le f, \quad 0 \le g}{e * g \le f * g}\ \text{(*-}\le\text{-congruence)} \qquad \frac{e \le f}{-f \le -e}\ \text{(--}\le\text{-congruence)}$$

$$0 \le 1 \quad \text{(01-axiom)}$$