Diagnostic Quiz

You will not be graded on this quiz. Take it sometime after class, and return it in class on Monday, September 11. Be sure to indicate your name, the date and "6.044 Diagnostic Quiz" on your answer sheet.

Problem 1. Have you taken 6.042, 18.063 or 18.310? If not, what are your qualifications for taking this course?

Problem 2. Let succ be the successor function on integers:

$$\mathrm{succ}(x) = x + 1$$

Describe the function $(\mathrm{succ} \circ \mathrm{succ})$, that is, the composition of succ with itself.

Problem 3. How many strings of length 4 are there over the alphabet $\{a, b, c\}$?

Problem 4. Define $\preceq$ to be the binary relation between sets such that $A \preceq B$ if and only if the cardinality of A is less than or equal to the cardinality of B.

(a) What is the definition of "uncountable set"? Now express the definition in terms of $\preceq$. Give an example of an uncountable set.

(b) For each of the following properties, state whether the relation $\preceq$ has the property, and if not, give a simple counterexample.

1. reflexivity
2. symmetricity
3. transitivity

Problem 5. For any set, A, let $\mathcal{P}\mathrm{ow}(A)$ be the powerset of A, namely, the set of all subsets of A. Exhibit the members of $\mathcal{P}\mathrm{ow}(\mathcal{P}\mathrm{ow}(\mathcal{P}\mathrm{ow}(\emptyset)))$.

Problem 6. Two Boolean formulas $F_1(x_1, \ldots, x_n)$ and $F_2(x_1, \ldots, x_n)$ are *equivalent* iff they yield the same truth value for all truth assignments to the variables $x_1, \ldots, x_n$.

(a) The Boolean binary operation *conjunction* (and), which our text writes as "&", is commutative, namely x_1 & x_2 is equivalent to x_2 & x_1. Describe a Boolean binary operation which is not commutative.

(b) Describe an infinite set of equivalent Boolean formulas.

(c) Explain why "equivalent" is actually an equivalence relation on formulas.

(d) Explain why there are only a finite number of equivalence classes of formulas with (at most) variables $x_1, \ldots, x_n$.

(e) How many are there?

Problem 7. A binary relation $\leq$ is *anti-symmetric* if $x = y$ whenever $x \leq y$ and $y \leq x$. A *partial order* of a set D is a binary relation $\leq$ that is reflexive, transitive, and anti-symmetric on D. A partial order is *total* if for every $d_1, d_2 \in D$, either $d_1 \leq d_2$ or $d_2 \leq d_1$. If $X \subseteq D$ and $y \leq x$ for every $x \in X$, then y is a *lower bound* of X. Similarly, if $x \leq y$ for every $x \in X$, then y is an *upper bound* of X. If y is a lower bound of X, and $z \leq y$ for every lower bound z of X, then y is the *greatest lower bound* ("glb" or "meet") of X. If y is an upper bound of X, and $y \leq z$ for every upper bound z of X, then y is the *least upper bound* ("lub") of X.

Let A be the set $\{1, 2, 3, 4, 5\}$.

(a) Give a total order of A. That is, describe a binary relation $\leq$ on A that is a total order.

(b) Give a partial order of A such that: (1) every pair of elements has a glb; and (2) there is no lub of $\{4, 5\}$.

(c) Give a partial order of A such that every pair of elements has both an lub and a glb, but the order is not total.

Problem 8. About how long did it take you to complete this quiz?