Structural Induction

To prove $P(x)$ holds for all $x$ in recursively defined set $R$, prove:

- $P(b)$ for each base case $b \in R$
- $P(c(x))$ for each constructor, $c$, assuming ind. hyp. $P(x)$

$E \subseteq \text{Even}$ by structural induction on $x \in E$ with ind. hyp. “$x$ is even”

- $0$ is even
- if $n$ is even, then so is $n+2$, $-n$

Lemma: Every $s$ in $M$ has the same number of ]'s and ['s.

Proof by structural induction on the definition of $M$
**Matched Paren Strings $M$**

**Lemma:** Every $s$ in $M$ has the same number of ]'s and ['s.

Let $EQ ::= \{\text{strings with same number of } ] \text{ and } [\}$

**Lemma (restated):** $M \subseteq EQ$

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**Structural Induction on $M$**

**Proof:**

Ind. Hyp. $P(s) ::= (s \in EQ)$

**Base case ($s = \lambda$):**

$\lambda$ has 0 ]'s and 0 ['s, so $P(\lambda)$ is true.

**base case is OK**

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**Structural Induction on $M$**

**Constructor step:** $s = [r]t$

can assume $P(r)$ and $P(t)$

$\#]$ in $s = \#]$ in $r + \#]$ in $t + 1$

$\#[ $ in $s = \#[ $ in $r + \#[ $ in $t + 1$

so $s = \text{ by } P(r) = \text{ by } P(t)$

so $P(s)$ is true **constructor case is OK**

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**Structural Induction on $M$**

so by struct. induct.

$M \subseteq EQ$

QED
Lemma.

F18 is closed under taking derivatives:
if \( f \in F18 \), then \( f' \in F18 \)

*Class Problem*