**Strong Induction**

Prove $P(0)$. Then prove $P(n+1)$ assuming all of $P(0), P(1), \ldots, P(n)$ (instead of just $P(n)$).

Conclude $\forall m. P(m)$

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**Postage by Strong Induction**

available stamps: $5\$ 3\

Thm: Get any amount $\geq 8\$

By strong induction with hyp: $P(n) ::= \text{can form } n + 8\$.$

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**Postage by Strong Induction**

available stamps: $5\$ 3\

Thm: Get any amount $\geq 8\$

base case $P(0)$: make $0 + 8\$

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**Postage by Strong Induction**

available stamps: $5\$ 3\

Thm: Get any amount $\geq 8\$

inductive step: Assume $m+8\$ for $n \geq m \geq 0$.

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**Postage by Strong Induction**

available stamps: $5\$ 3\

Thm: Get any amount $\geq 8\$

inductive step: Assume all from 8 to $n+8\$.
available stamps: 
5¢ 3¢

Thm: Get any amount \( \geq 8¢ \)

inductive step:
Assume all from 8 to \( n+8¢ \).
Prove can get \( n+9¢ \), for \( n \geq 0 \)

Postage by Strong Induction

We conclude by strong induction that, using 3¢ and 5¢ stamps, \( n + 8¢ \) postage can be formed for all \( n \geq 0 \).

Unstacking game

Start: a stack of boxes 
Move: split any stack into two of sizes \( a, b > 0 \)
Scoring: \( a \cdot b \) points 
Keep moving: until stuck 
Overall score: sum of move scores

Analyzing the Stacking Game

Claim: Every way of unstacking \( n \) blocks gives the same score:

\[
(n-1)+(n-2)+\ldots+1 = \frac{n(n - 1)}{2}
\]
Analyzing the Game

**Claim:** Starting with size $n$ stack, final score will be \[ \frac{n(n-1)}{2} \]

Proof: by **Strong induction** with Claim($n$) as hypothesis

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Proving the Claim by Induction

**Base case** $n = 0$:
\[ \text{score} = 0 = \frac{0(0-1)}{2} \]
Claim(0) is OK

**Inductive step.** Assume for stacks $\leq n$, and prove C($n+1$):
\[ (n+1)\text{-stack score} = \frac{(n+1)n}{2} \]

**Inductive step.**
**Case** $n+1 = 1$. verify for 1-stack:
\[ \text{score} = 0 = \frac{1(1-1)}{2} \]
C(1) is OK

**Inductive step.**
**Case** $n+1 > 1$. Split $n+1$ into an $a$-stack and $b$-stack, where $a + b = n + 1$.
\[ (a + b)\text{-stack score} = ab + a\text{-stack score} + b\text{-stack score} \]

**Proving the Claim by Induction**

by **strong induction**:
\[ a\text{-stack score} = \frac{a(a - 1)}{2} \]
\[ b\text{-stack score} = \frac{b(b - 1)}{2} \]
Proving the Claim by Induction

The total \((a + b)\)-stack score is:

\[
ab + \frac{a(a - 1)}{2} + \frac{b(b - 1)}{2} = \frac{(a + b)(a + b - 1)}{2} + \frac{(n+1)n}{2}
\]

so \(C(n+1)\) is \(\text{OK}\).

We're done!