Recursive Data

Recursive Definitions
Define something in terms of a simpler version of the same thing:
- **Base case(s)** that don't depend on anything else.
- **Constructor case(s)** that depend on simpler cases.

Example Definition: set $E$
Define set $E \subseteq \mathbb{Z}$, recursively:
- **Base case:** $0 \in E$
- **Constructor cases:**
  If $n \in E$, then
  1. $n + 2 \in E$, if $n \geq 0$;
  2. $-n \in E$, if $n > 0$.

Example Definition: set $E$
1. $n \in E$ and $n \geq 0$, then $n + 2 \in E$:
   0, 0+2, (0+2)+2, ((0+2)+2) +2
   0, 2, 4, 6, ...
2. $n \in E$ and $n > 0$, then $-n \in E$
   -2, -4, -6, ...
   all even numbers
Recursive Def: Extremal Clause

So, \( E \) contains the even integers
Anything Else?  No!

• \( 0 \in E \)
• If \( n \in E \) and \( n \geq 0 \), then \( n+2 \in E \)
• If \( n \in E \) and \( n > 0 \), then \( -n \in E \)
• That's All!

Extremal Clause
(Implicit part of definition)

Example Definition: set \( E \)

So \( E \) is exactly the Even Integers

Matched Paren Strings, \( M \)

set of strings, \( M \subseteq \{ \}, [ ] \) *
• Base: \( \lambda \in M \),
  (the empty string)
• Constructor:
  If \( s, t \in M \), then
  \[ s \] \( t \in M \)
Matched Paren Strings $M$

strings $[s] \in M$

- $[] \quad s = \lambda \quad t = \lambda$
- $[\[] \quad s = [] \quad t = \lambda$
- $[] \quad s = \lambda \quad t = [\]$  
- $[\[] \quad s = [] \quad t = [\]$  
- $[\[] \quad s = [\] \quad t = \lambda$
- $\vdots \quad \vdots \quad \vdots$

not in $M$

strings starting with $]$ are not in $M$ because

- $\lambda$ does not start with $]$
- $[s] \in M$ does not start with $]$

and everything in $M$ arises in one of these two ways

The 18.01 Functions, $F_{18}$

The set $F_{18}$ of functions on $\mathbb{R}$:

- $\text{Id}_{\mathbb{R}}$, constant functions, and $\sin x$

if $f, g \in F_{18}$, then

- $f + g$, $f \cdot g$, $2f$,
- the inverse, $f^{-1}$, of $f$, and
- $f \circ g$ (the composition of $f$ and $g$)

are in $F_{18}$.

Some functions in $F_{18}$:

- $-x = (-1) \cdot x$
- $\sqrt{x} = (x^2)^{(-1)}$ --- inverse
- $\cos x = (1 - (\sin x \cdot \sin x))^{1/2}$
- $\ln x = (2^x \log e)^{(-1)}$