Massachusetts Institute of Technology
6.042J/18.062J, Fall '09: Mathematics for Computer Science — November 4
Prof. Albert R. Meyer — revised November 5, 2009, 1176 minutes

Problem Set 9

Due: November 13

Reading: Notes Ch. 14.5; Ch. 15.1–15.2

Problem 1.
Indicate which of the following holds for each pair of functions $(f(n), g(n))$ in the table below. Assume $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Pick the four table entries you consider to be the most challenging or interesting and justify your answers to these.

$f(n)$	$g(n)$	$f = O(g)$	$f = o(g)$	$g = O(f)$	$g = o(f)$	$f = \Theta(g)$	$f \sim g$
2^n	$2^{n/2}$						
$\sqrt{n}$	$n^{\sin n\pi/2}$						
$\log(n!)$	$\log(n^n)$						
n^k	c^n						
$\log^k n$	n^ϵ						

Problem 2.
In a standard 52-card deck, each card has one of thirteen *rank*s in the set, R, and one of four *suit*s in the set, S, where

$$R ::= \{A, 2, \ldots, 10, J, Q, K\},$$
$$S ::= \{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}.$$

A 5-card *hand* is a set of five distinct cards from the deck.

For each part describe a bijection between a set that can easily be counted using the Product and Sum Rules of Ch. 15.2, and the set of hands matching the specification. *Give bijections, not numerical answers.*

For instance, consider the set of 5-card hands containing all 4 suits. Each such hand must have 2 cards of one suit. We can describe a bijection between such hands and the set $S \times R_2 \times R^3$ where R_2 is the set of two-element subsets of R. Namely, an element

$$(s, \{r_1, r_2\}, (r_3, r_4, r_5)) \in S \times R_2 \times R^3$$

indicates

1. the repeated suit, $s \in S$,

2. the set, $\{r_1, r_2\} \in R_2$, of ranks of the cards of suit, s, and
3. the ranks (r_3, r_4, r_5) of remaining three cards, listed in increasing suit order where $\clubsuit \prec \diamondsuit \prec \heartsuit \prec \spadesuit$.

For example,

$$(\clubsuit, \{10, A\}, (J, J, 2)) \longleftrightarrow \{A\clubsuit, 10\clubsuit, J\diamondsuit, J\heartsuit, 2\spadesuit\}.$$

(a) A single pair of the same rank (no 3-of-a-kind, 4-of-a-kind, or second pair).

(b) Three or more aces.

Problem 3.
An n-vertex *numbered tree* is a tree whose vertex set is $\{1, 2, \ldots, n\}$ for some $n > 2$. We define the *code* of the numbered tree to be a sequence of $n - 2$ integers from 1 to n obtained by the following recursive process:

> If there are more than two vertices left, write down the *father* of the largest leaf[1], delete this *leaf*, and continue this process on the resulting smaller tree.
> If there are only two vertices left, then stop —the code is complete.

For example, the codes of a couple of numbered trees are shown in the Figure 1.

(a) Describe a procedure for reconstructing a numbered tree from its code.

(b) Conclude there is a bijection between the n-vertex numbered trees and $\{1, \ldots, n\}^{n-2}$, and state how many n-vertex numbered trees there are.

[1]The necessarily unique node adjacent to a leaf is called its *father*.

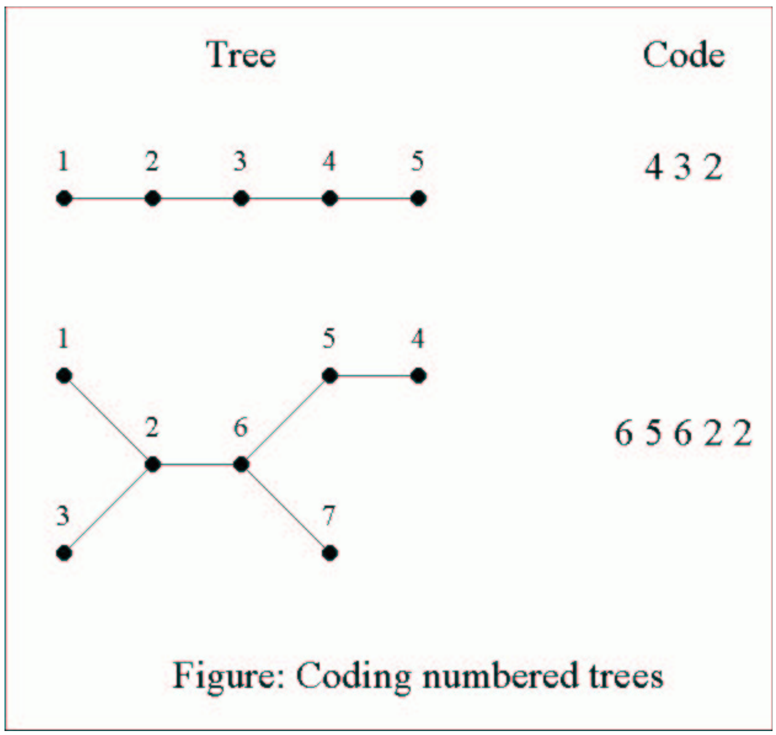
Tree
Code
1 2 3 4 5
4 3 2
1 5 4
2 6
3 7
6 5 6 2 2
Figure: Coding numbered trees

Figure 1:

Student's Solutions to Problem Set 9

Your name:

Due date: November 13

Submission date:

Circle your TA/LA: Jodyann Justin Megumi Rajeev Richard Steven Tom Eli

Collaboration statement: Circle one of the two choices and provide all pertinent info.

1. I worked alone and only with course materials.
2. I collaborated on this assignment with:

   got help from:[1]

   and referred to:[2]

DO NOT WRITE BELOW THIS LINE

Problem	Score
1	
2	
3	
Total	

[1]People other than course staff.

[2]Give citations to texts and material other than the Fall '09 course materials.