

## 6.042 Final Review: Topics

These are the main topics we have covered in 6.042 this semester. In studying for the final exam, you may find it useful to use this handout as a guide.

### 1 Proofs

- propositions, axioms, deductions
- predicates and quantifiers
- proof by contradiction
- proof by contrapositive

### 2 Induction

- proof by induction
- strong induction
- state machines and invariants
- the 8 puzzle (and friends)

### 3 Number theory

- divisibility
- Division Theorem
- GCD and linear combinations
- Pulverizer Algorithm
- modular arithmetic
- multiplicative inverses
- Fermat's Theorem
- relative primality and Euler's theorem

### 4 Graph theory

- graphs: simple, weighted, directed, trees
- graph colorings
- connectivity

- matchings, perfect matchings
- bipartite graphs
- stable marriage problem, the mating algorithm
- induction on graphs (build up error)
- minimum weight spanning trees, MST algorithm
- paths, cycles, walks, tours
- Hamiltonian paths, Euler circuits
- tournament graphs
- PageRank

## 5 Communication Networks

- definitions of latency, diameter, switch size, switch count, congestion
- network architectures: binary tree, 2D array, butterfly, Beneš

## 6 Relations

- basic definitions: reflexive, symmetric, antisymmetric, transitive
- equivalency relations and classes
- partial orders, total order, topological sorts, Hasse diagrams
- task scheduling theorems, chains, antichains, Dilworth's Theorem

## 7 Sums and asymptotics

- geometric sums, finite and infinite
- solving sums through differentiation and integration
- sums of powers (e.g.  $\sum i$ ,  $\sum i^2$ )
- integration bounds for sums
- harmonic numbers
- solving products through logarithms

## 8 Recurrences

- guess-and-check
- plug-and-chug
- Akra-Bazzi theorem
- linear recurrences

## 9 Asymptotic Notation

- asymptotic notation
- induction involving asymptotics

## 10 Counting

- functions, bijections
- sum rule
- product rule
- the pigeonhole principle
- generalized product rule
- bookkeeper formula
- permutations
- division rule
- inclusion-exclusion principle
- Stirling's approximation
- counting bit sequences
- counting poker hands
- combinatorial proofs

## 11 Basic Probability

- Monty Hall problem
- tree diagrams
- sample points/outcomes, events
- product rule for intersection of events
- inclusion-exclusion for unions of events
- independence (pairwise and mutual)
- conditional probability, Bayes rule
- union bound
- law of total probability
- birthday paradox

## 12 Random Variables

- random variables, indicator random variables
- probability distributions: Bernoulli, binomial, uniform and their expectation, variance
- independence of random variables
- expectation, conditional expectation, expectation of functions
- expectation of sums (distributive for all random variables)

- expectation of products (distributive for independent random variables)
- expectation of quotients (in general, not distributive)
- law of total expectation
- mean time to failure
- variance, different definitions
- variance of sums (distributive for independent random variables)
- polling, approximating the binomial distribution
- expectation bounds: Markov, Chebyshev, Chernoff, Murphy's
- random walks: gambler's ruin, random walks on graphs