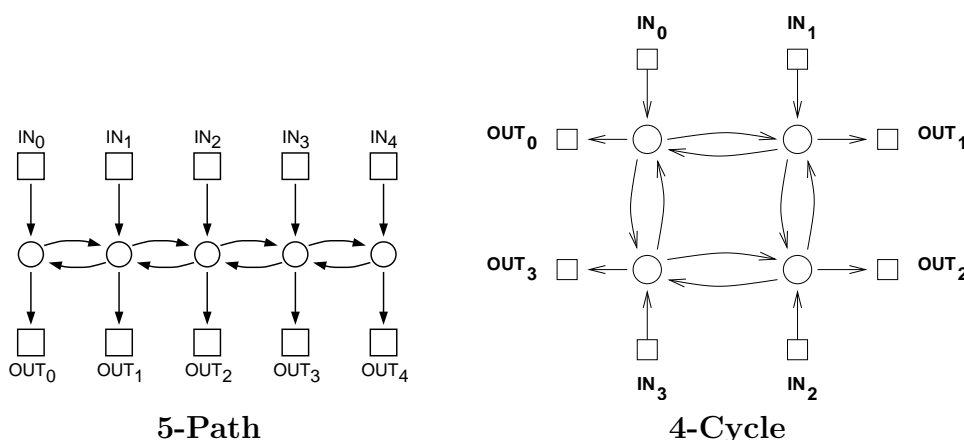


Notes for Recitation 10

Analysis of Two Networks

Two communication networks are shown below. Complete the table of properties and be prepared to justify your answers.



network	# switches	switch size	diameter	max congestion
5-path	5	3×3	6	5
4-cycle	4	3×3	4	3

Recall that the *diameter* of a communication network is the number of edges on the shortest path between the input and output that are farthest apart. The *max congestion* of a network is the largest number of packets that pass through any switch in the best solution to the hardest permutation routing problem. You might imagine that your enemy picks a permutation and then you pick the path taken by each packet. (Her goal is to cause congestion, and yours is to eliminate it.) Assuming you both do your best, the max congestion is then equal to the largest number of packets passing through a single switch.

Solution. The congestion of the 5-path is at least 5, because every path must contain the central switch when $\pi(i) = 4 - i$. The congestion is at most 5, because there are only 5 paths in total.

The congestion of the 4-cycle is at least 3. Consider the permutation routing problem in which each input sends a packet to the diagonally opposite output: $\pi(0) = 2$, $\pi(1) = 3$,

$\pi(2) = 0, \pi(3) = 1$. Packets 0 and 2 must pass through the switches on the upper left and lower right in order to access the appropriate input and output terminals. Packet 1 must pass through one of these switches as well, so at least three packets pass through either the upper-left switch or the lower-left switch.

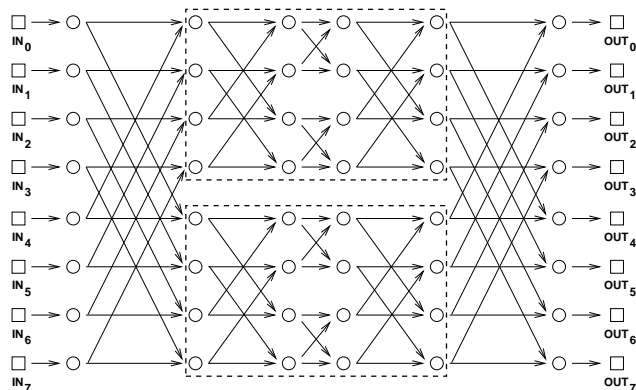
The congestion of the 4-cycle is at most 3. Suppose we route each packet by the shortest path and break ties by routing clockwise around the cycle. Now consider any particular switch, say the one in the upper right. At worst, this switch sees three packets: the one from input 1, the one destined for output 1, and one passing through from input 0 to output 2. By symmetry, every switch sees at most 3 packets. ■

Routing in a Beneš Network

In lecture, we saw that the Beneš network has a max congestion of 1; that is, every permutation can be routed in such a way that a single packet passes through each switch. Let's work through an example. A Beneš network of size $N = 8$ is attached.

1. Within the Beneš network of size $N = 8$, there are two subnetworks of size $N = 4$. Put boxes around these. Hereafter, we'll refer to these as the *upper* and *lower* subnetworks.

Solution.



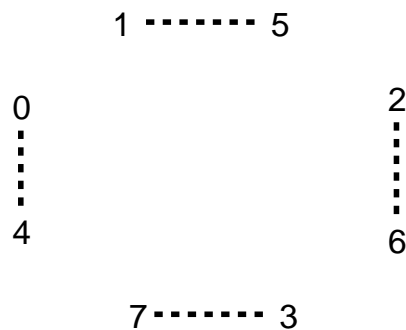
■

2. Now consider the following permutation routing problem:

$$\begin{array}{ll}
 \pi(0) = 3 & \pi(4) = 2 \\
 \pi(1) = 1 & \pi(5) = 0 \\
 \pi(2) = 6 & \pi(6) = 7 \\
 \pi(3) = 5 & \pi(7) = 4
 \end{array}$$

Each packet must be routed through either the upper subnetwork or the lower subnetwork. Construct a graph with vertices 0, 1, ..., 7 and draw a *dashed* edge between each pair of packets that can not go through the same subnetwork because a collision would occur in the second column of switches.

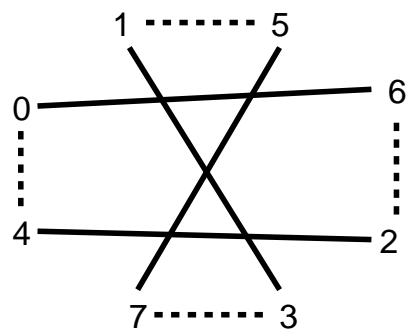
Solution.



■

3. Add a *solid* edge in your graph between each pair of packets that can not go through the same subnetwork because a collision would occur in the next-to-last column of switches.

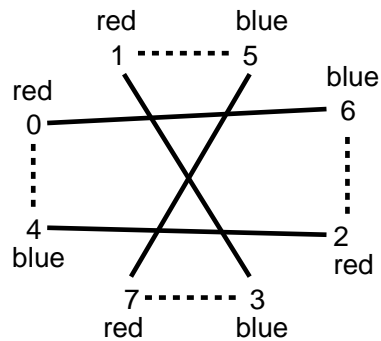
Solution.



■

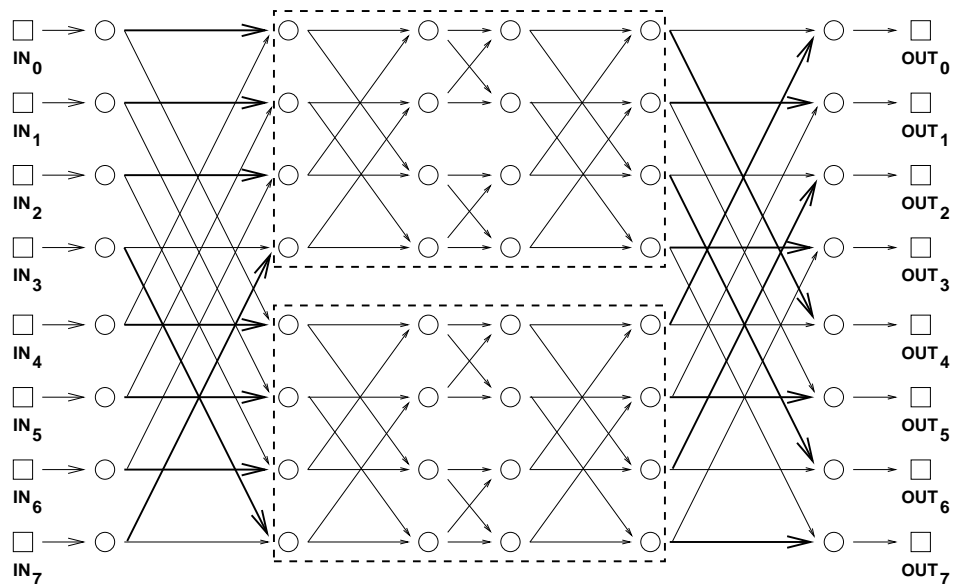
4. Color (i.e., label) the vertices of your graph red and blue so that adjacent vertices get different colors. Why must this be possible, regardless of the permutation π ?

Solution. This must be possible, because the dashed edges form a matching and the solid edges form another matching. Because of the result you proved in homework, when you combine the edges, the result is a bipartite graph, which must be 2-colorable.



5. Suppose that red vertices correspond to packets routed through the upper subnetwork and blue vertices correspond to packets routed through the lower subnetwork. On the attached copy of the Beneš network, highlight the first and last edge traversed by each packet.

Solution.



6. All that remains is to route packets through the upper and lower subnetworks. One way to do this is by applying the procedure described above recursively on each subnetwork. However, since the remaining problems are small, see if you can complete all the paths on your own.

Solution.

