

## Problem Set 9

*Due:* November 4

**Reading:** [Week 9 Notes](#), Optional: Rosen §4.3,4.6

**Problem 1.** A state Senate has fifteen Republicans and ten Democrats. They are creating a committee, which must have six people in it. Since the Republicans have a majority in the Senate, they insist on having a majority on the committee. How many ways are there to form a committee that satisfies the above conditions?

**Problem 2.** How many positive integers less than 1,000,000 have exactly one digit equal to 9 and have a sum of digits equal to 13.

**Problem 3.** Let  $B(n)$  denote the number of equivalence relations on  $n$  elements.

- (a) Show that  $B(n) \leq n!$ .
- (b) Show that  $B(n) \geq 2^{n-1}$ .

**Problem 4.** Prove the following identity both by (a) algebraic manipulation and by (b) giving a combinatorial argument:

$$\binom{2n}{2} = 2\binom{n}{2} + n^2$$

**Problem 5.** George's 6.042 table has 12 students, who are supposed to break up into 4 groups of 3 students each. George has observed that the students waste too much time trying to form balanced groups, so he decided to pre-assign students to groups and email the group assignments to his students.

How many *different* group assignments are possible? (Clarification: There is no "numbering" of the groups. All that matters is who ends up with whom. Two group assignments are different if they differ in at least one group.)

**Problem 6.** The *king positioning* in an arrangement of a deck of 52 playing cards is the sequence of numerical positions in the deck, from one to 52, of the four successive kings. For example, the king positioning  $(1, 2, 3, 4)$  means all the kings come at the beginning of the deck. The king positioning  $(1, 18, 35, 52)$  describes the situation in which the kings are spaced uniformly—with exactly 16 cards between successive kings.

- (a) How many king positionings are there?
- (b) How many king positionings are there in which *no two kings are adjacent*?
- (c) Of the  $52!$  possible arrangements of the deck, how many have no two kings adjacent?

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## Student's Solutions to Problem Set 9

**Your name:**

**Due date:** November 4

**Submission date:**

**Circle your TA:** Adrian George Josh Karen Lee Min Nikos Tina

**Collaboration statement:** Circle one of the two choices and provide all pertinent info.

1. I worked alone and only with course materials.

2. I collaborated on this assignment with:

got help from:<sup>1</sup>

and referred to:<sup>2</sup>

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**DO NOT WRITE BELOW THIS LINE**

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Problem	Score
1	
2	
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