

In-Class Problems — Week 9, Fri

Problem 1. Recall the definition of a multinomial coefficient:

$$\binom{n}{r_1, r_2, \dots, r_k} ::= \frac{n!}{r_1! r_2! \dots r_k!}. \quad (1)$$

An alternative definition is

$$\binom{n}{r_1, r_2, \dots, r_k} ::= \binom{n}{r_1} \cdot \binom{n-r_1}{r_2} \binom{n-r_1-r_2}{r_3} \dots \binom{n-r_1-r_2-\dots-r_{k-1}}{r_k} \quad (2)$$

Prove that the two definitions are equivalent using both a (i) combinatorial and (ii) an algebraic argument.

Problem 2. Suppose that we have a generalized n -letter “MISSISSIPPI” word whose letters break down as follows: m occurrences of “M”, i occurrences of “I”, s of “S”, and p of “P”. For example, for the word “MISSISSIPPI” breaks down as $(n; m, i, s, p) = (11, 1, 4, 4, 2)$. For each of the following formulas, identify which correctly describe the number of different permutations of the generalized “MISSISSIPPI” word. For the correct formulas, justify your answer using a combinatorial (not an algebraic) argument.

(a) $\binom{n}{m, i, s, p}$.

(b) $\binom{n}{p, i, m, s}$

(c) $\binom{n}{m, i, s, n-m-i-s}$

(d) $\binom{n}{m} \binom{n}{i} \binom{n}{s} \binom{n}{p}$

(e) $\binom{n}{m} \binom{m}{i} \binom{i}{s} \binom{s}{p}$

Problem 3. The U.S. House of Representatives has 435 seats. (In this context, a “seat” is not a chair, but a right to belong to the legislative body. Thus, all seats are indistinguishable.)

- (a) In how many ways can the seats be distributed among three political parties?
- (b) In how many ways can the seats be distributed among three political parties so that no party has a majority? (This implies that a coalition of any two parties *does* form a majority.)
- (c) How do your solutions to parts (a) and parts (b) change if the “seats” are actually distinguishable chairs (numbered 1 through 435)?