

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
6.042J/18.062J

WELCOME!
Prof. Albert R. Meyer
Dr. Radhika Nagpal

<http://theory.lcs.mit.edu/classes/6.042>

“Proof, Proofs & More Proofs”

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Quick Summary

1. Fundamental concepts of Mathematics.
2. Discrete structures.
3. Discrete probability theory.

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DUE FRIDAY: Online Tutor
Reading Problem 1 (RP1):

- *Course Registration*
- *Diagnostic Survey*

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Course Organization

- **“Paperless.”** All handouts online -- no take-home handouts
- **Studio-Lecture Style:** mixture of mini-lectures & team problem-solving sessions

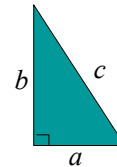
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Studio Style

Say “hello” to your TA
& the people next to you.

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**Getting started:
Pythagorean theorem**



$$a^2 + b^2 = c^2$$

Familiar? **Yes!**
Obvious? **No!**

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A Cool Proof

Rearrange into: (i) a $c \times c$ square, and then
(ii) an $a \times a$ & a $b \times b$ square

(Many many proofs: <http://www.cut-the-knot.com>)

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A Cool Proof

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A Cool Proof

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A False Proof: Getting Rich By Diagram

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A false proof: getting rich by diagram

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Getting Rich

The bug:

↑ ↓ are not right triangles!

The top and bottom line of the rectangle is not straight!

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Another False proof

Theorem: Every quadratic polynomial over \mathbb{C} has two roots.

Proof (by calculation):

The polynomial $ax^2 + bx + c$ has roots

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

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Another False proof

Counter-example:

$$0x^2 + 0x + 1 \text{ has 0 roots.}$$

$$0x^2 + 1x + 1 \text{ has 1 root.}$$

The bug: divide by zero error.

The fix: assume $a \neq 0$.

(Could also say def of “quadratic” requires $a \neq 0$.)

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Another false proof

Counter-example:

$$1x^2 + 0x + 0 \text{ has 1 root.}$$

The bug: $r_1 = r_2$

The fix: need hypothesis $D \neq 0$ where

$$D ::= \sqrt{b^2 - 4ac}$$

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Another false proof

Ambiguity when $D < 0$:

$$x^2 + 1 \text{ has roots } i, -i.$$

Which is r_1 , which is r_2 ?

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Another false proof

The ambiguity causes problems:

$$1 = \sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{-1}\sqrt{-1} = (\sqrt{-1})^2 = -1$$

Moral: “mindless” calculation not safe.

1. Be sure rules are properly applied.
2. Calculation is a risky substitute for understanding.

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Consequences of $1 = -1$

$$\frac{1}{2} = -\frac{1}{2} \quad (\text{multiply by } \frac{1}{2})$$

$$2 = 1 \quad (\text{add } \frac{3}{2})$$

“Since I and the Pope are clearly 2, we conclude that

I and the Pope are 1.

That is, I am the Pope.”

-- Bertrand Russell

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Consequences of $1 = -1$



Bertrand Russell (1872 - 1970)

(Picture source: <http://www.usnews.edu/~jlanier/brow.html>)

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Proof by Contradiction

Theorem: $\sqrt{2}$ is irrational.

Proof (by contradiction):

- Suppose $\sqrt{2}$ was rational.
- Choose m, n without common prime factors (always possible) such that

$$\sqrt{2} = \frac{m}{n}$$

- Show that m & n are both even, a contradiction.

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Indirect Proof

Theorem: $\sqrt{2}$ is irrational.

Proof (by contradiction):

$$\sqrt{2} = \frac{m}{n}$$

$$\sqrt{2}n = m$$

$$2n^2 = m^2$$

so m is even.

so can assume $m = 2l$

$$m^2 = 4l^2$$

$$2n^2 = 4l^2$$

$$n^2 = 2l^2$$

so n is even.

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Short exercise

Proof assumes that if m^2 is even, then m is even.

Prove it!

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Generalizations

Can you prove $\sqrt{3}$ is irrational?

How about $\sqrt[3]{2}$?

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CLASS PROBLEMS

1 & 2