Recitation 7 Solutions

1. **6.4 a.** For rectilinear floor-planning, one possibility is to have a variable for each of the small rectangles, with the value of each variable being a 4-tuple consisting of the $x$ and $y$ coordinates of the upper left and lower right corners of the place where the rectangle will be located. The domain of each variable is the set of 4-tuples that are the right size for the corresponding small rectangle and that fit within the large rectangle. Constraints say that no two rectangles can overlap; for example if the value of variable $R_1$ is $[0, 0, 5, 8]$, then no other variable can take on a value that overlaps with the 0, 0 to 5, 8 rectangle.

   **b.** For class scheduling, one possibility is to have three variables for each class, one with times for values (e.g. MWF8:00, TuTh8:00, MWF9:00, ...), one with classrooms for values (e.g. Wheeler110, Evans330, ...) and one with instructors for values (e.g. Abelson, Bibel, Canny, ...). Constraints say that only one class can be in the same classroom at the same time, and an instructor can only teach one class at a time. There may be other constraints as well (e.g. an instructor should not have two consecutive classes).

   **c.** For Hamiltonian tour, one possibility is to have one variable for each stop on the tour, with binary constraints requiring neighboring cities to be connected by roads, and an AllDiff constraint that all variables have a different value.

2. **6.11** We’ll trace through each iteration of the **while** loop in AC-3 (for one possible ordering of the arcs):

   **a.** Remove $SA - WA$, delete $G$ from $SA$.
   **b.** Remove $SA - V$, delete $R$ from $SA$, leaving only $B$.
   **c.** Remove $NT - WA$, delete $G$ from $NT$.
   **d.** Remove $NT - SA$, delete $B$ from $NT$, leaving only $R$.
   **e.** Remove $NSW - SA$, delete $B$ from $NSW$.
   **f.** Remove $NSW - V$, delete $R$ from $NSW$, leaving only $G$.
   **g.** Remove $Q - NT$, delete $R$ from $Q$.
   **h.** Remove $Q - SA$, delete $B$ from $Q$.
   **i.** remove $Q - NSW$, delete $G$ from $Q$, leaving no domain for $Q$. 


3.  
6.5 The exact steps depend on certain choices you are free to make; here are the ones I made:

a. Choose the \( X_3 \) variable. Its domain is \{0, 1\}.
b. Choose the value 1 for \( X_3 \). (We can’t choose 0; it wouldn’t survive forward checking, because it would force \( F \) to be 0, and the leading digit of the sum must be non-zero.)
c. Choose \( F \), because it has only one remaining value.
d. Choose the value 1 for \( F \).
e. Now \( X_2 \) and \( X_1 \) are tied for minimum remaining values at 2; let’s choose \( X_2 \).
f. Either value survives forward checking, let’s choose 0 for \( X_2 \).
g. Now \( X_1 \) has the minimum remaining values.
h. Again, arbitrarily choose 0 for the value of \( X_1 \).
i. The variable \( O \) must be an even number (because it is the sum of \( T + T \) less than 5 (because \( O + O = R + 10 \times 0 \)). That makes it most constrained.
j. Arbitrarily choose 4 as the value of \( O \).
k. \( R \) now has only 1 remaining value.
l. Choose the value 8 for \( R \).
m. \( T \) now has only 1 remaining value.

n. Choose the value 7 for \( T \).
o. \( U \) must be an even number less than 9; choose \( U \).
p. The only value for \( U \) that survives forward checking is 6.
q. The only variable left is \( W \).
r. The only value left for \( W \) is 3.
s. This is a solution.

This is a rather easy (under-constrained) puzzle, so it is not surprising that we arrive at a solution with no backtracking (given that we are allowed to use forward checking).