7.4 In all cases, the question can be resolved easily by referring to the definition of entailment.

a. $False \models True$ is true because $False$ has no models and hence entails every sentence AND because $True$ is true in all models and hence is entailed by every sentence.

b. $True \models False$ is false.

c. $(A \wedge B) \models (A \Leftrightarrow B)$ is true because the left-hand side has exactly one model that is one of the two models of the right-hand side.

d. $A \Leftrightarrow B \models A \vee B$ is false because one of the models of $A \Leftrightarrow B$ has both A and B false, which does not satisfy $A \vee B$.

e. $A \Leftrightarrow B \models \neg A \vee B$ is true because the RHS is $A \Rightarrow B$, one of the conjuncts in the definition of $A \Leftrightarrow B$.

f. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$ is true because the RHS is false only when both disjuncts are false, i.e., when A and B are true and C is false, in which case the LHS is also false. This may seem counterintuitive, and would not hold if $\Rightarrow$ is interpreted as "causes."

g. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$ is true; proof by truth table enumeration, or by application of distributivity (Fig 7.11).

j. $(A \vee B) \wedge \neg(A \Rightarrow B)$ *is* satisfiable; model has A and $\neg B$.

k. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ *is* satisfiable; RHS is entailed by LHS so models are those of $A \Leftrightarrow B$.

7.20 The CNF representations are as follows:

S1: $(\neg A \vee B \vee E) \wedge (\neg B \vee A) \wedge (\neg E \vee A)$.
S2: $(\neg E \vee D)$.
S3: $(\neg C \vee \neg F \vee \neg B)$.
S4: $(\neg E \vee B)$.
S5: $(\neg B \vee F)$.
S6: $(\neg B \vee C)$.

7.12 To prove the conjunction, it suffices to prove each literal separately. To prove $\neg B$, add the negated goal S7: B.

- Resolve S7 with S5, giving S8: F.
- Resolve S7 with S6, giving S9: C.
- Resolve S8 with S3, giving S10: $(\neg C \vee \neg B)$.
- Resolve S9 with S10, giving S11: $\neg B$.
- Resolve S7 with S11 giving the empty clause.

To prove $\neg A$, add the negated goal S7: A.

- Resolve S7 with the first clause of S1, giving S8: $(B \vee E)$.
- Resolve S8 with S4, giving S9: B.
- Proceed as above to derive the empty clause.

4.
(a) not unsatisfiable
(b) valid
(c) not valid
(d) not unsatisfiable
(e) not unsatisfiable
(f) not valid
(g) unsatisfiable