CS 336/436 – Algorithms for Sensor-based Robotics

Lecture I – Introduction and Course Overview

Erion Plaku

Department of Computer Science
Laboratory for Computational Sensing and Robotics
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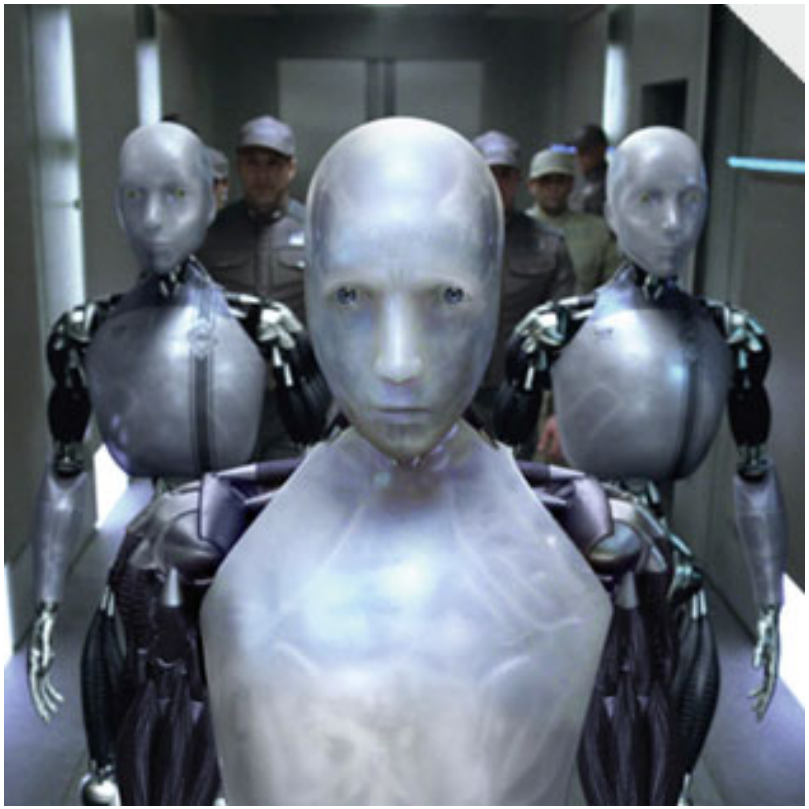
January 26, 2010

What is a Robot?



I, Robot (2004)

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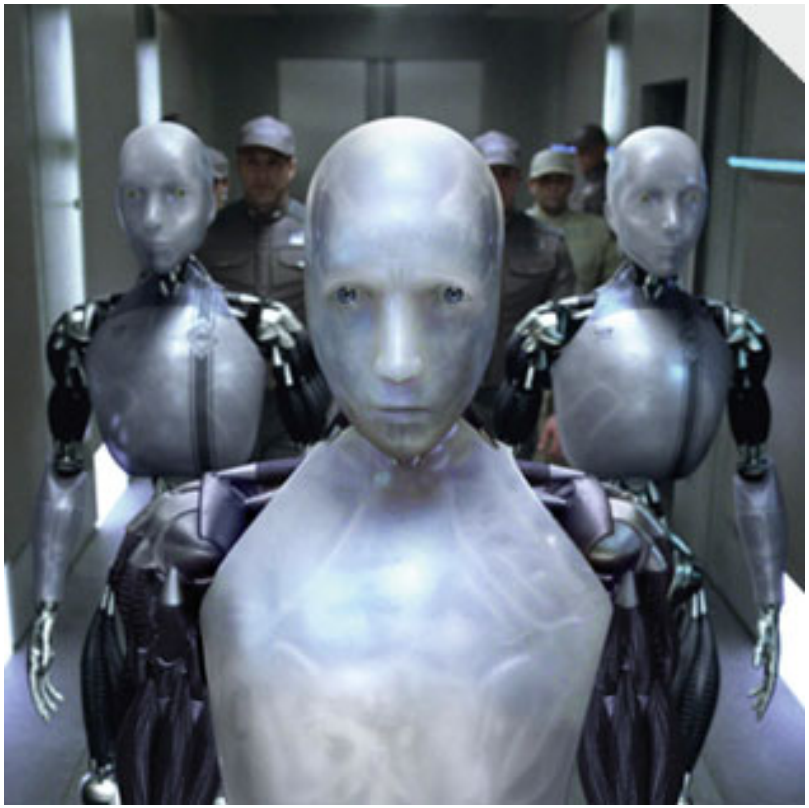

I, Robot (2004)

Automaton (Greek, autos "self" + matos "thinking, animated, willing")

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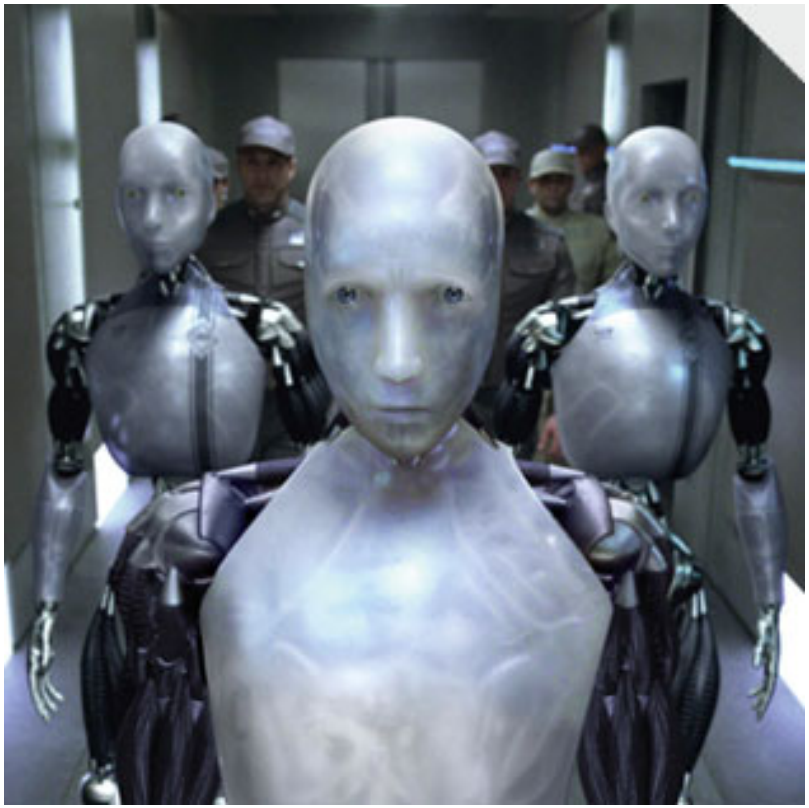
- from English translation of 1920 play "Rossum's Universal Robots" by Karel Capek
- from Czech robotnik (slave), robota (forced labor, drudgery), robotiti (to work, drudge)
- from Slavic (arabeit) related to German Arbeit (work)
- Word coined by Capek's brother Josef, who used it initially in a short story
- Robotics coined in 1941 in a science fiction context by Isaac Asimov

What is a Robot?



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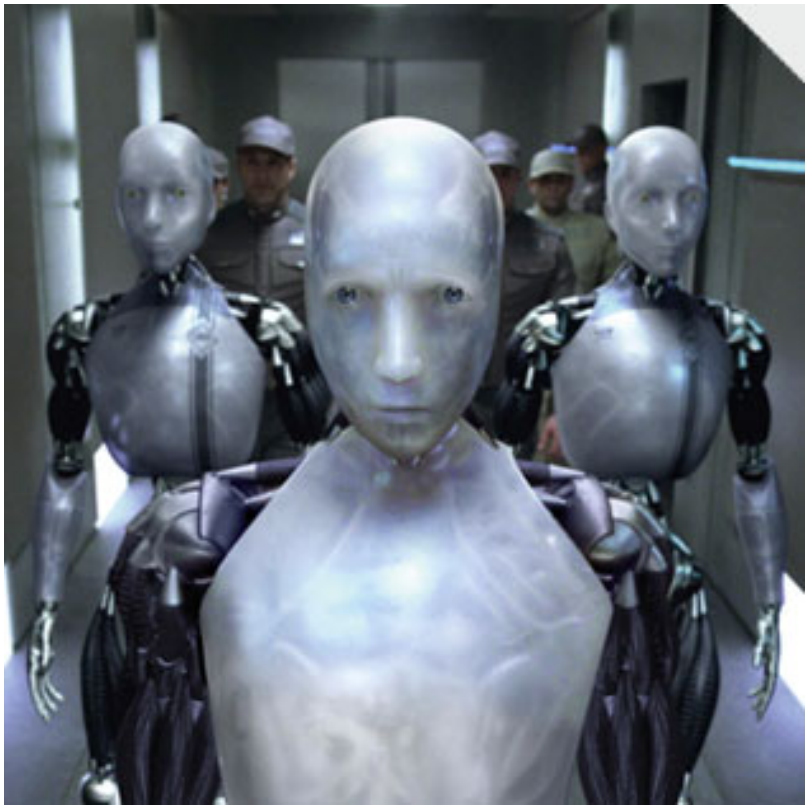

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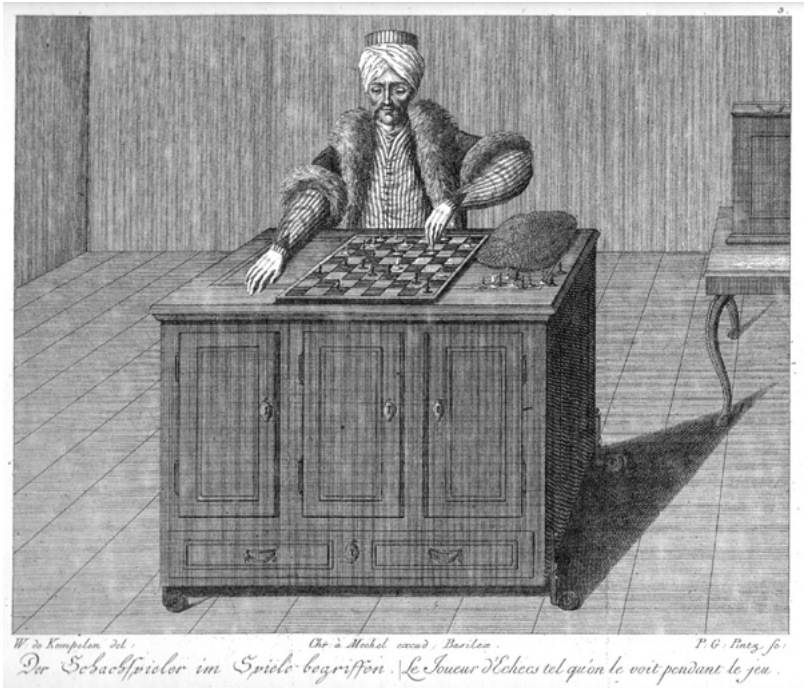
- Robotics Institute of America: re-programmable multi-functional manipulator designed to move materials, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks
- "device that automatically performs complicated often repetitive tasks," or a "mechanism guided by automatic controls"

First Robot – The Turk / Automaton Chess Player (1770)



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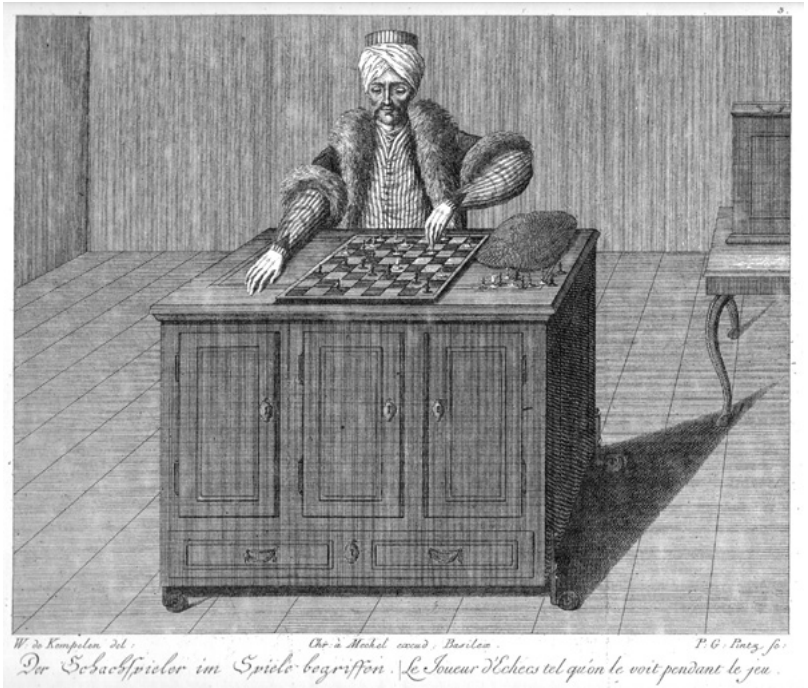

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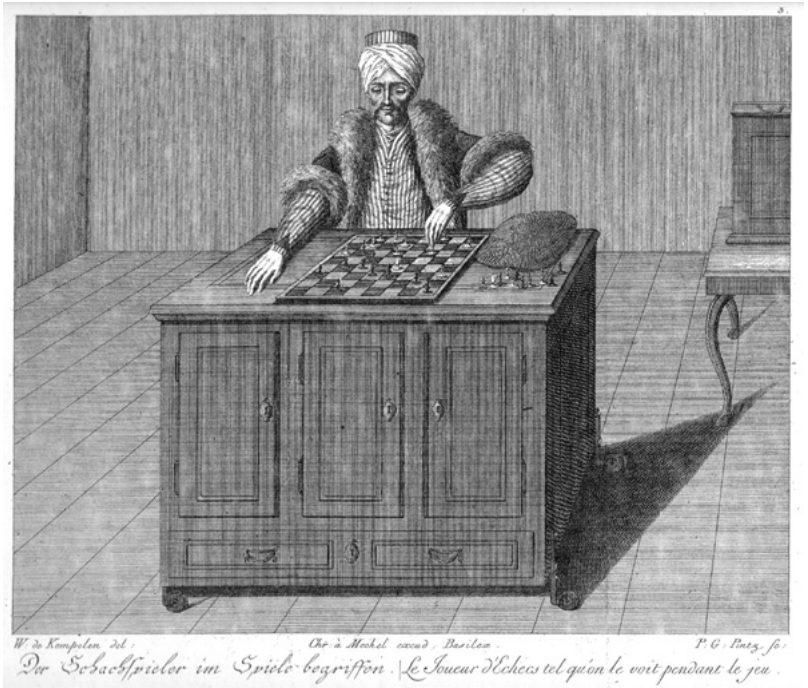
- Constructed by Wolfgang von Kempelen in 1770
- Played many exhibition chess games
- Solved the knight-tour problem
- Even played against Benjamin Franklin in France

First Fake Robot – The Turk / Automaton Chess Player (1770)



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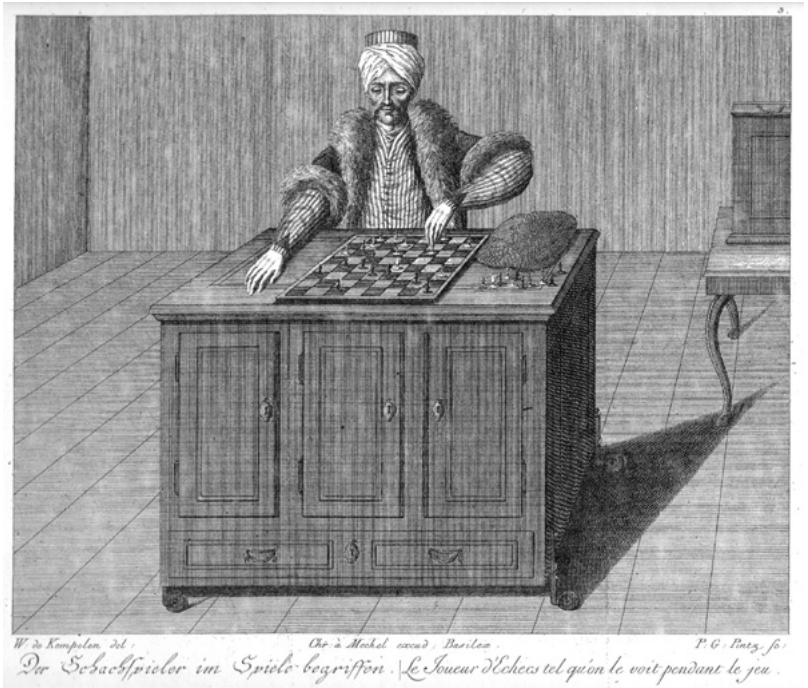

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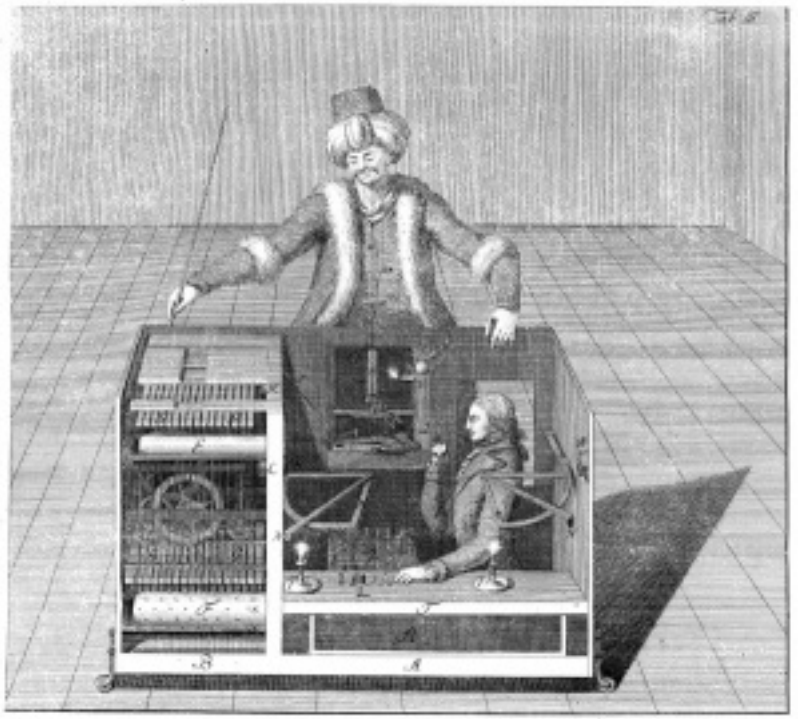
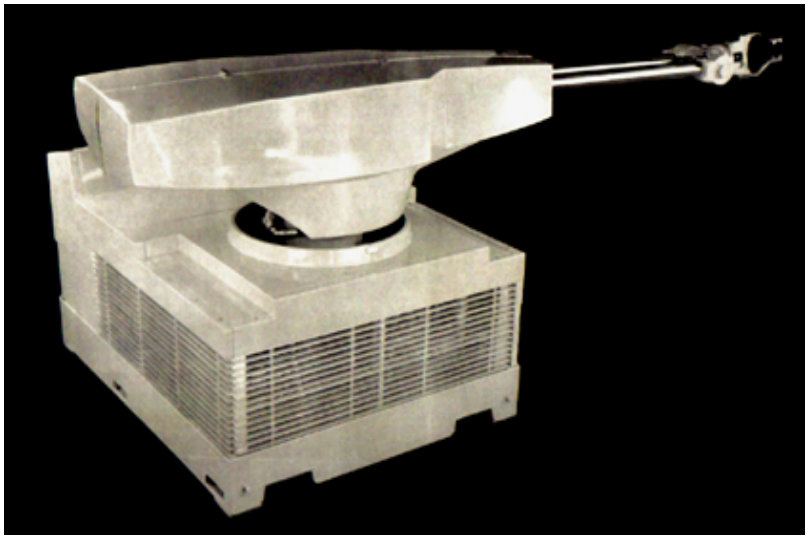


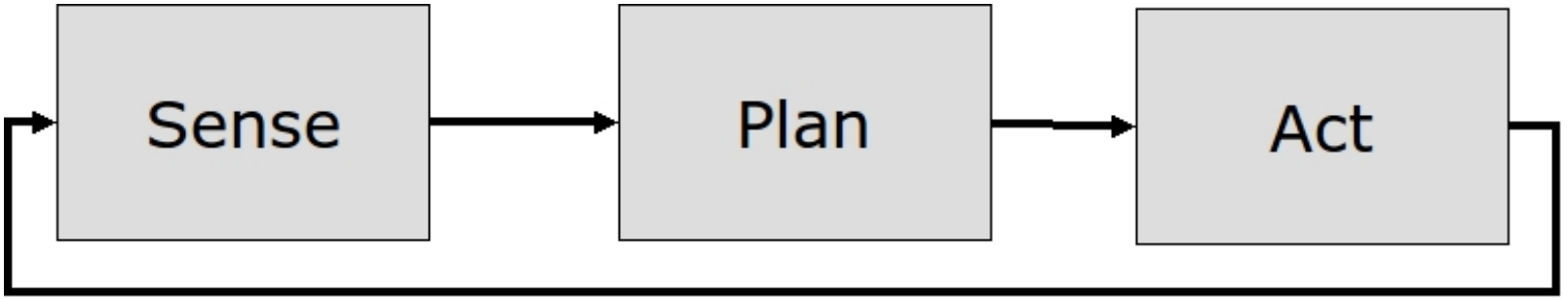
First Real Robot – Unimate (1961)



http://en.wikipedia.org/wiki/Unimate

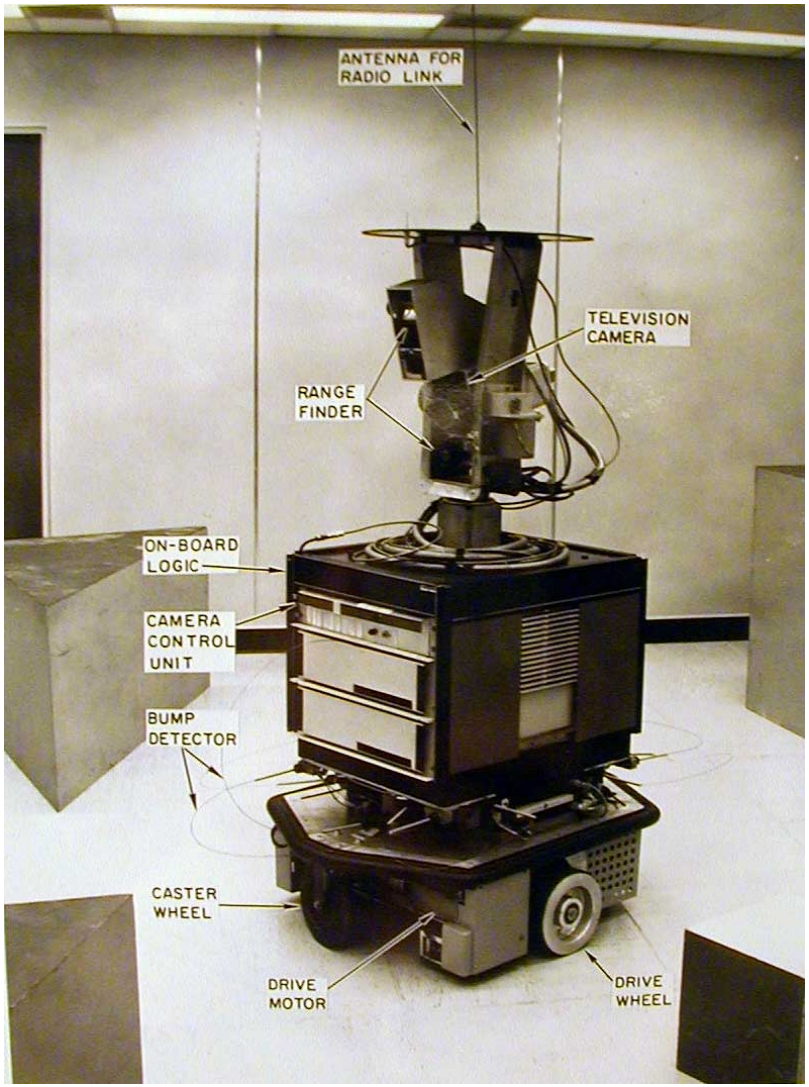
- Created by George Devol
- Worked on a General Motors assembly line in New Jersey in 1961
- Job consisted of transporting die castings from an assembly line and welding these parts on auto bodies
- Conducted in Robot Hall of Fame in 2003

Classical Paradigm



- Focus on automated reasoning and knowledge representation
- Perfect world model
- Closed world assumption: "what is not currently known to be true, is false"
- STRIPS (Stanford Research Institute Problem Solver)

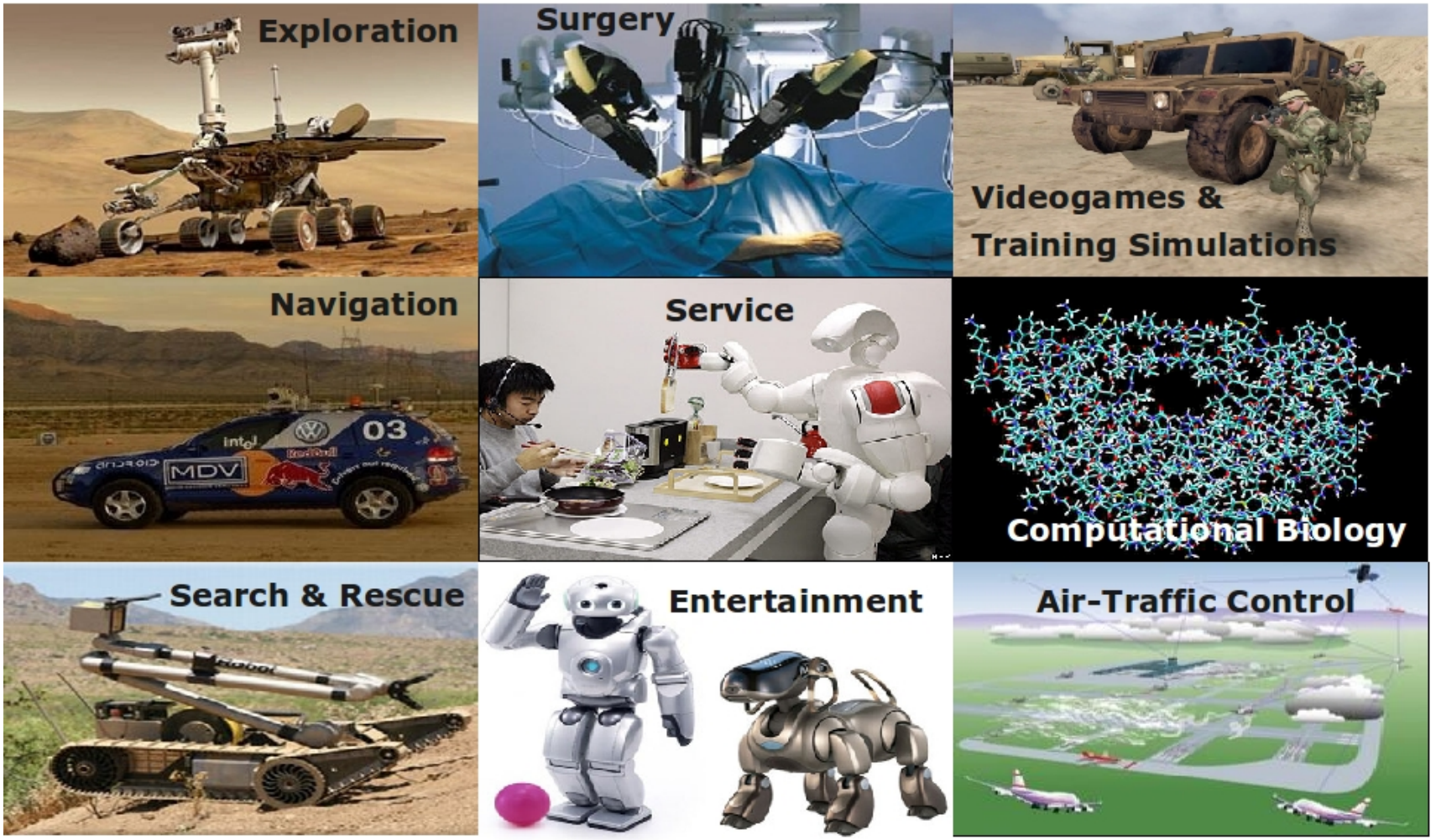
Shakey (Stanford Research Institute, 1966)



http://en.wikipedia.org/wiki/Shakey_the_robot

- First mobile robot to reason about its own actions
- Programs for "seeing," "reasoning," and "acting"
- Triangulating range-finder for sensing obstacles
- Wireless radio and video camera
- Used STRIPS to perform "block-worlds" tasks
- Conducted in Robot Hall of Fame in 2004

Robots Today



[stanley] [bdog] [ldog] [rhex] [heli] [snake] [hand] [asimo]

CS 336/436 – Algorithms for Sensor-based Robotics

Lecture IV – Configuration Space

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February 4, 2010

Path Planning: From Point Robots to Robots with Geometric Shapes

- We have seen path-planning algorithms when a robot is a point
- How can we plan a collision-free path when the robot has a geometric shape?

. . . a key concept in path planning is the notion of a *configuration space*

Configuration (denoted by q)

- a complete specification of the position of every point of the robot

Configuration Space or C-Space (denoted by Q)

- space of all possible configurations of the robot, i.e., $Q = \{q : q \textit{ is a configuration}\}$

Collision-Free Configuration

- q is collision free iff the robot does not collide with any obstacles when in configuration q, i.e., $\texttt{Robot}(q) \cap \left(\bigcup_{i=1} \texttt{Obstacle}_i\right) = \emptyset$

Collision-Free Configuration Space

- $Q_{free} = \{q \in Q : q \textit{ is collision free}\}$

Path-Planning Problem: Compute collision-free path from q_{init} to q_{goal}

- $\texttt{path} : [0, 1] \rightarrow Q_{free}$ is a continuous function with $\texttt{path}(0) = q_{init}$, $\texttt{path}(1) = q_{goal}$

Examples of Configuration Spaces

disk robot with radius r that can translate without rotating in the plane:

- How can the configuration be represented?

  as the two-dimensional position (c_x, c_y) of the robot's center

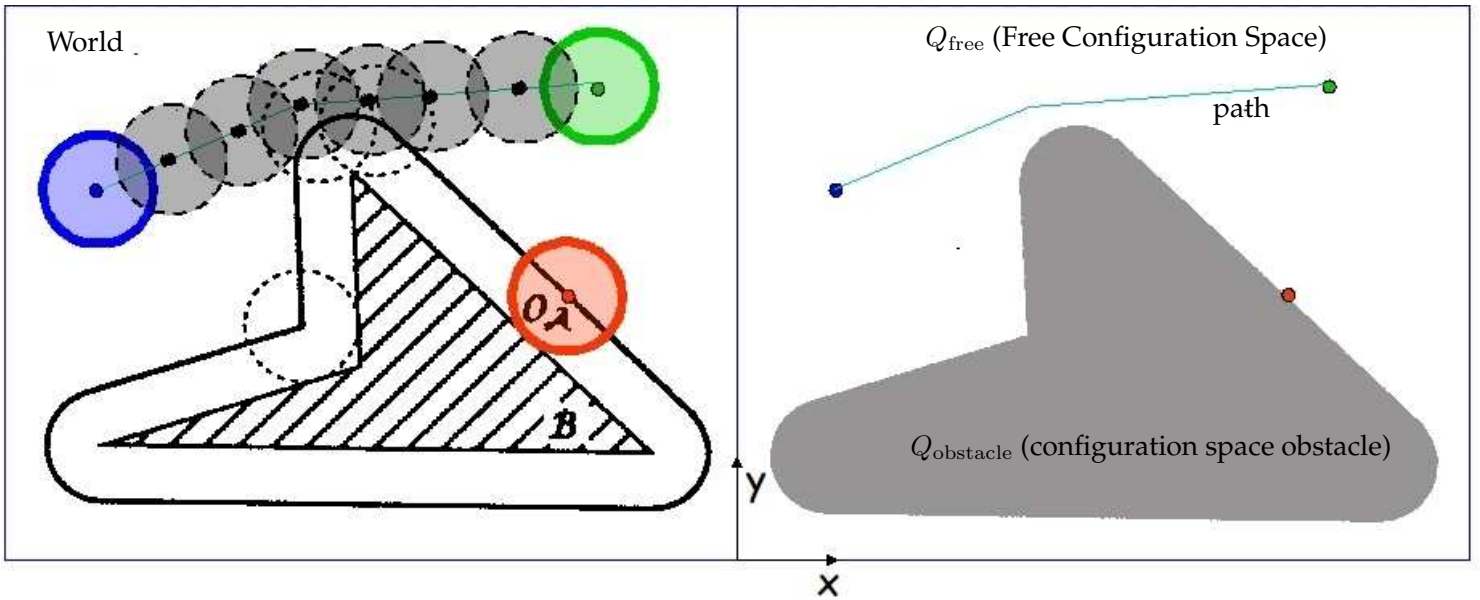
- How can the points on the robot be expressed as a function of its configuration?

$$\texttt{Robot}(c_x, c_y) = \{(x, y) : (x - c_x)^2 + (y - c_y) \leq r^2\}$$

- What is the configuration space Q?

$$Q = \mathbb{R}^2 \text{ (same as that of a point robot)}$$

- What is the free configuration space Q_{free}? Is it the same as that of a point robot?



[Fig. courtesy of Latombe]

- How would you compute Q_{free}?

Examples of Configuration Spaces

polygon P that can translate and rotate in the plane:

- How can the configuration be represented?

  (c_x, c_y, θ): position + orientation

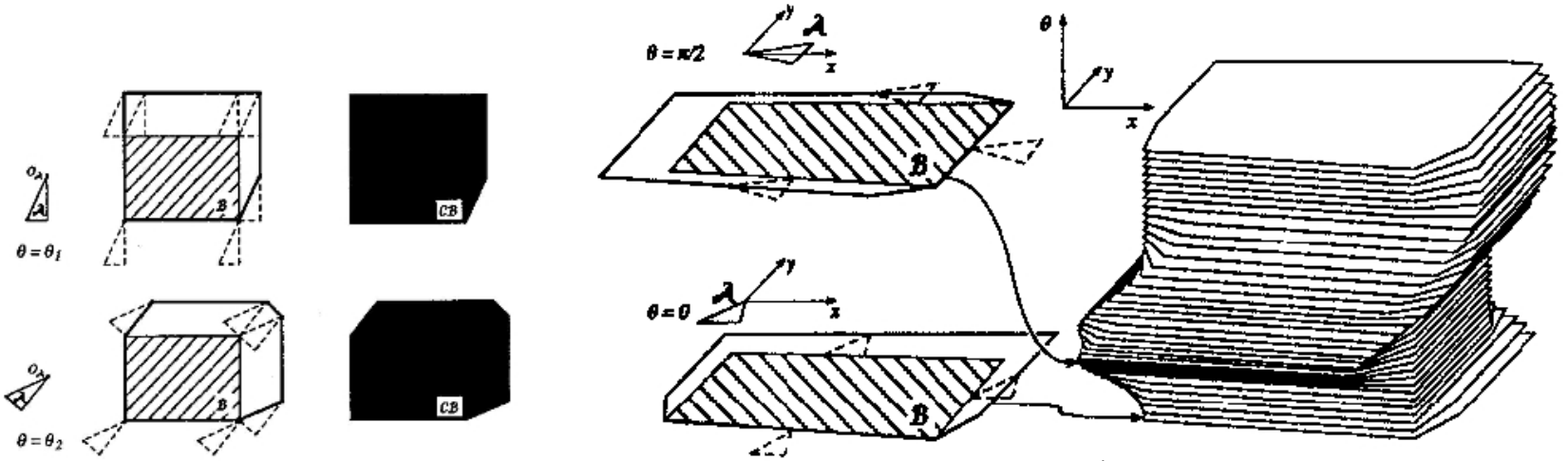
- How can the points on the robot be expressed as a function of its configuration?

$$\texttt{Robot}(c_x, c_y, \theta) = \left\{ \begin{pmatrix} \cos\theta & -\sin\theta & c_x \\ \sin\theta & \cos\theta & c_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} : (x, y) \in P \right\}$$

- What is the configuration space Q?

  $Q = \mathbb{R}^2 \times S^1$ (S^1 refers to the unit circle)

- What is the free configuration space Q_{free}?



[Fig. courtesy of Latombe]

- How would you compute Q_{free}?

Examples of Configuration Spaces

manipulator with revolute joints:

- How can the configuration be represented?

  $(\theta_1, \theta_2, \ldots, \theta_n)$: vector of joint values

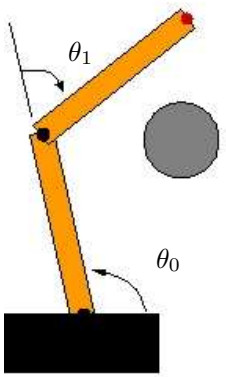
- How can the points on the robot be expressed as a function of its configuration?

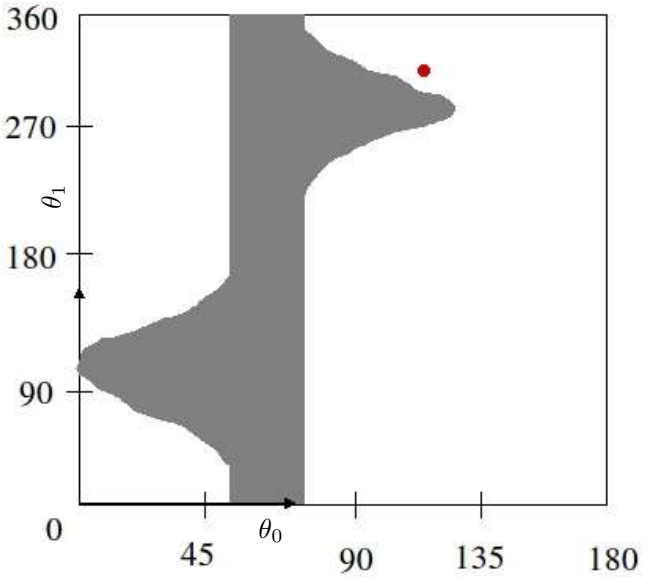
  forward kinematics (more later in the course)

- What is the configuration space Q?

  $Q = \overbrace{S^1 \times S^1 \ldots \times S^1}^{n}$ (S^1 refers to the unit circle)

- What is the free configuration space Q_{free}?





[Fig. courtesy of Hager]

- How would you compute Q_{free}?

Minkowski Sums

- The Minkowski sum of two sets A and B, denoted by $A \oplus B$, is defined as

$$A \oplus B = \{a + b : a \in A, b \in B\}$$

- The Minkowski difference of two sets A and B, denoted by $A \ominus B$, is defined as

$$A \ominus B = \{a - b : a \in A, b \in B\}$$

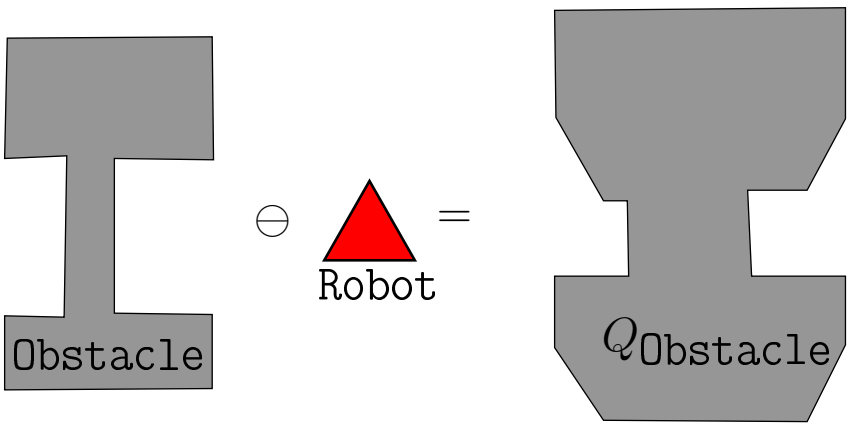
How does it relate to path planning?

- Recall the definition of the configuration-space obstacle

$$Q_{\texttt{Obstacle}} = \{q : q \in Q \textit{ and } \texttt{Robot}(q) \cap \texttt{Obstacle} \neq \emptyset\}$$

  (set of all robot configurations that collide with the obstacle)
- Classical result shown by Lozano-Perez and Wesley 1979

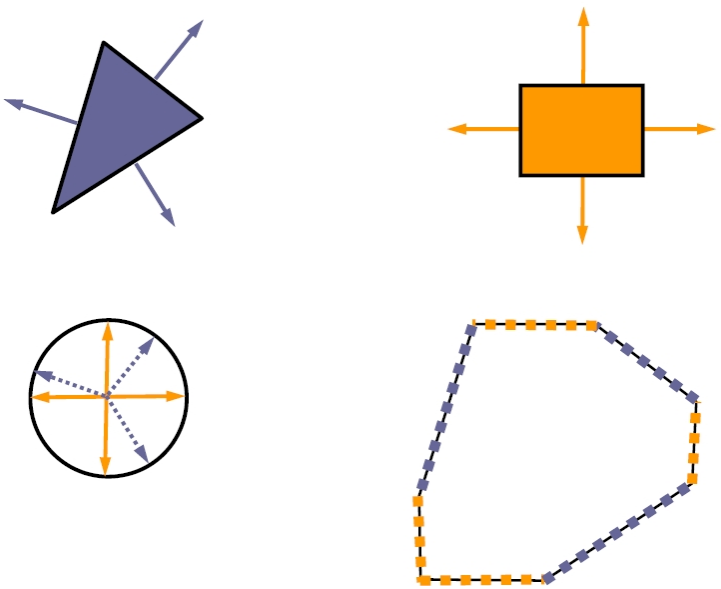
for polygons and polyhedra : $Q_{\texttt{Obstacle}} = \texttt{Obstacle} \ominus \texttt{Robot}$



[Fig. courtesy of Manocha]

Properties of Minkowski Sums

- Minkowski sum of two convex sets is convex
- Minkowski sum of two convex polygons A and B with m and n vertices ...
  - ... is a convex polygon with $m + n$ vertices
  - ... vertices of $A \oplus B$ are "sums" of vertices of A and B
  - ... $A \oplus B$ can be computed in linear time and space $O(n + m)$



[Fig. courtesy of Manocha]

Algorithm

- sort edges according to angle between x-axis and edge normal
- let the sorted edges be $e_1, e_2, \ldots, e_{n+m}$
- attach edges one after the other so that edge e_{i+1} starts where edge e_i ends

- Minkowski sum for nonconvex polygons
  - Decompose into convex polygons (e.g., triangles, trapezoids)
  - Compute the minkowski sums of the convex polygons and take their union
  - Complexity: $O(n^2 m^2)$ (4-th order polynomial)
- 3D Minkowski sums: [convex: $O(nm)$ complexity] [nonconvex: $O(n^3 m^3)$ complexity]

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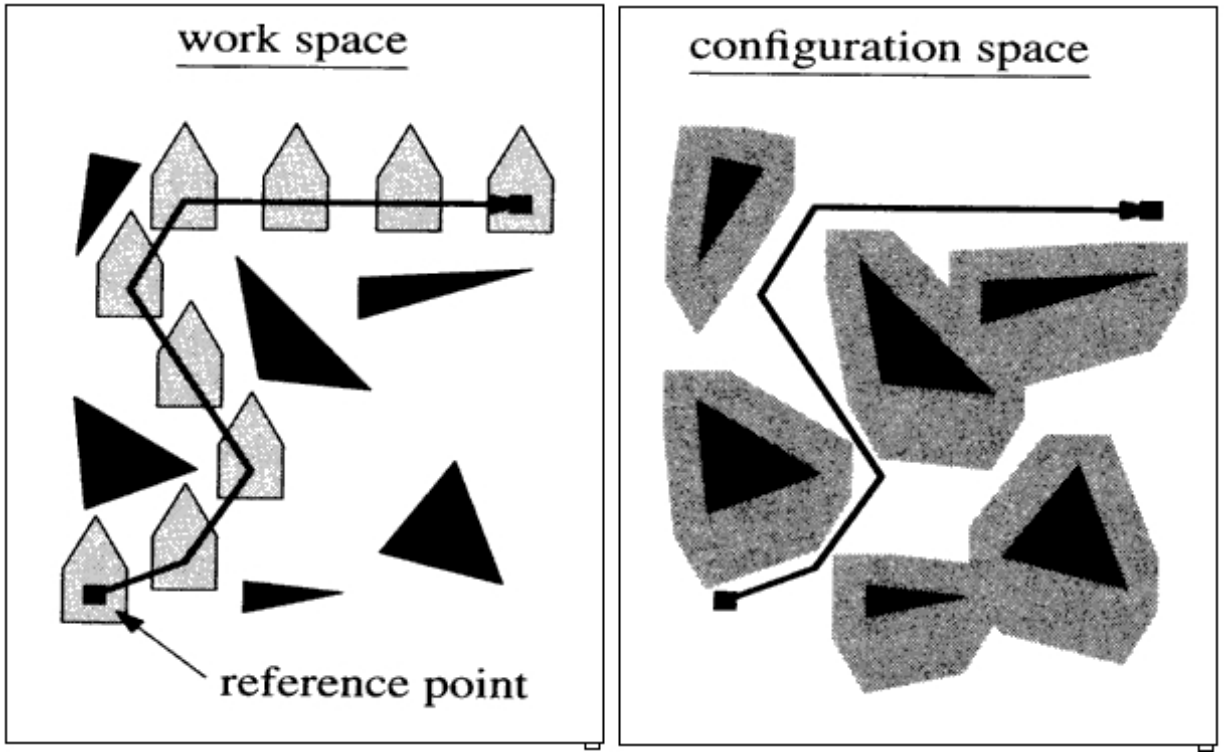

- reduce robot to a point in the configuration space
- compute configuration-space obstacles (difficult to do in general)
- search for a path for the point robot in the free configuration space

Topology of Configuration Spaces

Why study it?

- Extend results from one configuration space to another
- Design specialized algorithms that take advantage of certain topologies

What about the topology?

- Topology is the "intrinsic character" of a space
- Two spaces have different topologies if cutting and pasting is required to make them the same (think of rubber figures — if we can stretch and reshape "continuously" without tearing, one into the other, they have the same topology)
- Mathematical mechanisms for talking about topology: homeomorphism/diffeomorphism

  $f : X \to Y$ is called a homeomorphism iff
  - f is a bijection (one-to-one and onto)
  - f is continuous
  - f^{-1} (the inverse of f) is continuous

  examples of homeomorphisms: [disc to square]; [$(-1, 1)$ to $\mathbb{R}$]

  X is diffeomorphic to Y iff exists $f : X \to Y$ such that
  - f is a homeomorphism where f and f^{-1} are smooth (derivatives of all orders exist)

  example of diffeomorphism: circle to ellipse
- An n-dimensional configuration space Q is a *manifold* if it locally looks like $\mathbb{R}^n$, i.e., every $q \in Q$ has a neighborhood homeomorphic to $\mathbb{R}^n$
- A manifold is path-connected if there is a path between any two points

CS 336/436 – Algorithms for Sensor-based Robotics

Sampling-based Motion Planning - Parts I-IV

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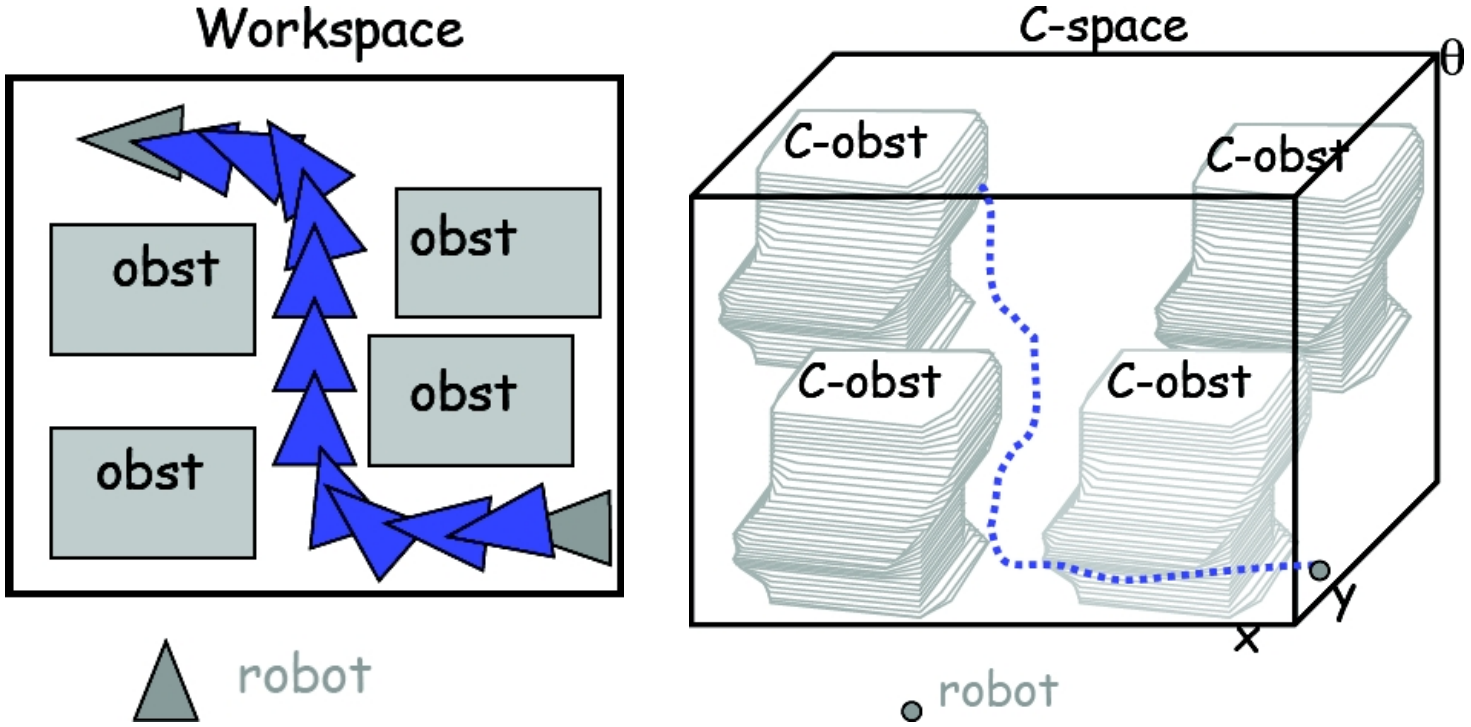
February 25, 2010 March 4, 2010 March 9, 2010 March 11, 2010

From Workspace to Configuration Space

- simple workspace obstacle transformed into complex configuration-space obstacle
- robot transformed into point in configuration space
- path transformed from swept volume to 1d curve



[fig from Jyh-Ming Lien]

Explicit Construction of Configuration Space/Roadmaps

- PSPACE-complete
- Exponential dependency on dimension
- No practical algorithms

Motion Planning for a Point Robot in 2D

- Robotic system: Single point
- Task: Compute collision-free path from initial to goal position

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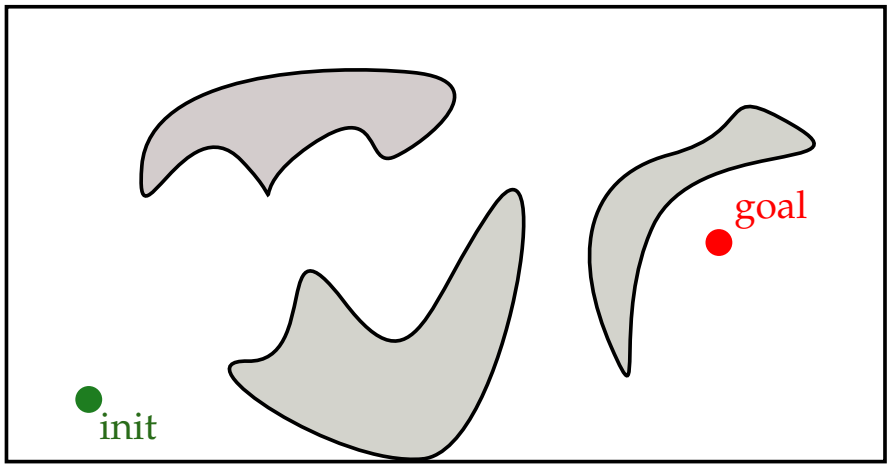
How would you solve it?



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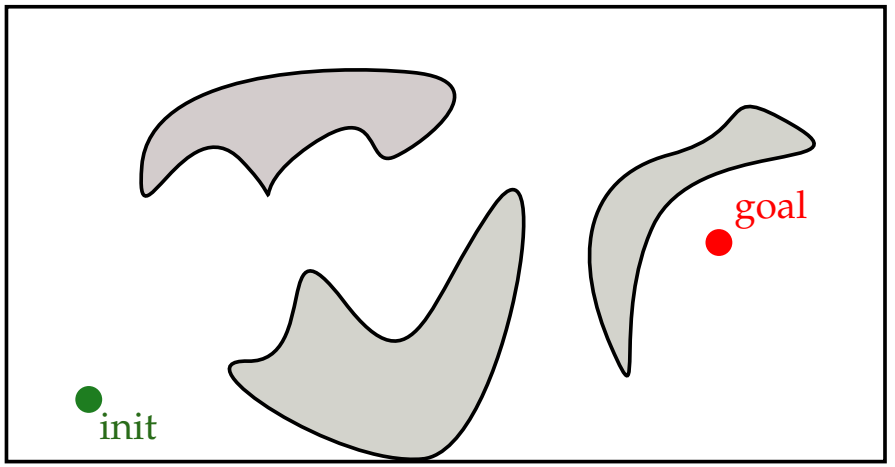

Hint: How would you approximate π?

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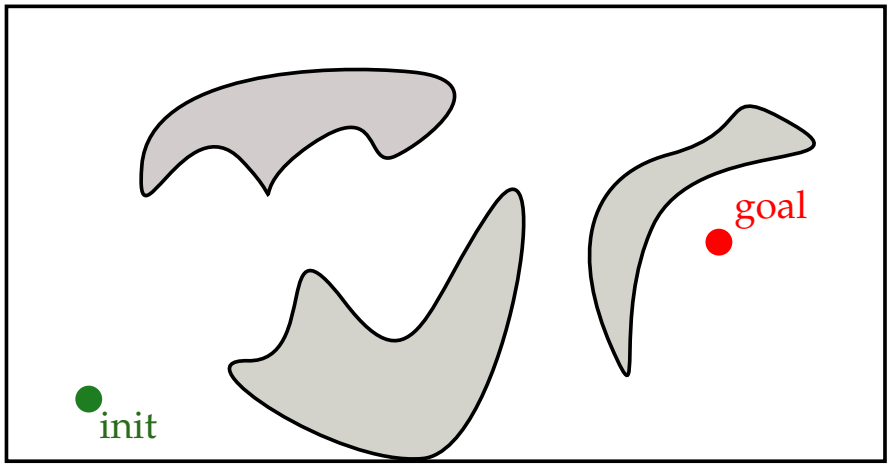

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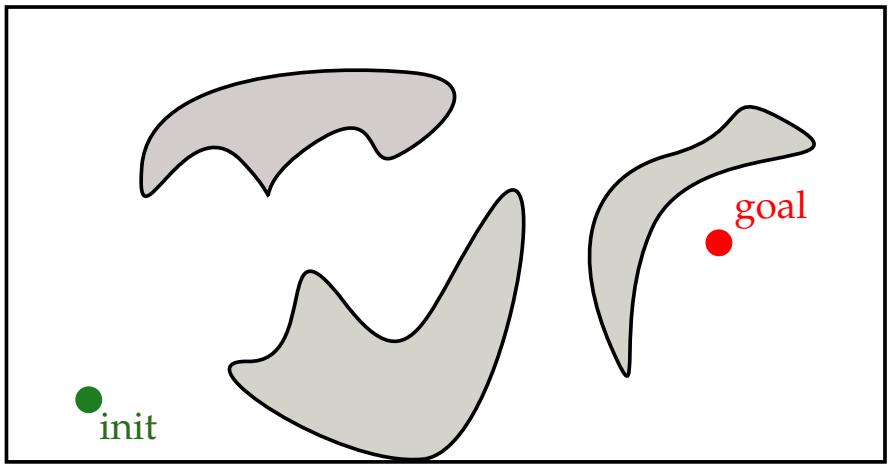

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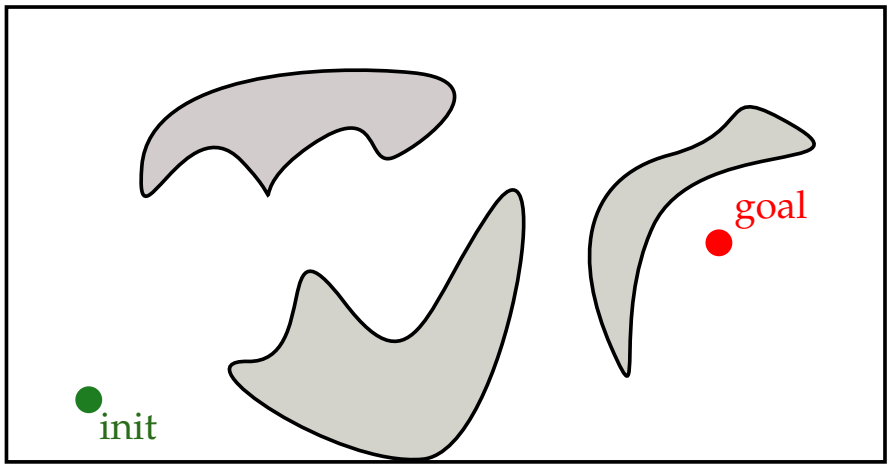

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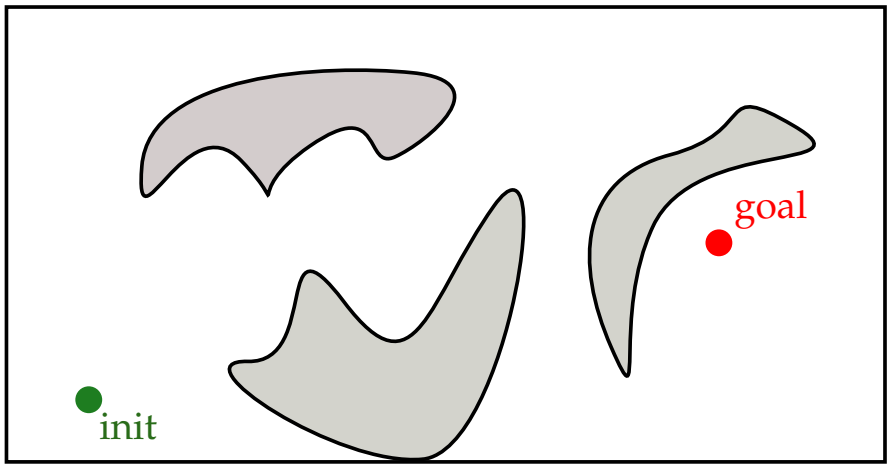

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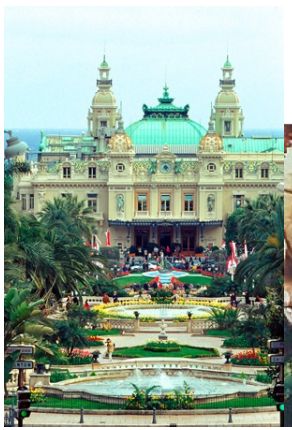


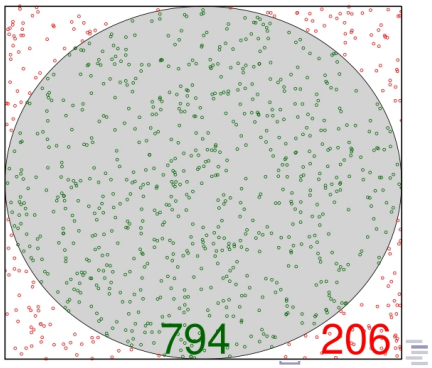

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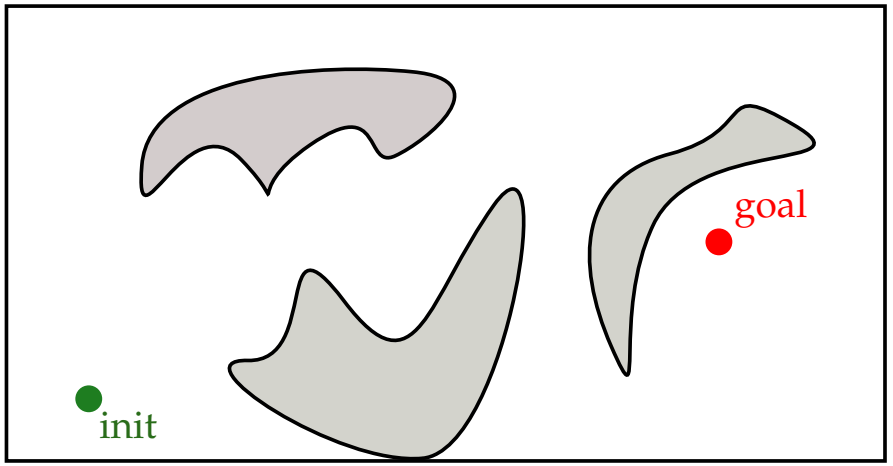

Monte-Carlo Idea:

- Define input space
- Generate inputs at random by *sampling* the input space
- Perform a deterministic computation using the input samples
- Aggregate the partial results into final result

Sampling-based Motion Planning

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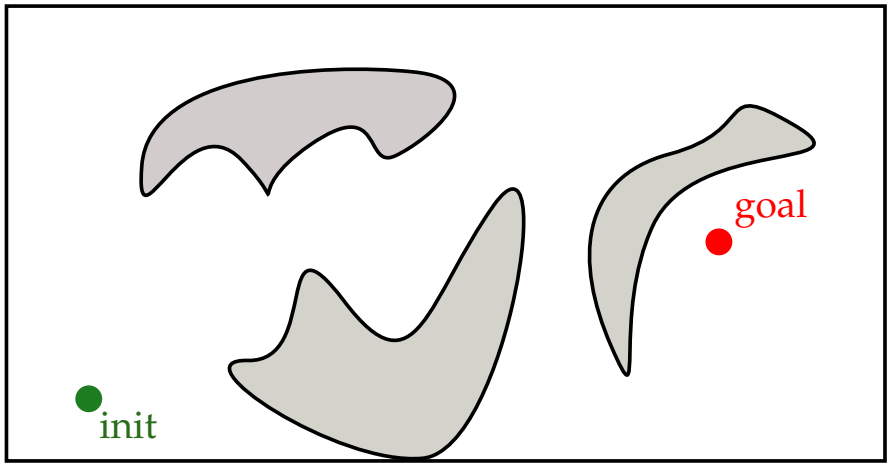

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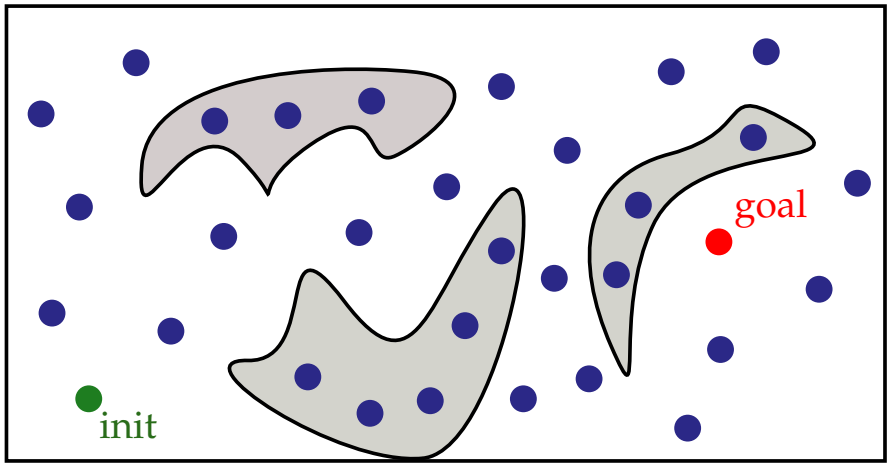

- Sample points

Sampling-based Motion Planning

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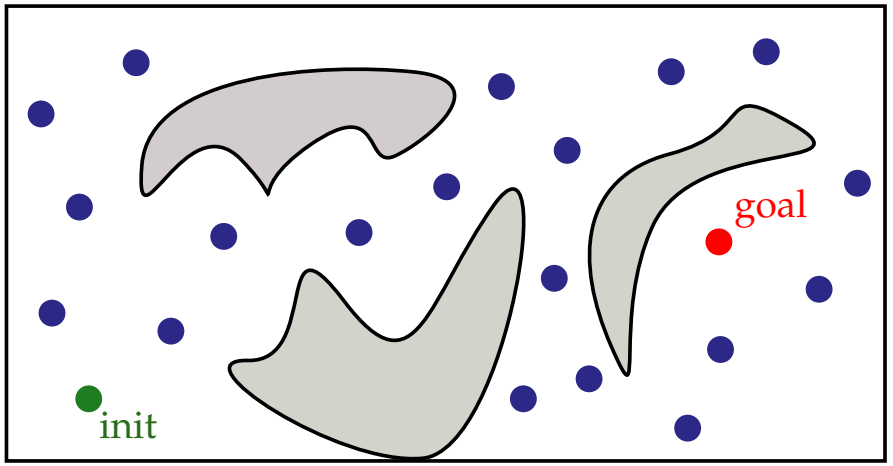

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Sampling-based Motion Planning

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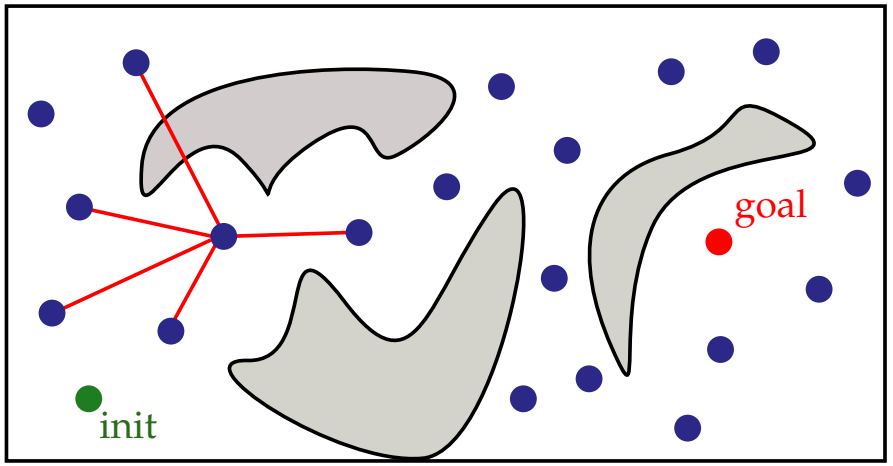

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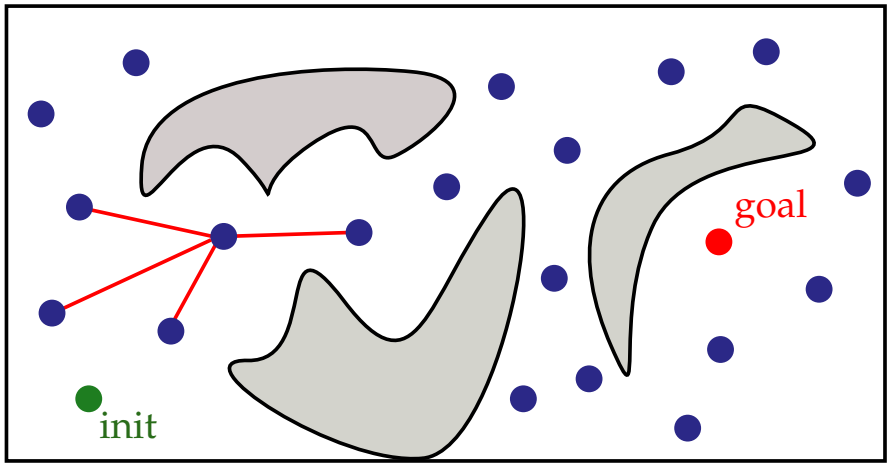

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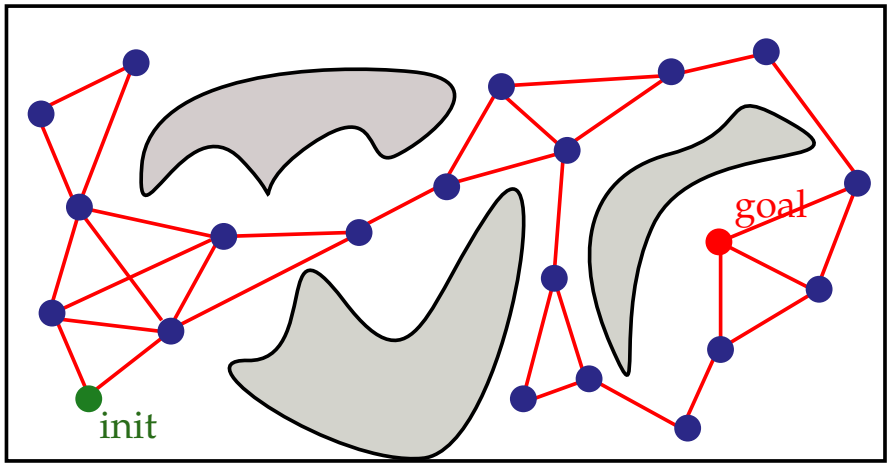

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$\Rightarrow$ Gives rise to a graph, called the *roadmap*

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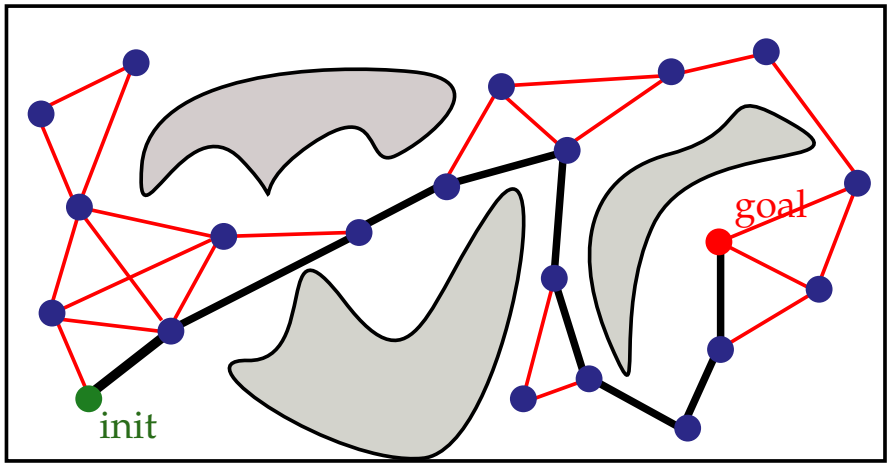

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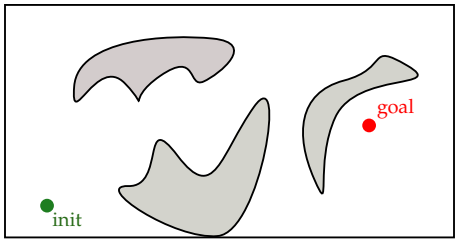
$\Rightarrow$ Collision-free path can be found by performing graph search on the roadmap

Probabilistic RoadMap (PRM) Method

[Kavraki, Švestka, Latombe, Overmars 1996]

0. Initialization

add q_{init} and q_{goal} to roadmap vertex set V



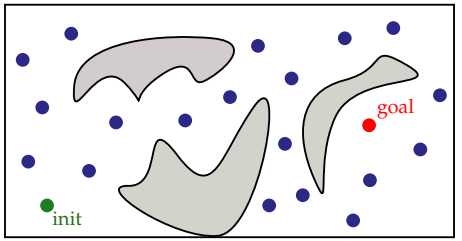
1. Sampling

repeat several times

$q \leftarrow \textsc{Sample}()$

if $\textsc{IsCollisionFree}(q) = \texttt{true}$

add q to roadmap vertex set V



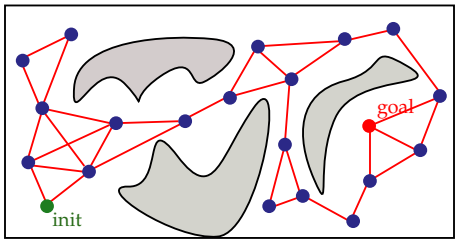
2. Connect Samples

for each pair of neighboring samples $(q_a, q_b) \in V \times V$

$\text{path} \leftarrow \textsc{GenerateLocalPath}(q_a, q_b)$

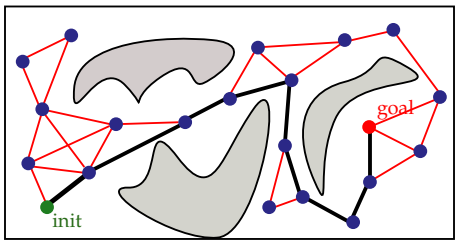
if $\textsc{IsCollisionFree}(\text{path}) = \texttt{true}$

add (q_a, q_b) to roadmap edge set E



3. Graph Search

search graph (V, E) for path from q_{init} to q_{goal}



Sampling-based Path Planning

Advantages

- Computationally efficient
- Solves high-dimensional problems (with hundreds of DOFs)
- Easy to implement
- Applications in many different areas

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It offers *probabilistic completeness*

- When a solution exists, a probabilistically complete planner finds a solution with probability as time goes to infinity.
- When a solution does not exists, a probabilistically complete planner may not be able to determine that a solution does not exist.

PRM Applied to 2D-point Robot

$q = (x, y) \leftarrow \text{SAMPLE}()$

- $x \leftarrow \text{RAND}(\min_x, \max_x)$
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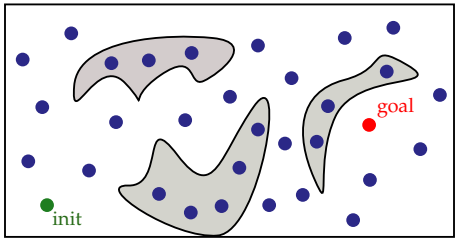

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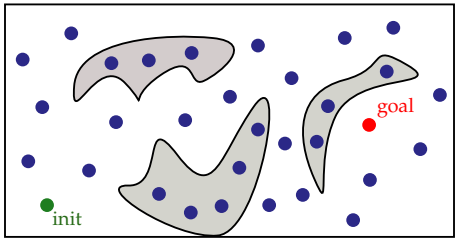
ISSAMPLECOLLISIONFREE(q)
- Point inside/outside polygon test



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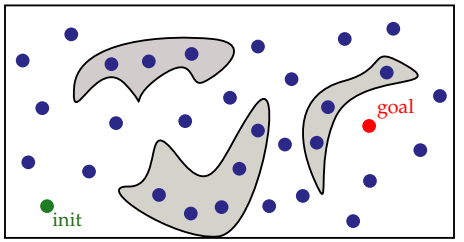

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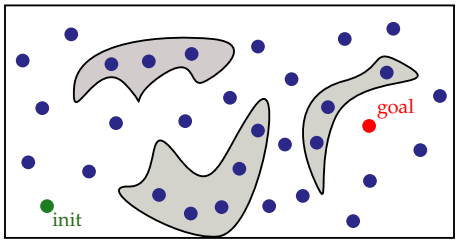

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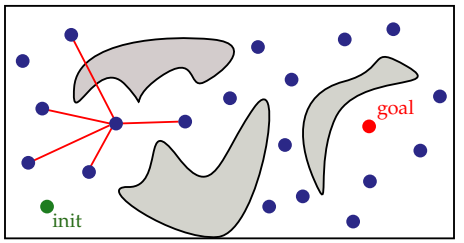

PRM Applied to 2D Rigid-Body Robot

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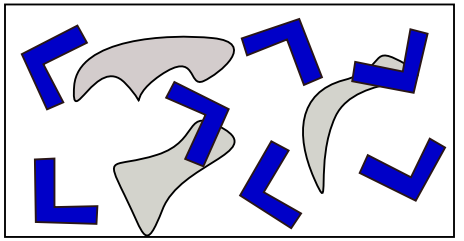

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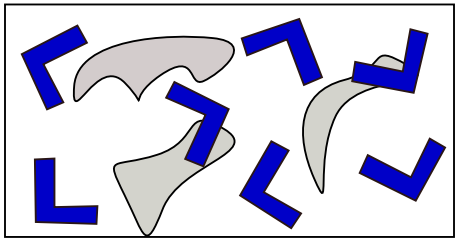

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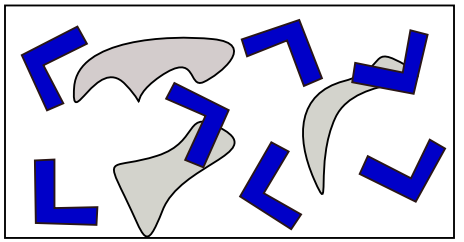

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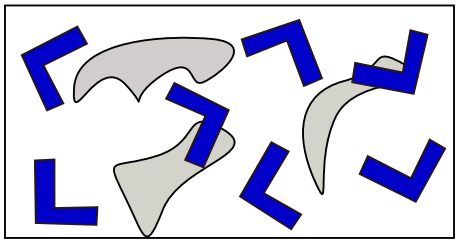

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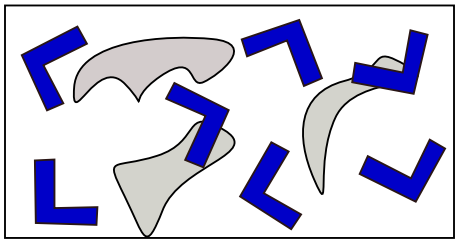

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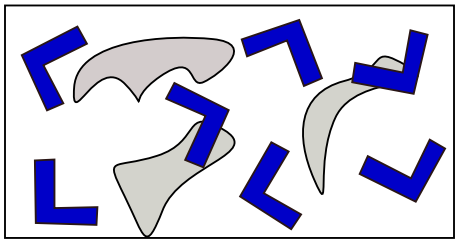

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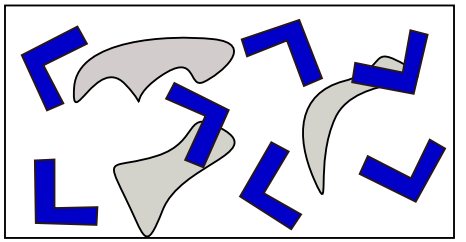

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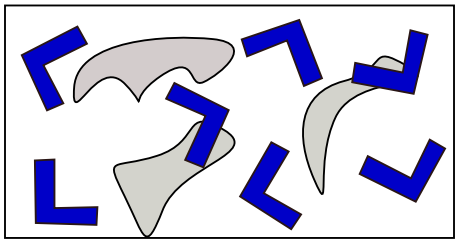

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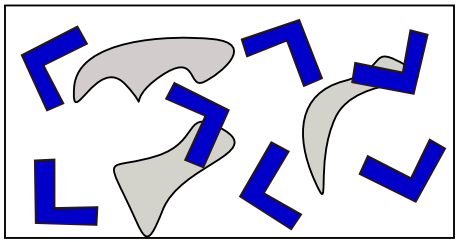

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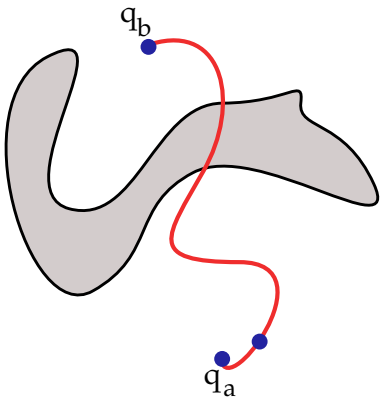

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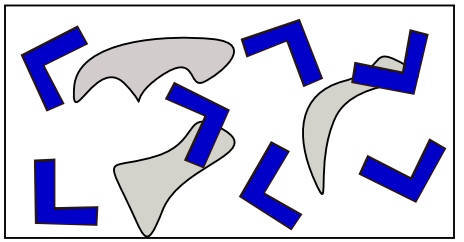

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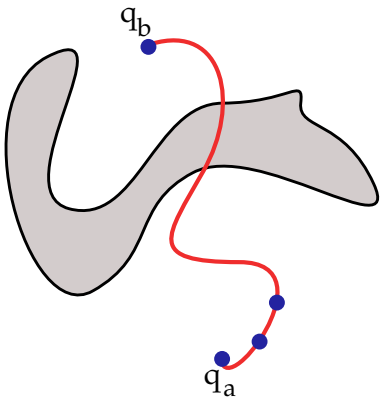

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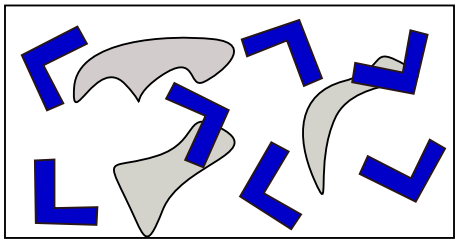

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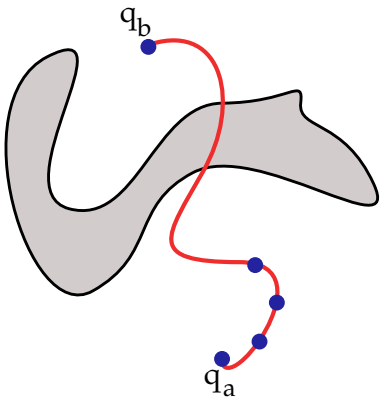

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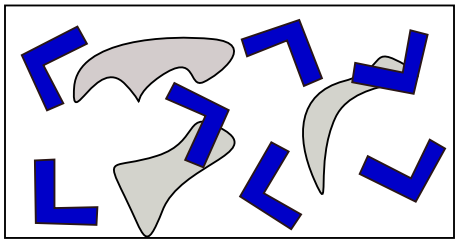

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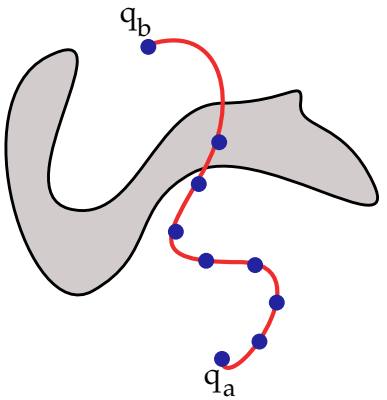

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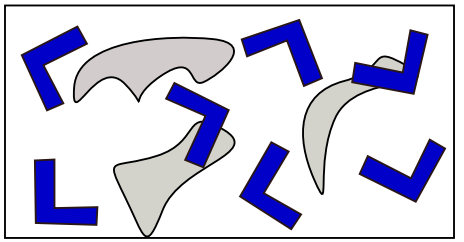
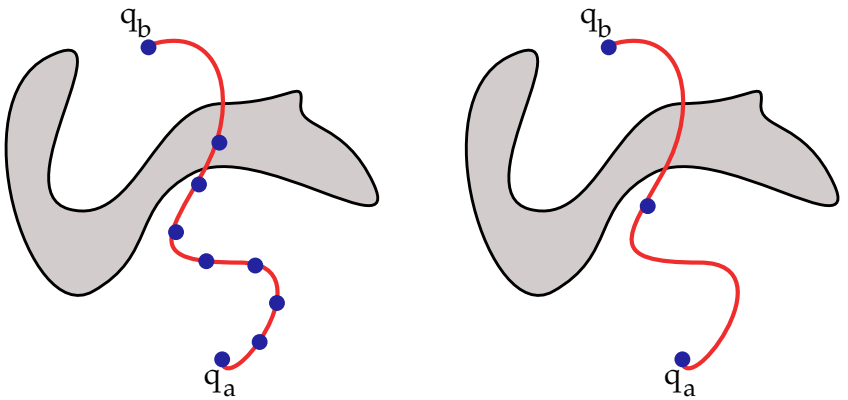


IsSampleCollisionFree(q)
- Place rigid body in position and orientation specified by q
- Polygon-polygon intersection test

path $\leftarrow$ GenerateLocalPath(q_a, q_b)
- Continuous function parameterized by time: $\text{path} : [0, 1] \rightarrow Q$
- Starts at q_a and ends at q_b: $\text{path}(0) = q_a, \text{path}(1) = q_b$
- Many possible ways of defining it, e.g., by linear interpolation

$$\text{path}(t) = (1 - t) * q_a + t * q_b$$

IsPathCollisionFree(path)
- Incremental approach
- Subdivision approach



PRM Applied to 2D Rigid-Body Robot

$q = (x, y, \theta) \leftarrow \text{SAMPLE}()$
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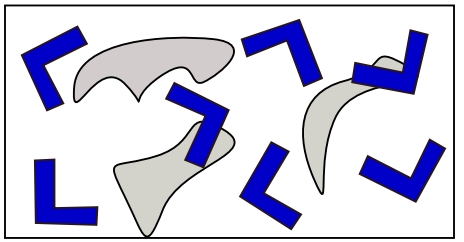

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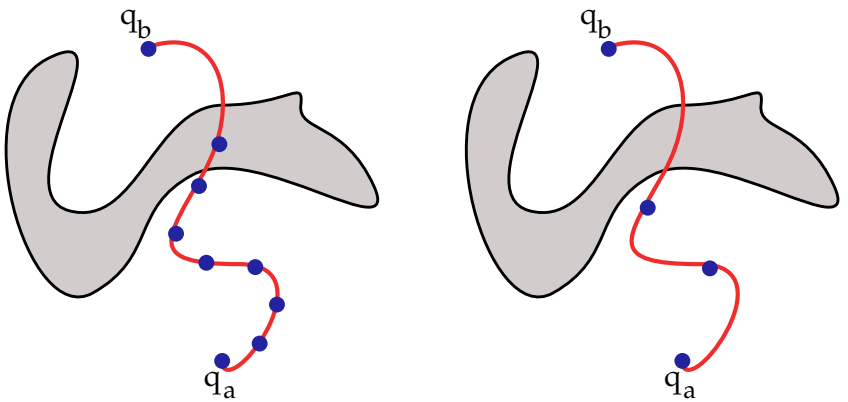

PRM Applied to Articulated Chain

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$q = (\theta_1, \theta_2, \ldots, \theta_n) \leftarrow \textsc{Sample}()$
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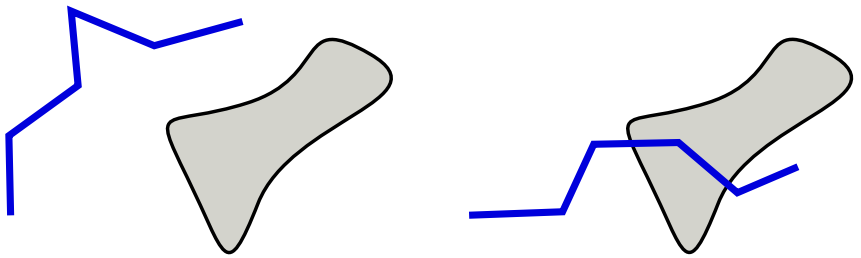

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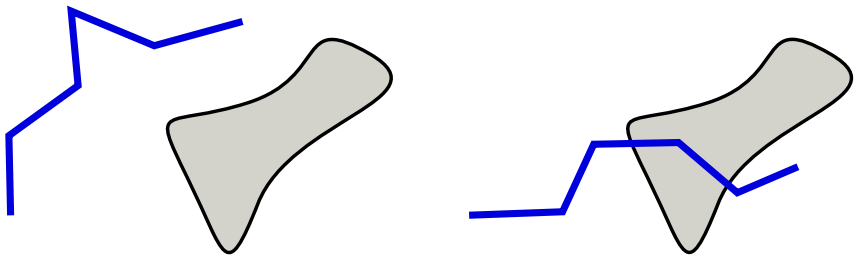

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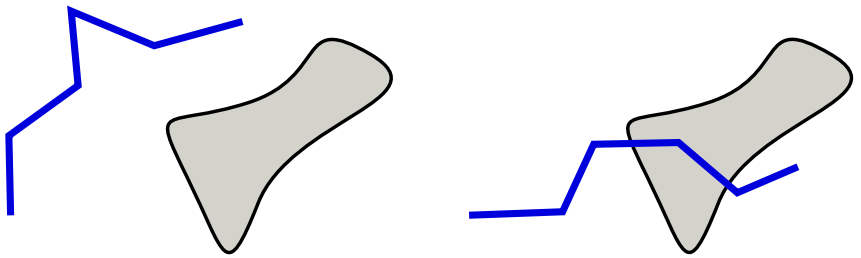

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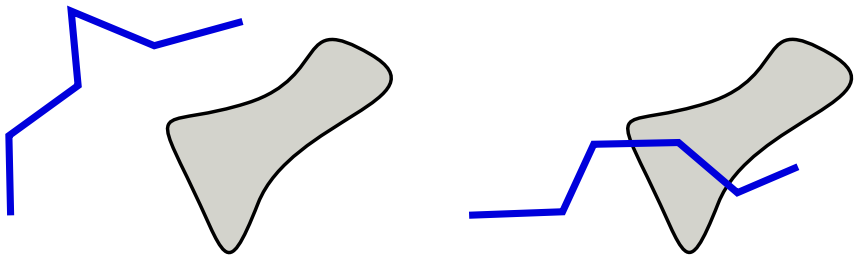

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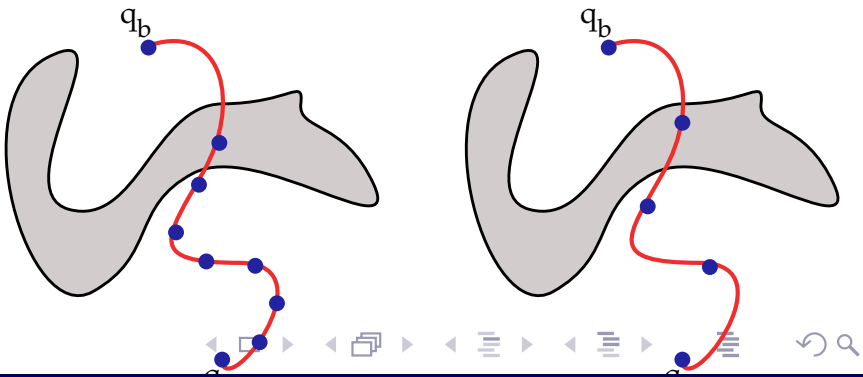

[everest] [skeleton] [knot] [manip]

Path Smoothing

- Solution paths produced by PRM planners tend to be long and non-smooth (due to sampling and edge connections)
- Post processing is commonly used to improve the quality of the paths
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SmoothPath($q_1, q_2, \ldots, q_n$) – one version

1: **for** several times **do**
2: select i and j uniformly at random from $1, 2, \ldots, n$
3: attempt to directly connect q_i to q_j
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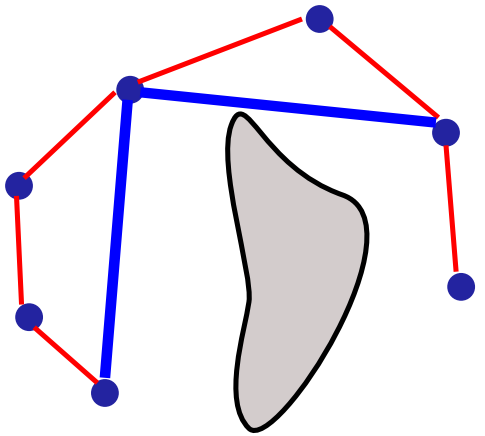

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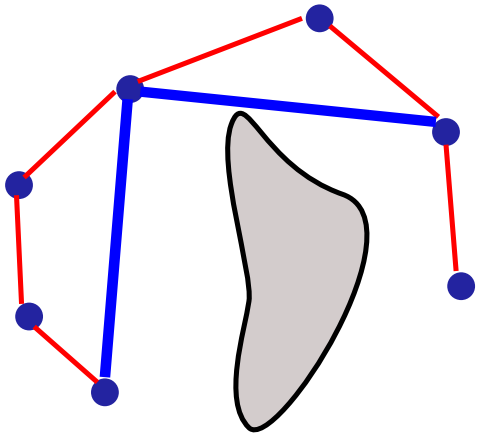

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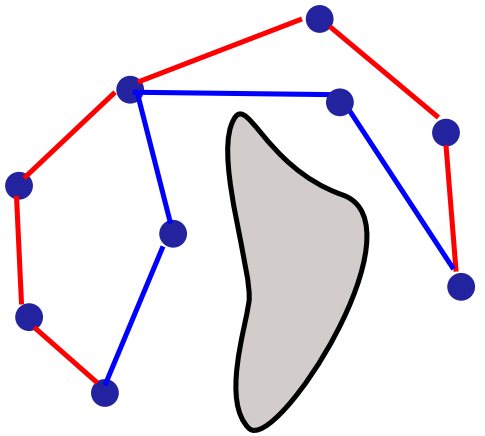
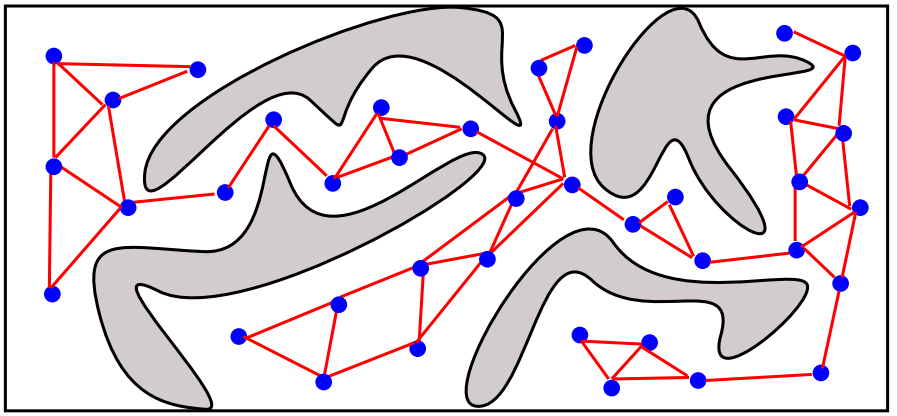
3: $q \leftarrow$ generate collision-free sample
4: attempt to connect q_i to q_j through q
5: if successful, replace the in-between nodes $q_{i+1}, \ldots, q_j$ by q



Motivation: Tree-based Motion Planning

- PRM-based planners aim to construct a roadmap that captures the whole connectivity of the configuration space



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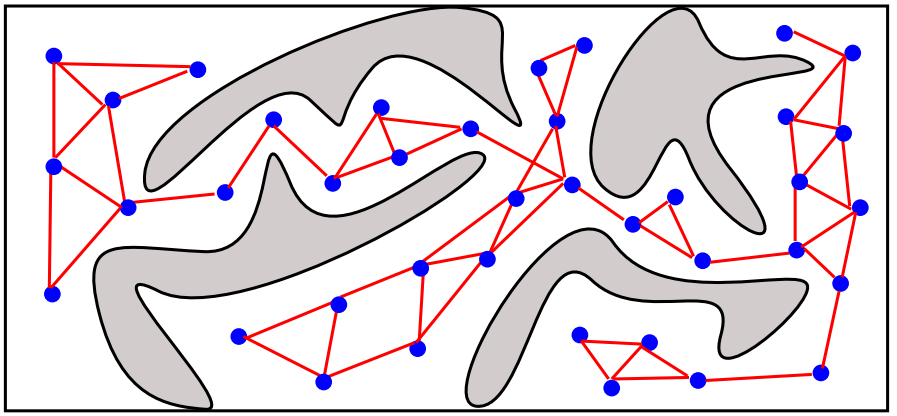

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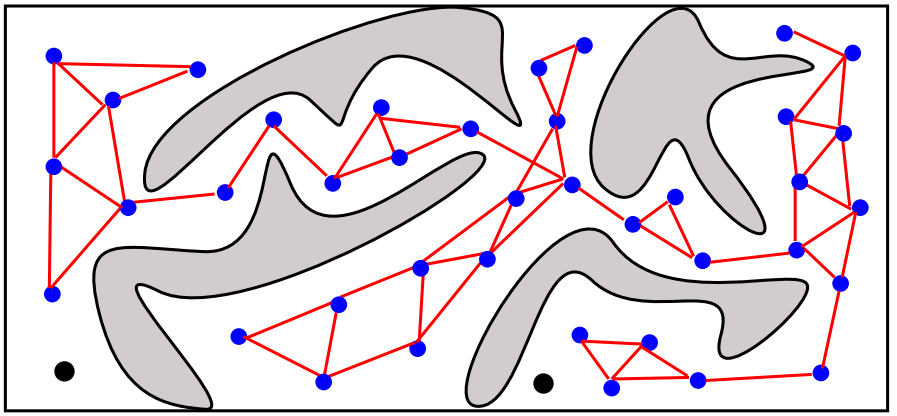

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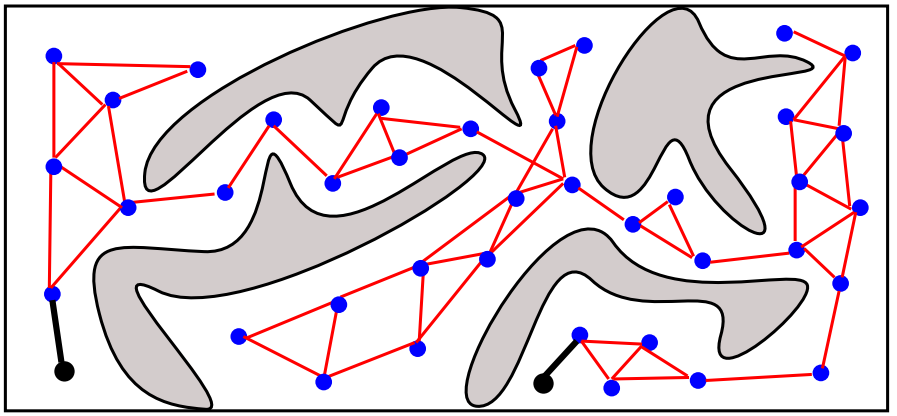

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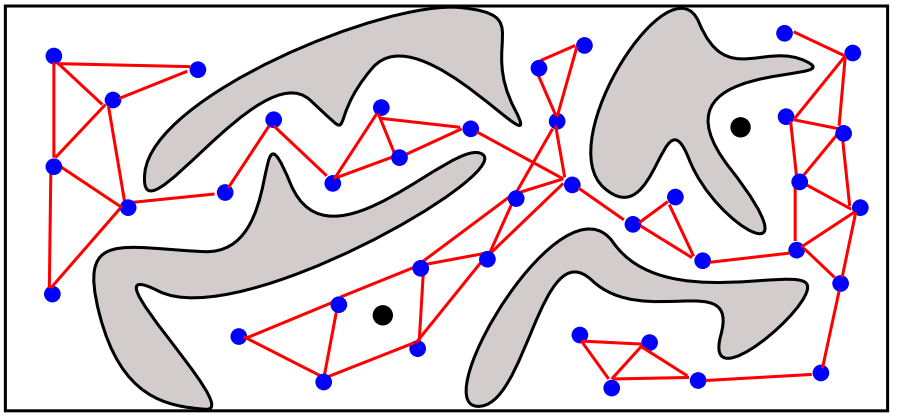

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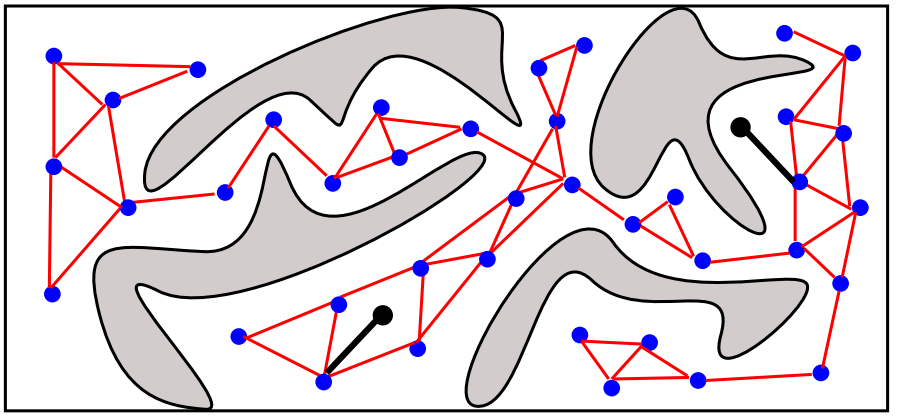

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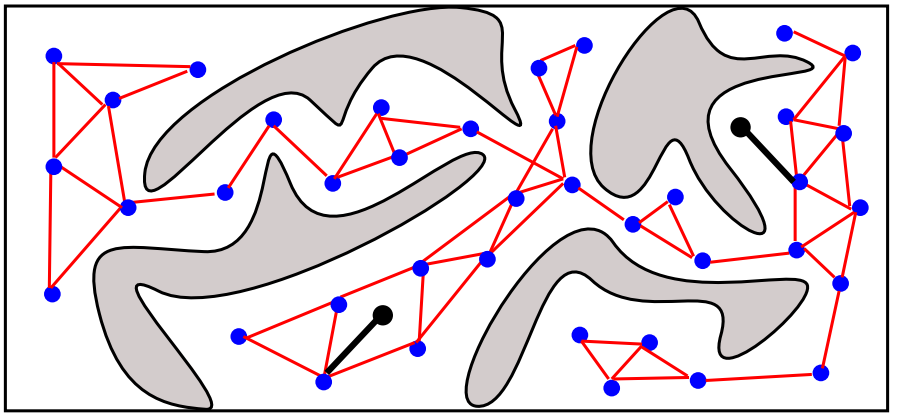
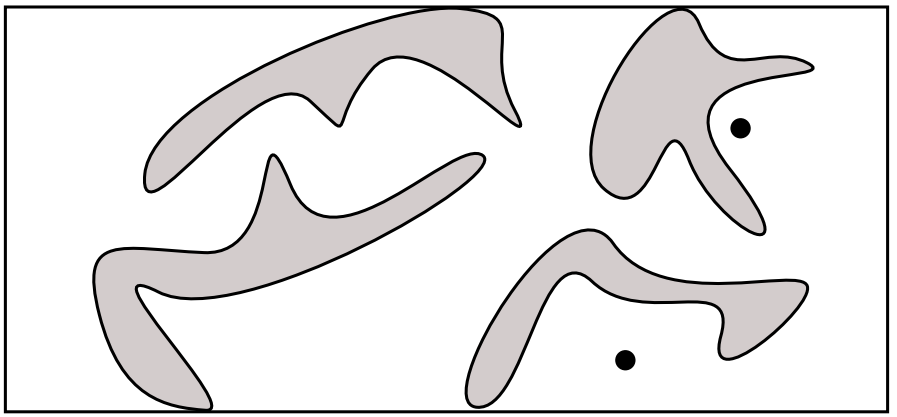
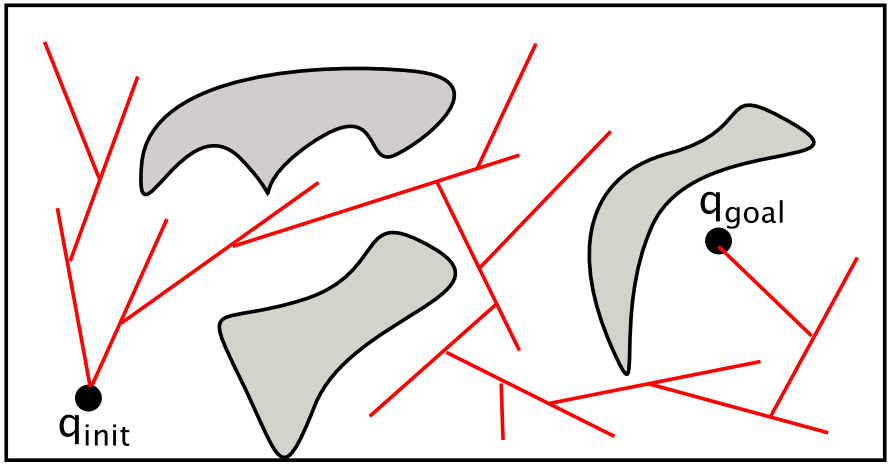


- Good when the objective is to solve *multiple* queries
- Maybe a bit too much when the objective is to solve a *single* query



General Idea

Grow a tree in the free configuration space from q_{init} toward q_{goal}



TREESEARCHFRAMEWORK($q_{\text{init}}, q_{\text{goal}}$)
1: $\mathcal{T} \leftarrow$ ROOTTREE(q_{init})
2: **while** q_{goal} has not been reached **do**
3: $q \leftarrow$ SELECTCONFIGFROMTREE($\mathcal{T}$)
4: ADDTREEBRANCHFROMCONFIG($\mathcal{T}, q$)

Critical Issues

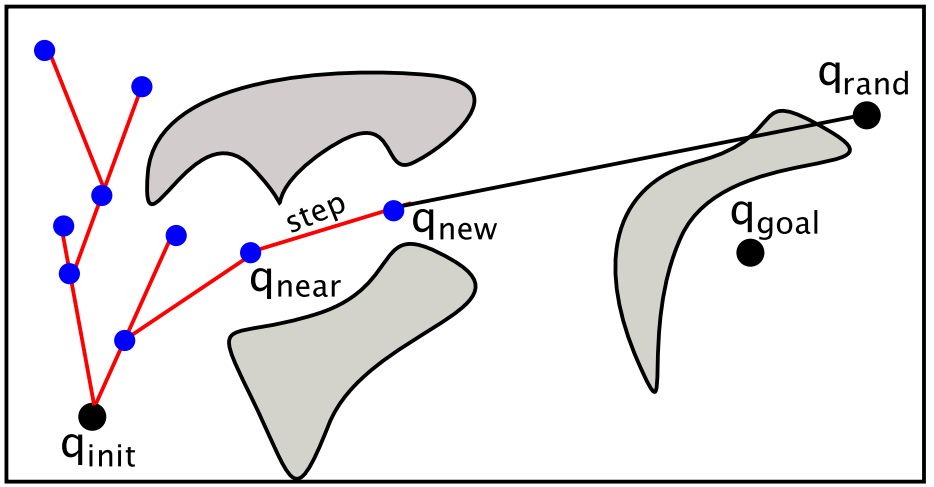
- How should a configuration be selected from the tree?
- How should a new branch be added to the tree from the selected configuration?

Rapidly-exploring Random Tree (RRT)

Pull the tree toward random samples in the configuration space

- RRT relies on nearest neighbors and distance metric $\rho : Q \times Q \leftarrow \mathbb{R}^{\geq 0}$
- RRT adds Voronoi bias to tree growth

[LaValle, Kuffner: 1999]



$\mathrm{RRT}(q_{\mathrm{init}}, q_{\mathrm{goal}})$

▷ *initialize tree*
1: $\mathcal{T} \leftarrow$ create tree rooted at q_{init}

2: **while** solution not found **do**

▷ *select configuration from tree*
3: $q_{\mathrm{rand}} \leftarrow$ generate a random sample
4: $q_{\mathrm{near}} \leftarrow$ nearest configuration in $\mathcal{T}$ to q_{rand} according to distance ρ

▷ *add new branch to tree from selected configuration*
5: $\mathrm{path} \leftarrow$ generate path (not necessarily collision free) from q_{near} to q_{ra}
6: **if** $\textsc{IsSubpathCollisionFree}(\mathrm{path}, 0, \mathtt{step})$ **then**
7: $q_{\mathrm{new}} \leftarrow \mathrm{path}(\mathtt{step})$
8: add configuration q_{new} and edge $(q_{\mathrm{near}}, q_{\mathrm{new}})$ to $\mathcal{T}$

▷ *check if a solution is found*
9: **if** $\rho(q_{\mathrm{new}}, q_{\mathrm{goal}}) \approx 0$ **then**
10: **return** solution path from root to q_{new}

Rapidly-exploring Random Tree (RRT) (cont.)

Aspects for Improvement

Suggested Improvements in the Literature

Rapidly-exploring Random Tree (RRT) (cont.)

Aspects for Improvement

- BasicRRT does not take advantage of q_{goal}
- Tree is pulled towards random directions based on the uniform sampling of Q
- In particular, tree growth is not directed towards q_{goal}

Suggested Improvements in the Literature

Rapidly-exploring Random Tree (RRT) (cont.)

Aspects for Improvement

- BasicRRT does not take advantage of q_{goal}
- Tree is pulled towards random directions based on the uniform sampling of Q
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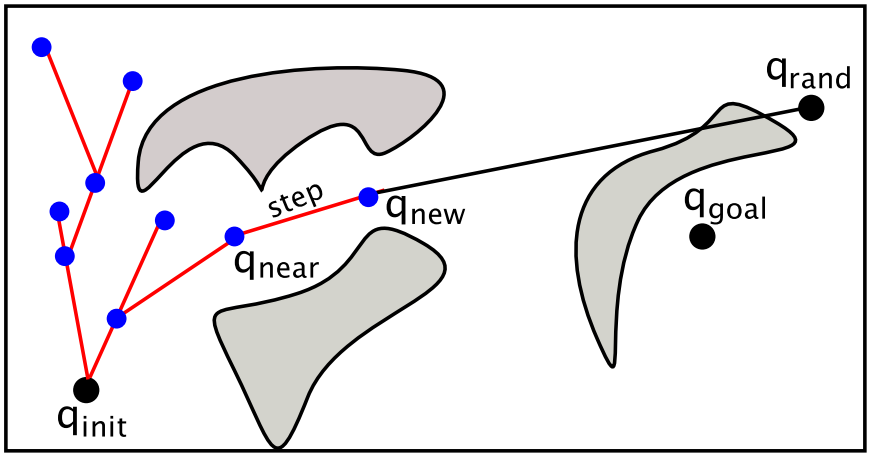
Suggested Improvements in the Literature

- Introduce goal-bias to tree growth (known as GoalBiasRRT)
  - q_{rand} is selected as q_{goal} with probability p
  - q_{rand} is selected based on uniform sampling of Q with probability $1 - p$
  - Probability p is commonly set to ≈ 0.05

Rapidly-exploring Random Tree (RRT) (cont.)

Aspects for Improvement

- BasicRRT takes only one small step when adding a new tree branch



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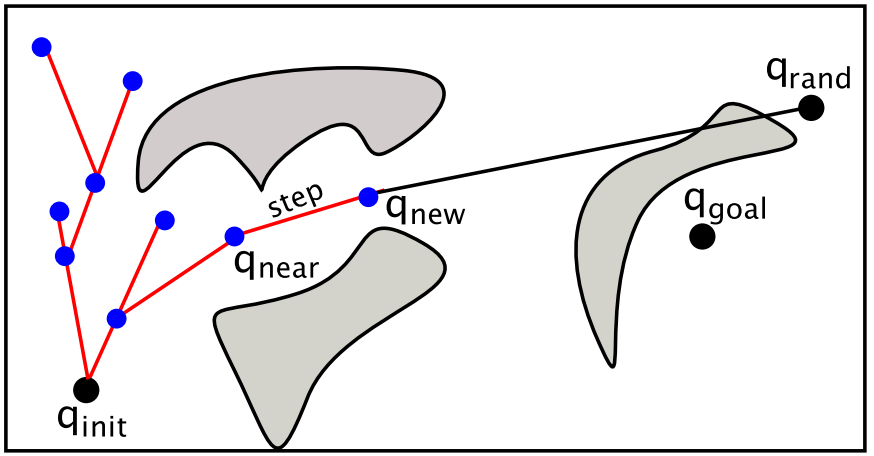

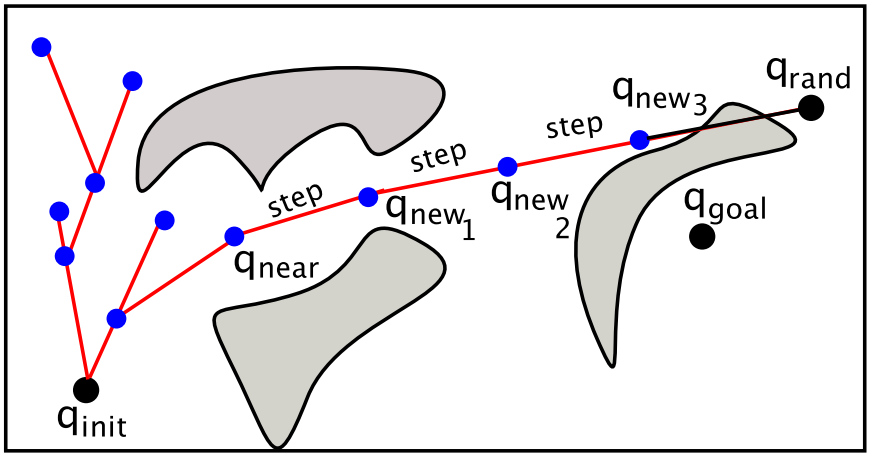

- This slows down tree growth

Suggested Improvements in the Literature

- Take several steps until q_{rand} is reached or a collision is found (ConnectRRT)
- Add all the intermediate nodes to the tree

Observations in High-Dimensional Problems

- Tree generally grows rapidly for the first few thousand iterations
- Tree growth afterwards slows down quite significantly
- Large number of configurations increases computational cost
- It becomes increasingly difficult to guide the tree towards previously unexplored parts of the free configuration space

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Possible improvements?

Bi-directional Trees

Grow two trees, rooted at q_{init} and q_{goal}, towards each other

- Bi-directional trees improve computational efficiency compared to a single tree
- Growth slows down significantly later than when using a single tree
- Fewer configurations in each tree, which imposes less of a computational burden
- Each tree explores a different part of the configuration space

BITREE($q_{\text{init}}, q_{\text{goal}}$)

1: $\mathcal{T}_{\text{init}} \leftarrow$ create tree rooted at q_{init}
2: $\mathcal{T}_{\text{goal}} \leftarrow$ create tree rooted at q_{goal}
3: **while** solution not found **do**
4: add new branch to $\mathcal{T}_{\text{init}}$
5: add new branch to $\mathcal{T}_{\text{goal}}$
6: attempt to connect neighboring configurations from the two trees
7: if successful, return path from q_{init} to q_{goal}

Advantages

- Explores small subset of possibilities by sampling
- Computationally efficient
- Solves high-dimensional problems (with hundreds of DOFs)
- Easy to implement
- Applications in many different areas

Disadvantages

- Does not guarantee completeness (a complete planner always finds a solution if there exists one, or reports that no solution exists)

Is it then just a heuristic approach? No. It's more than that

It offers *probabilistic completeness*

- When a solution exists, a probabilistically complete planner finds a solution with probability as time goes to infinity.
- When a solution does not exists, a probabilistically complete planner may not be able to determine that a solution does not exist.

Motion-Planning Problem for Systems with Differential Constraints

Given

- State space S
- Control space U
- Equations of motions as differential equations $f : S \times U \rightarrow \dot{S}$
- State-validity function $\textsc{valid} : S \rightarrow \{\texttt{true}, \texttt{false}\}$, e.g, check collisions
- Goal function $\textsc{goal} : S \rightarrow \{\texttt{true}, \texttt{false}\}$
- Initial state s_0

Compute a control trajectory $u : [0, T] \rightarrow U$ such that the resulting state trajectory $s : [0, T] \rightarrow S$ obtained by integration is valid and reaches the goal, i.e.,

$$s(t) = s_0 + \int_{h=0}^{h=t} f(s(t), u(t))dh \qquad (1)$$

$$\forall t \in [0, T] : \textsc{valid}(s(t)) = \texttt{true} \qquad (2)$$

$$\exists t \in [0, T] : \textsc{goal}(s(t)) = \texttt{true} \qquad (3)$$

Motion-Planning Methods for Systems with Differential Constraints

Decoupled approach

1. Compute a geometric solution path ignoring differential constraints
2. Transform the geometric path into a trajectory that satisfies the differential constraints

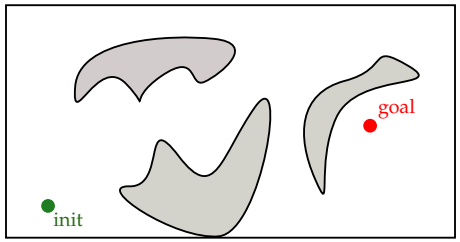
Sampling-based Motion Planning

- Take the differential constraints into account during motion planning

Roadmap Approaches

0. Initialization

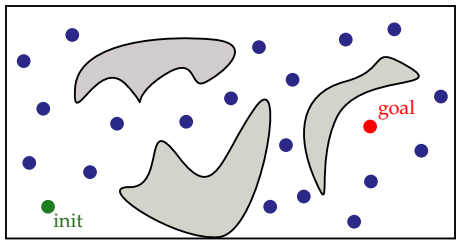
add s_{init} and s_{goal} to roadmap vertex set V



1. Sampling

repeat several times

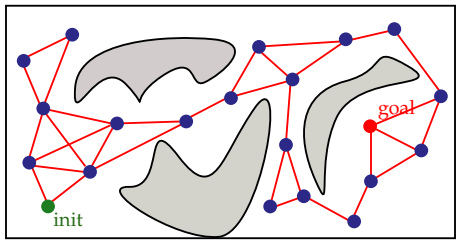
$s \leftarrow \textsc{StateSample}()$
if $\textsc{IsStateValid}(s) = \texttt{true}$
add s to roadmap vertex set V



2. Connect Samples

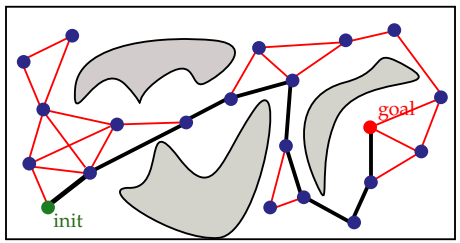
for each pair of neighboring samples $(s_a, s_b) \in V \times V$

$\lambda \leftarrow \textsc{GenerateLocalTrajectory}(s_a, s_b)$
if $\textsc{IsTrajectoryValid}(\lambda) = \texttt{true}$
add (s_a, s_b) to roadmap edge set E



3. Graph Search

search graph (V, E) for path from s_{init} to s_{goal}



Implementation of Roadmap Approaches with Differential Constraints

$s \leftarrow$ StateSample()

- generate random values for all the state components

IsStateValid(s)

- place the robot in the configuration specified by the position and orientation components of the state
- check if the robot collides with the obstacles
- check if velocity and other state components are within desired bounds

IsTrajectoryValid(s)

- use subdivision or incremental approach to check if intermediate states are valid

$\lambda \leftarrow$ GenerateLocalTrajectory(s_a, s_b)

- linear interpolation between s_a and s_b won't work as it does not respect underlying differential constraints
- need to find control function $u : [0, T] \rightarrow U$ such that trajectory obtained by applying u to s_a for T time units ends at s_b
- known as two-point boundary value problem
- cannot always be solved analytically, and numerical solutions increase computational cost

Tree Approaches with Differential Constraints

RRT

1: $\mathcal{T} \leftarrow$ create tree rooted at s_{init}
2: **while** solution not found **do**
▷ *select state from tree*
3: $s_{\text{rand}} \leftarrow \textsc{StateSample}()$
4: $s_{\text{near}} \leftarrow$ nearest configuration in $\mathcal{T}$ to q_{rand} according to distance ρ
▷ *add new branch to tree from selected configuration*
5: $\lambda \leftarrow \textsc{GenerateLocalTrajectory}(s_{\text{near}}, s_{\text{rand}})$
6: **if** $\textsc{IsSubTrajectoryValid}(\lambda, 0, \texttt{step})$ **then**
7: $s_{\text{new}} \leftarrow \lambda(\texttt{step})$
8: add configuration s_{new} and edge $(s_{\text{near}}, s_{\text{new}})$ to $\mathcal{T}$
▷ *check if a solution is found*
9: **if** $\rho(s_{\text{new}}, s_{\text{goal}}) \approx 0$ **then**
10: **return** solution trajectory from root to s_{new}

✓ $\textsc{StateSample}()$: random values for state components
✓ $\rho : S \times S \rightarrow \mathbb{R}^{\geq 0}$: distance metric between states
✓ $\textsc{IsSubTrajectoryValid}(\lambda, 0, \texttt{step})$: incremental approach

$\lambda \leftarrow \textsc{GenerateLocalTrajectory}(s_{\text{near}}, s_{\text{rand}})$

- will it not create the same two-boundary value problems as in PRM?
- is it necessary to connect to s_{rand}?
- would it suffice to just come close to s_{rand}?

Avoiding Two-Boundary Value Problem

Rather than computing a trajectory from s_{near} to s_{rand} compute a trajectory that starts at s_{near} and extends toward s_{rand}

Approach 1 – extend according to random control

- Sample random control u in U
- Integrate equations of motions when applying u to s_{near} for Δt units of time, i.e.,

$$\lambda \rightarrow s(t) = s_{\text{near}} + \int_{h=0}^{h=\Delta t} f(s(t), u) dh$$

Approach 2 – find the best-out-of-many random controls

1. for $i = 1, \ldots, m$ do
   1. $u_i \leftarrow$ sample random control in U
   2. $\lambda_i \rightarrow s(t) = s_{\text{near}} + \int_{h=0}^{h=\Delta t} f(s(t), u_i) dh$
   3. $d_i \leftarrow \rho(s_{\text{rand}}, \lambda_i(\Delta t))$
2. return λ_i with minimum d_i

Tree approaches require only the ability to simulate robot motions

- Physics engines can be used to simulate robot motions
- Physics engines provide greater simulation accuracy
- Physics engines can take into account friction, gravity, and interactions of the robot with objects in the evironment