6.034 Introduction to Artificial Intelligence

Tommi Jaakkola
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The world is drowning in data...
The world is drowning in data...

... access to information is based on recommendations
Recommending news feeds

- Lots of venues (and articles) ... challenging to find the few articles that you are actually interested in reading
Recommending news feeds

- Training examples and corresponding ratings

news articles

\[ x_1 \quad x_2 \quad x_3 \quad x_4 \quad \ldots \]

ingrating

\[ y_1 \quad y_2 \quad y_3 \quad y_4 \quad \ldots \]
Recommending news feeds

- Training examples and corresponding ratings

news articles

```
Romney Tells Evangelicals Their Values Are His, Too
By ASHLEY PARKER
Speaking at Liberty University, Mitt Romney sought to quell concerns among evangelical voters by offering a forceful defense of Christian values and faith in public life.
```

```
U.S. May Scrap Costly Efforts to Train Iraqi Police Force
By NICHOLAS THOMAS
The State Department could jettison a multimillion-dollar training effort by the end of 2013 that has struggled to deliver the latest high-profile example of America’s wasting influence in the country.
```

```
Candidate in Egypt Makes an Insider’s Run for President
By KAREEM FAHIM
Amr Moussa, a former Egyptian foreign minister who served under President Hosni Mubarak, is trying to make a strength from the liability of his long government career.
```

```
Member of Afghan Peace Council Is Assassinated
By ABDUL RAHEEM AND ABDUL MAJEED
An Afghan police official for the Afghan Peace Council, was shot dead by an unknown gunman in Kabul on Tuesday morning, a Kabul police official confirmed.
```

feature vectors

```
x_1
\phi(x_1)
```

```
x_2
\phi(x_2)
```

```
x_3
\phi(x_3)
```

```
x_4
\phi(x_4)
```

rating

```
y_1
+1
```

```
y_2
-1
```

```
y_3
+1
```

```
y_4
-1
```

```
\cdots
```

\cdots
Recommending news feeds

- Training examples and corresponding ratings

news articles

\[ x_1 \quad x_2 \quad x_3 \quad x_4 \quad \ldots \]

feature vectors

\[ \phi(x_1) \quad \phi(x_2) \quad \phi(x_3) \quad \phi(x_4) \quad \ldots \]

rating

\[ +1 \quad -1 \quad +1 \quad -1 \quad \ldots \]

\[ y_1 \quad y_2 \quad y_3 \quad y_4 \quad \ldots \]
Articles as feature vectors

• Does the word order matter?

White House officials consulted with the Justice Department in preparing a list of U.S. attorneys who would be removed.

(NYT 03/13/07)
Articles as feature vectors

• Does the word order matter?

White House officials consulted with the Justice Department in preparing a list of U.S. attorneys who would be removed.

(NYT 03/13/07)
Does the word order matter?

- Not for every task...

(Wolf et al. 2006)
White House officials consulted with the Justice Department in preparing a list of U.S. attorneys who would be removed.

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(NYT 03/13/07)
Articles as feature vectors

White House officials consulted with the Justice Department in preparing a list of U.S. attorneys who would be removed.

(NYT 03/13/07)

\( x \)
Recommending news feeds

• A few examples of articles that we’d like to read (+1)
• Potentially a large number of unwanted articles (-1)

\[
\phi(x) = \begin{bmatrix}
0 \\
1 \\
0 \\
0 \\
1 \\
\ldots
\end{bmatrix}
\begin{array}{c}
\text{politics} \\
\text{Justice} \\
\text{government} \\
\text{president} \\
\text{House} \\
\ldots
\end{array}
\]
Recommending news feeds

• A few examples of articles that we’d like to read (+1)
• Potentially a large number of unwanted articles (-1)

linear preferences \( y(x) = \theta \cdot \phi(x) + b \)

\[ \phi(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ \ldots \end{bmatrix} \]

\[ \theta \cdot \phi + b = 0 \]
Recommending news feeds

• Why is the problem challenging?
  - lots of possible words
  - only a small subset appears in any particular article
  - most frequent words are not content words
  - meaningful classes of articles are typically tied to words that occur relatively infrequently
  - any two articles in the same meaningful class may have only a few content words in common
Some tricks

• We can transform the counts in the feature vectors so as to emphasize more “relevant” words
• TFIDF weighting

\[
\phi_w(x) = \left( \frac{\text{freq. of word } w \text{ in doc. } x}{\text{TF}} \right) \cdot \log \left[ \frac{\# \text{ of docs}}{\# \text{ of docs with word } w} \right]
\]
Recommending news feeds

linear preferences \( y(x) = \theta \cdot \phi(x) + b \)

\[
\phi(x) = \begin{bmatrix}
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Recommending news feeds

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\[
\phi(x) = \begin{bmatrix}
0 \\
1 \\
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\end{bmatrix}
\]

\( J(\theta, b) = \sum_{t=1}^{n} (y_t - \theta \cdot \phi(x_t) - b)^2 \)

sum over the training examples

error on each example
Linear regression, complexity

- We can easily obtain (too) complex regression functions by considering different feature mappings.

\[ \phi(x) = [1, x, x^2, x^3]^T \]
Recommending news feeds

linear preferences  \( y(x) = \theta \cdot \phi(x) + b \)

\( \phi(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ \ldots \end{bmatrix} \)

\( J(\theta, b) = \sum_{t=1}^{n} (y_t - \theta \cdot \phi(x_t) - b)^2 \)

sum over the training examples

squared prediction error on each example
Recommending news feeds

Linear preferences

\[ y(x) = \theta \cdot \phi(x) + b \]

\[ \phi(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ \ldots \end{bmatrix} \]

\[ J(\theta, b) = \sum_{t=1}^{n} (y_t - \theta \cdot \phi(x_t) - b)^2 + \lambda \|\theta\|^2 \]

- Sum over the training examples
- Squared prediction error on each example
- Regularization term
Recommending news feeds

linear preferences $y(x) = \theta \cdot \phi(x) + b$

$$\phi(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ \ldots \end{bmatrix}$$

$$\begin{array}{l} \text{politics} \\ \text{Justice} \\ \text{government} \\ \text{president} \\ \text{House} \\ \ldots \end{array}$$

$$J(\theta, b) = \sum_{t=1}^{n} (y_t - \theta \cdot \phi(x_t) - b)^2 + \lambda \|\theta\|^2$$

- sum over the training examples
- squared prediction error on each example
- regularization term
Today’s topics

• Preface: regression for recommendation problems

• Collaborative filtering
  - setup, regression formulation
  - matrix factorization
## Collaborative filtering

- Consider the problem of predicting how \( n \) users rate \( m \) movies.
- Known ratings (training data) are arranged in a partially filled \( nxm \) data matrix.
- The goal is to predict the remaining entries.

<table>
<thead>
<tr>
<th>n users</th>
<th>m movies</th>
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Collaborative filtering

- Consider the problem of predicting how $n$ users rate $m$ movies.
- Known ratings (training data) are arranged in a partially filled $n \times m$ data matrix.
- The goal is to predict the remaining entries.
- Basic intuition: similar users can complete each others experience.

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### n users

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### m movies

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Collaborative filtering

- Consider the problem of predicting how n users rate m movies
- Known ratings (training data) are arranged in a partially filled nxm data matrix
- The goal is to predict the remaining entries
- Basic intuition: similar users can complete each others experience
- Key part of the problem is to couple the estimation tasks across users / movies
Collaborative filtering

• Our goal is to fill the data matrix, i.e., accurately predict values for unobserved entries

• Computational issues:
  - a typical matrix is very large, e.g., n=400K, m=17K

• Statistical issues:
  - the matrix is very sparse, e.g., 1% known ratings
  - ratings may be diverse and under-sampled (?)

• Formulation issues:
  - many interpretations for missing entries
Single user predictions

- We could try to solve the problem separately for each user using simple linear regression models for ratings.

\[ J_i(\theta_i) = \sum_{j \in M_i} (Y_{ij} - \theta_i \cdot \phi_j)^2 + \lambda \|\theta_i\|^2 \]

\[
\begin{array}{cccc}
\phi_1 & \cdots & \phi_j & \phi_m \\
5 & 4 & 5 & 1 & 4 \\
\vdots & & & & \\
\end{array}
\]

- m movies
- User i
- Known entries for user i
- Rating matrix
- Parameters
- Feature vector for movie j
- \( \text{dim}(\theta_i) = \text{dim}(\phi_j) = d \)
Single user predictions

- We could try to solve the problem separately for each user using simple linear regression models for ratings.

\[
\begin{array}{cccc}
\Phi_1 & \cdots & \Phi_j & \Phi_m \\
5 & 4 & 5 & 1 & 4 \\
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\[
dim(\theta_i) = dim(\phi_j) = d
\]

\[
J_i(\theta_i) = \sum_{j \in M_i} (Y_{ij} - \theta_i \cdot \phi_j)^2 + \lambda \|\theta_i\|^2
\]

- But
  - reasonable feature vectors may be hard to obtain
  - each user may have only a few ratings
  - no help from similar users
Matrix factorization

- We can approximate the rating matrix as a product of two lower rank matrices.

\[ Y_{ij} \approx [UV^T]_{ij} \]
• We can approximate the rating matrix as a product of two lower rank matrices

$$\min_{U,V} \sum_{ij \in D} (Y_{ij} - [UV^T]_{ij})^2.$$
Matrix factorization

- We can approximate the rating matrix as a product of two lower rank matrices.

\[ Y_{ij} \approx [UV^T]_{ij} \]

\[ \min_{U, V} \sum_{i,j \in D} (Y_{ij} - [UV^T]_{ij})^2. \]

The only complexity control would be the rank \( d \).
Matrix factorization

- We can approximate the rating matrix as a product of two lower rank matrices

\[ Y_{ij} \approx [UV^T]_{ij} \]

\[
\begin{array}{cccccc}
5 & 5 & & & & 5 \\
3 & 5 & 1 & 3 & 4 & 4 & 4 \\
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4 & & & & & 4 \\
5 & & 4 & & & 4 \\
5 & & & 5 & & 3 \\
\end{array}
\]

\[
\min_{U,V} \sum_{i,j \in D} (Y_{ij} - [UV^T]_{ij})^2 + \lambda \|U\|_F^2 + \lambda \|V\|_F^2
\]
Matrix factorization

• The matrix factorization approach can be interpreted as iteratively solving regression problems for users/movies

\[ Y_{ij} \approx \hat{\theta}_i \cdot \hat{\phi}_j \]
Matrix factorization

- The matrix factorization approach can be interpreted as iteratively solving regression problems for users/movies.

\[ Y_{ij} \approx \theta_i \cdot \phi_j \]
Matrix factorization

- The matrix factorization approach can be interpreted as iteratively solving regression problems for users/movies.

\[ Y_{ij} \approx \theta_i \cdot \phi_j \]

\[ J_i(\theta_i) = \sum_{j:ij \in D} (Y_{ij} - \theta_i \cdot \phi_j)^2 + \lambda \|\theta_i\|^2 \]

\[ U \times V^T \approx \begin{bmatrix} \theta_{1}^T \\ \vdots \\ \theta_{n}^T \end{bmatrix} \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_m \end{bmatrix} \]

regression problem for each user with fixed movie features
The matrix factorization approach can be interpreted as iteratively solving regression problems for users/movies.

\[ Y_{ij} \approx \theta_i \cdot \phi_j \]

\[ J_j(\phi_j) = \sum_{i:i \in D} (Y_{ij} - \theta_i \cdot \phi_j)^2 + \lambda \| \phi_j \|^2 \]

regression problem for each movie with fixed user features
Matrix factorization cont’d

- We can approximate the rating matrix as a product of two lower rank matrices

\[
Y_{ij} \approx [UV^T]_{ij}
\]

\[
\min_{U,V} \sum_{i,j \in D} (Y_{ij} - [UV^T]_{ij})^T + \lambda \|U\|_F^2 + \lambda \|V\|_F^2
\]

observed entries
CF and the Netflix Price

• Progress using different matrix factorization methods

(Koren et al., 2009)

• (to win the price, one had to combine hundreds of different methods)
Matrix factorization

• We try to find the best rank d approximation to the rating matrix based on the observed entries

\[
\text{minimize} \quad \frac{1}{2} \sum_{i,j \in D} (Y_{ij} - [UV^T]_{ij})^2 + \frac{\lambda}{2} \|U\|_F^2 + \frac{\lambda}{2} \|V\|_F^2
\]

where \( U \) is \( n \times d \) and \( V \) is \( m \times d \)

- rank d can be used for complexity control along with the regularization parameter lambda
- the optimization problem is not jointly convex in \( U \) and \( V \). However, it is convex in \( U \) if we fix \( V \), and vice versa
- an alternating minimization algorithm, i.e., iteratively solving user / movie regression problems, may get stuck in a locally optimal solution (initialization is important)
- algorithms that sequentially add simple rank-1 components at a time are typically better.