1 FOL Proof - Rags ∨ Riches?

(a) Unification

Find a most general unifier for each of the following pairs of sentences. Bob and Alice are constants.

<table>
<thead>
<tr>
<th>$S_1$</th>
<th>$S_2$</th>
<th>Unifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H(Bob)$</td>
<td>$H(x)$</td>
<td>$x$/$Bob$</td>
</tr>
<tr>
<td>$Eq(f(Bob), Alice)$</td>
<td>$Eq(x, y)$</td>
<td>$x$/$f(Bob)$, $y$/$Alice$</td>
</tr>
<tr>
<td>$P(f(x), Bob)$</td>
<td>$P(Alice, x)$</td>
<td>none</td>
</tr>
<tr>
<td>$Eq(f(f(Bob)), f(Bob))$</td>
<td>$Eq(f(x), x)$</td>
<td>$x$/$f(Bob)$</td>
</tr>
<tr>
<td>$P(y, Bob)$</td>
<td>$P(f(Bob), x)$</td>
<td>$x$/$Bob$, $y$/$f(Bob)$</td>
</tr>
</tbody>
</table>

(b) Proof

Say we have the following predicates:

- $H(x)$ x is an heir.
- $M(x)$ x is male.
- $P(y, x)$ y is in the Trump family and is the parent of x.
- $Eq(x, y)$ x and y are equal.
Convert the following English sentence into first-order logic clausal form.

1. A person is an heir if and only if he or she is male, and has a parent from the Trump family who is an heir.

Solution:

\[ \forall x. H(x) \leftrightarrow M(x) \land (\exists y. P(y, x) \land H(y)) \]

\[ \Rightarrow (\neg H(x_1) \lor M(x_1)) \land (\neg H(x_2) \lor P(f(x_2), x_2)) \land (\neg H(x_3) \lor H(f(x_3))) \land (\neg M(x_4) \lor \neg P(y_4, x_4) \lor \neg H(y_4) \lor H(x_4)) \]

2. A person only has one parent from the Trump family.

Solution:

\[ \forall x. \forall y. \forall z. (p(y, x) \land P(z, x)) \rightarrow Eq(y, z) \]

\[ \Rightarrow \neg P(y_1, x_1) \lor \neg P(z_1, x_1) \lor Eq(y_1, z_1) \]

3. If two people are Eq and one of them is an heir, the other is an heir. (That is, allow substitutions for predicate H)

Solution:

\[ \forall x. \forall y. (H(x) \land Eq(x, y)) \rightarrow H(y) \]

\[ \Rightarrow \neg H(x_1) \lor \neg Eq(x_1, y_1) \lor H(y_1) \]
4. Eq is symmetric.

Solution:

\[ \forall x. \forall y. Eq(x, y) \rightarrow Eq(y, x) \]

\[ \Rightarrow \neg Eq(x_1, y_1) \lor Eq(y_1, x_1) \]

On the next pages are a framework for conducting two proofs about Bob and Alice. A few specifics about them have been stated along with axioms from the given knowledge base. All axioms that we converted to clausal form have already been stated. You need only provide the proofs. Be sure to note any unification substitutions you make.
Prove that Alice is not an heir.

<table>
<thead>
<tr>
<th>step</th>
<th>reason</th>
<th>result</th>
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</thead>
<tbody>
<tr>
<td>1.</td>
<td>KB 4</td>
<td>$\neg Eq(x_1, y_1) \lor Eq(y_1, x_1)$</td>
</tr>
<tr>
<td>2.</td>
<td>KB 1</td>
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</tr>
<tr>
<td>3.</td>
<td>KB 1</td>
<td>$\neg H(x_3) \lor H(f(x_3))$</td>
</tr>
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<td>4.</td>
<td>KB 1</td>
<td>$\neg H(x_4) \lor P(f(x_4), x_4)$</td>
</tr>
<tr>
<td>5.</td>
<td>KB 1</td>
<td>$H(x_5) \lor \neg M(x_5) \lor \neg P(y_5, x_5) \lor \neg H(y_5)$</td>
</tr>
<tr>
<td>6.</td>
<td>KB 2</td>
<td>$\neg P(y_6, x_6) \lor \neg P(z_6, x_6) \lor Eq(y_6, z_6)$</td>
</tr>
<tr>
<td>7.</td>
<td>KB 3</td>
<td>$\neg H(x_7) \lor \neg Eq(x_7, y_7) \lor H(y_7)$</td>
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</tr>
<tr>
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<td>$\neg M(Alice)$</td>
</tr>
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<td>10.</td>
<td>neg. concl.</td>
<td>$H(Alice)$</td>
</tr>
<tr>
<td>11.</td>
<td>2,10 ${x_2/Alice}$</td>
<td>$M(Alice)$</td>
</tr>
<tr>
<td>12.</td>
<td>9,11</td>
<td>false</td>
</tr>
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Assume that Alice is not an heir. Prove that Bob is not an heir.

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<td>(\neg H(x_4) \lor P(f(x_4), x_4))</td>
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<td>(H(x_5) \lor \neg M(x_5) \lor \neg P(y_5, x_5) \lor \neg H(y_5))</td>
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<td>10,4 ({x_4/Bob})</td>
<td>(P(f(Bob), Bob))</td>
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<tr>
<td>12.</td>
<td>11,6 ({x_6/Bob, y_6/f(Bob)})</td>
<td>(\neg P(z_6, Bob) \lor Eq(f(Bob), z_6))</td>
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<tr>
<td>13.</td>
<td>12,8 ({z_6/Alice})</td>
<td>(Eq(f(Bob), Alice))</td>
</tr>
<tr>
<td>14.</td>
<td>13,7 ({x_7/f(Bob), y_7/Alice})</td>
<td>(\neg H(f(Bob)) \lor H(Alice))</td>
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<tr>
<td>15.</td>
<td>14,9</td>
<td>(\neg H(f(Bob)))</td>
</tr>
<tr>
<td>16.</td>
<td>15,3 ({x_3/Bob})</td>
<td>(\neg H(Bob))</td>
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<td>17.</td>
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